

MATHS



MODEL ANSWERS

THREE INDEPENDENT EVENTS

SCORE

/133

PERCENTAGE

%

STEMUP

- 1 Bag A contains 5 red counters and 1 blue counter. Bag B contains 5 red counters and 5 blue counters.

Bag C contains 1 red counter and 4 blue counters.

One counter is taken at random from each bag.

Work out the probability that all three counters are red.

$$\frac{5}{6} \times \frac{1}{2} \times \frac{1}{5} = \frac{1}{12}$$

$$\frac{1}{12}$$

/ 1

- 2 Three fair six-sided dice are rolled at the same time.

Work out the probability that none of the dice shows a prime number.

$$P(\text{not}) = \frac{1}{2}$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$\frac{1}{8}$$

/ 1

- 3 A bag contains 2 black counters and 6 white counters.

A counter is taken at random, its colour is noted and it is put back. This is done three times.

Work out the probability that none of the three counters is black.

$$P(\text{not}) = \frac{3}{4}$$

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

$$\frac{27}{64}$$

/ 1

- 4 A fair coin is thrown, a fair six-sided dice is rolled and a fair spinner with 4 equal sections numbered 1 to 4 is spun.

Work out the probability of getting a head, a 1 or a 2 and the number 4.

$$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$$

$$\frac{1}{24}$$

/ 1

- 5 Bethan rolls a fair six-sided dice three times.

Work out the probability that Bethan gets a 1 or a 2 at least once.

$$P(\text{none}) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

$$1 - \frac{8}{27} = \frac{19}{27}$$

$$\frac{19}{27}$$

/ 1

- 6 In Wrexham, the probability of frost on any day in winter is $\frac{1}{3}$.

Whether it frosts on one day does not depend on the other days.

Work out the probability that there is a frost on at least one of the three days, Friday, Saturday and Sunday.

$$P(\text{none}) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

$$1 - \frac{8}{27} = \frac{19}{27}$$

$$\frac{19}{27}$$

/ 1

- 7 A spinner has three colours, red, white and green.
 The probability of landing on red is 0.4 and the probability of landing on white is 0.2.
 The spinner is spun three times. Each spin is independent of the others.
 Work out the probability that the spinner lands on red at least once.

$$P(\text{none}) = 0.6 \times 0.6 \times 0.6 = 0.216$$

$$1 - 0.216 = 0.784$$

0.784

/ 1

- 8 A fair coin is thrown three times.
 Calculate the probability of getting exactly two heads.

*HHT, HTH, THH are 3 ways
 each $\frac{1}{8}$*

$$3 \times \frac{1}{8} = \frac{3}{8}$$

$\frac{3}{8}$

/ 1

- 9 In Aberystwyth, the probability of frost on any day in winter is $\frac{1}{5}$.
 Whether it frosts on one day does not depend on the other days.
 Work out the probability that there is a frost on exactly one of the three days, Thursday, Friday and Saturday.

3 ways

$$3 \times \frac{1}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{48}{125}$$

$\frac{48}{125}$

/ 1

- 10** A factory uses three machines, A, B and C. They work independently of each other. On any day, the probability that A works is 0.9, the probability that B works is 0.85 and the probability that C works is 0.6.

Calculate the probability that at least one of the machines works on a day.

$$P(\text{none}) = 0.1 \times 0.15 \times 0.4 = 0.006$$

$$1 - 0.006 = 0.994$$

0.994

/ 1

- 11** Bag A contains 7 red counters and 1 blue counter. Bag B contains 3 red counters and 1 blue counter.
Bag C contains 3 red counters and 5 blue counters.
One counter is taken at random from each bag.
Work out the probability that at least one of the three counters is red.

$$P(\text{none}) = \frac{1}{8} \times \frac{1}{4} \times \frac{5}{8} = \frac{5}{256}$$

$$1 - \frac{5}{256} = \frac{251}{256}$$

$\frac{251}{256}$

/ 1

- 12** In Llandudno, the probability of rain on any day in winter is 0.2. Whether it rains on one day does not depend on the other days.
Work out the probability that it rains on exactly one of the three days, Thursday, Friday and Saturday.

3 ways

$$3 \times 0.2 \times 0.8 \times 0.8 = 0.384$$

0.384

/ 1

- 13 A fair coin is thrown, a fair six-sided dice is rolled and a fair spinner with 4 equal sections numbered 1 to 4 is spun.

Work out the probability of getting a head, not a 1 or a 2 and the number 1.

$$P(\text{not a 1 or a 2}) = \frac{2}{3}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\frac{1}{12}$$

1 / 1

- 14 A spinner has three colours, blue, yellow and white.

The probability of landing on blue is 0.4 and the probability of landing on yellow is 0.4.

The spinner is spun three times. Each spin is independent of the others.

Work out the probability that the spinner lands on blue exactly once.

3 ways

$$3 \times 0.4 \times 0.6 \times 0.6 = 0.432$$

$$0.432$$

2 / 2

- 15 Gwen takes three penalty kicks. The probability that she scores with each kick is 85%, whatever happens with the other kicks.

Work out the probability that Gwen scores with at least one of the kicks.

$$P(\text{none}) = 15\% \times 15\% \times 15\% = 0.3375\%$$

$$100\% - 0.3375\% = 99.6625\%$$

$$99.6625\%$$

2 / 2

- 16** A fair coin is thrown, a fair six-sided dice is rolled and a fair spinner with 4 equal sections numbered 1 to 4 is spun.

Work out the probability of getting a tail, not an even number and the number 1.

$$P(\text{not an even number}) = \frac{1}{2}$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{16}$$

$$\frac{1}{16}$$

2 / 2

- 17** A spinner has three colours, red, green and white.
The probability of landing on red is 0.2 and the probability of landing on green is 0.3.
The spinner is spun three times. Each spin is independent of the others.
Work out the probability that the spinner lands on red exactly twice.

3 ways

$$3 \times 0.2 \times 0.2 \times 0.8 = 0.096$$

$$0.096$$

2 / 2

- 18** An archer fires three arrows at a target. The probability that each arrow hits the target is 0.85, independent of the other arrows.

Work out the probability that at least one arrow hits the target.

$$P(\text{none}) = 0.15 \times 0.15 \times 0.15 = 0.003375$$

$$1 - 0.003375 = 0.996625$$

$$0.996625$$

2 / 2

- 19 A factory uses three pumps, P, Q and R. They work independently of each other. On any day, the probability that P works is 0.85, the probability that Q works is 0.8 and the probability that R works is 0.95.

Calculate the probability that exactly one of the pumps works on a day.

$$0.85 \times 0.2 \times 0.05 + 0.15 \times 0.8 \times 0.05 + 0.15 \times 0.2 \times 0.95$$

$$\text{total} = 0.043$$

0.043

2 / 2

- 20 Each situation below involves finding the probability of more than one event happening.
- A Two sweets are taken, one after the other, from a bag and eaten. Find the probability that both are green.
- B A counter is taken from a bag and put back. A second counter is then taken. Find the probability that both are red.
- C Three spinners are each spun once. Find the probability that all three land on 4.
- In which of A, B and C can you find the answer by multiplying the probabilities of independent events?

A: not independent (the first result changes the second probability)

B: independent (the first result does not change the second probability)

C: independent (the first result does not change the second probability)

B, C

2 / 2

- 21 Gwen takes three penalty kicks. The probability that she scores with each kick is 0.8, whatever happens with the other kicks.

(a) Work out the probability that Gwen scores with all three kicks.

(b) Work out the probability that Gwen misses with at least one of the kicks.

$$(a) 0.8 \times 0.8 \times 0.8 = 0.512$$

$$(b) P(\text{all three}) = 0.8 \times 0.8 \times 0.8 = 0.512$$

$$1 - 0.512 = 0.488$$

(a) 0.512 (b) 0.488

3 / 3

- 22 A spinner lands on red with probability 0.2. It is spun three times. This is done 250 times.
How many times would you expect the spinner to land on red every time?

$$0.2 \times 0.2 \times 0.2 = 0.008 = \frac{1}{125}$$

$$\text{expected number} = \frac{1}{125} \times 250 = 2$$

2

3 / 3

- 23 Each situation below involves finding the probability of more than one event happening.
- A Two cards are taken, one after the other, from a pack and not replaced. Find the probability that both are hearts.
- B Three spinners are each spun once. Find the probability that all three land on 4.
- C It is more likely to rain on Tuesday if it rained on Monday. Find the probability that it rains on both Monday and Tuesday.
- In which of A, B and C can you find the answer by multiplying the probabilities of independent events?

A: not independent (the first result changes the second probability)

B: independent (the first result does not change the second probability)

C: not independent (the first result changes the second probability)

B

2 / 2

- 24 The probability that a train is on time is 0.2. Rhian catches the train on three days.
- (a) List all the different ways in which the train is on time on exactly one of the three days, using O for on time and L for late.
- (b) Work out the probability that the train is on time on exactly one of the three days.

(b) 3 ways

$$3 \times 0.2 \times 0.8 \times 0.8 = 0.384$$

(a) OLL, LOL, LLO (b) 0.384

3 / 3

25 A fair coin is thrown three times. This is done 800 times.

(a) Work out the probability of getting three heads in one set of three throws.

(b) How many times would you expect to get three heads in the 800 sets?

$$(a) \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$(b) \text{ expected number} = \text{probability} \times \text{number of sets} = \frac{1}{8} \times 800 = 100$$

$$(a) \frac{1}{8} \quad (b) 100$$

3 / 3

26 A fair six-sided dice is rolled three times.

(a) Work out the probability that the dice shows a six each time.

(b) Work out the probability that the dice never shows a six.

$$(a) \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$(b) P(\text{not}) = \frac{5}{6}$$

$$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$$(a) \frac{1}{216} \quad (b) \frac{125}{216}$$

3 / 3

27 A biased coin lands on heads with probability $\frac{1}{3}$. It is thrown three times.

(a) List all the different ways of getting exactly one head in the three throws, using H for heads and T for tails.

(b) Work out the probability of getting exactly one head.

(b) 3 ways

$$3 \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

$$(a) HTT, THT, TTH \quad (b) \frac{4}{9}$$

3 / 3

28 A fair six-sided dice is rolled three times. This is done 400 times.

(a) Work out the probability of getting an even number every time in one set of three rolls.

(b) How many times would you expect to get an even number every time in the 400 sets?

(a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(b) $\text{expected number} = \text{probability} \times \text{number of sets}$
 $= \frac{1}{8} \times 400 = 50$

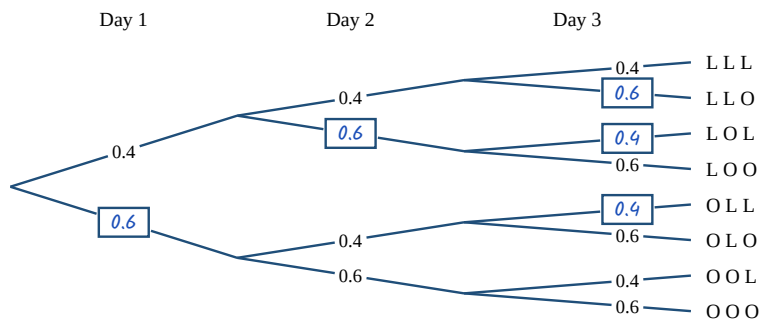
(a) $\frac{1}{8}$ (b) 50

3 / 3

29 The probability that Dan's bus is late is 0.4 on each of three days.

(a) Complete the tree diagram. The upper branch of each pair is for "late" (L).

(b) Work out the probability that the bus is late on at least one day.



(a) the two branches from each point add up to 1, so 0.4 and 0.6
 (b) $P(\text{none}) = 0.6 \times 0.6 \times 0.6 = 0.216$
 $1 - 0.216 = 0.784$

(b) 0.784

3 / 3

30 The probability that it rains on any day is 0.4. Whether it rains on one day does not depend on the other days.

Sion says, "The probability that it rains on at least one of the next three days is $0.4 + 0.4 + 0.4 = 1.2$."

Is Sion correct? You must show your working.

$P(\text{no rain on a day}) = 0.6$

$P(\text{no rain on all three days}) = 0.6 \times 0.6 \times 0.6 = 0.216$

$P(\text{at least one rainy day}) = 1 - 0.216 = 0.784$
 this is not 1.2

No. The probability is 0.784, not 1.2

3 / 3

- 31 In Bangor, the probability of rain on any day in winter is $\frac{3}{10}$.

Whether it rains on one day does not depend on the other days.

(a) Work out the probability that it does not rain on any of the three days, Friday, Saturday and Sunday.

(b) Work out the probability that at least one of the three days, Friday, Saturday and Sunday, is dry.

$$(a) P(\text{not}) = \frac{7}{10}$$

$$\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{343}{1000}$$

$$(b) P(\text{all three}) = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{27}{1000}$$

$$1 - \frac{27}{1000} = \frac{973}{1000}$$

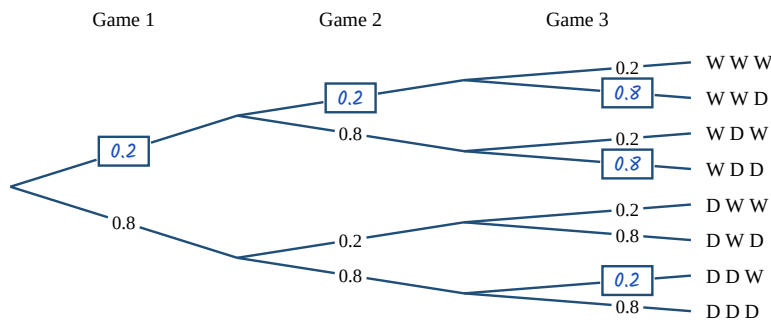
$$(a) \frac{343}{1000} \quad (b) \frac{973}{1000}$$

3 / 3

- 32 The probability that Sara wins a game is 0.2. She plays three games.

(a) Complete the tree diagram. The upper branch of each pair is for "wins" (W).

(b) Work out the probability that Sara wins all three games.



(a) the two branches from each point add up to 1, so 0.2 and 0.8

(b) $0.2 \times 0.2 \times 0.2 = 0.008$

(b) 0.008

3 / 3

- 33 Rhian says, "A fair dice is rolled three times. The probability of getting at least one six is $\frac{1}{6} +$

$$\frac{1}{6} + \frac{1}{6} = \frac{1}{2}."$$

Is Rhian correct? You must show your working.

$$P(\text{no six on a roll}) = \frac{5}{6}$$

$$P(\text{no sixes}) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$$P(\text{at least one six}) = 1 - \frac{125}{216} = \frac{91}{216}$$

this is not $\frac{1}{2}$

No. The probability is $\frac{91}{216}$, not $\frac{1}{2}$

3 / 3

34 A fair six-sided dice is rolled three times.

(a) Work out the probability that the dice shows a number less than 3 at least once.

(b) Work out the probability that the dice never shows a number less than 3.

$$(a) P(\text{none}) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

$$1 - \frac{8}{27} = \frac{19}{27}$$

$$(b) P(\text{not}) = \frac{2}{3}$$

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

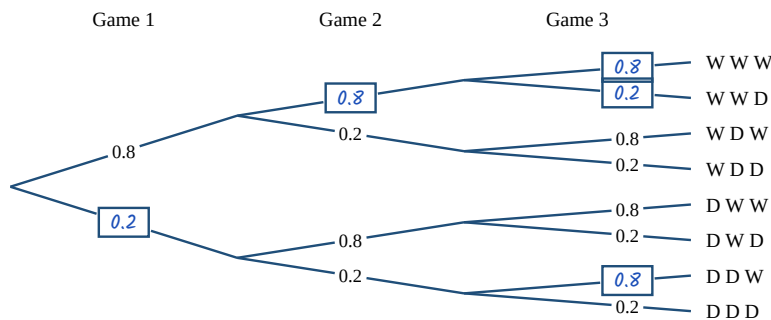
$$(a) \frac{19}{27} \quad (b) \frac{8}{27}$$

4 / 4

35 The probability that Sara wins a game is 0.8. She plays three games.

(a) Complete the tree diagram. The upper branch of each pair is for "wins" (W).

(b) Work out the probability that Sara wins at least one of the games.



(a) the two branches from each point add up to 1, so 0.8 and 0.2
 (b) $P(\text{none}) = 0.2 \times 0.2 \times 0.2 = 0.008$
 $1 - 0.008 = 0.992$

(b) 0.992

3 / 3

36 Dylan says, "A fair coin is thrown three times. The probability of getting exactly one head is $\frac{1}{2}$

$$\times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}."$$

Is Dylan correct? Give a reason for your answer.

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \text{ is for one particular order}$$

exactly one head can happen in 3 ways: HTT,

THT, TTH

$$3 \times \frac{1}{8} = \frac{3}{8}$$

No. There are 3 ways, so the probability is $\frac{3}{8}$

3 / 3

37 Carys throws a fair coin three times.

(a) Work out the probability of getting a tail, then a tail, then a head, in that order.

(b) Work out the probability of getting at least one head.

$$(a) \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$(b) P(\text{no heads}) = \frac{1}{8}$$

$$1 - \frac{1}{8} = \frac{7}{8}$$

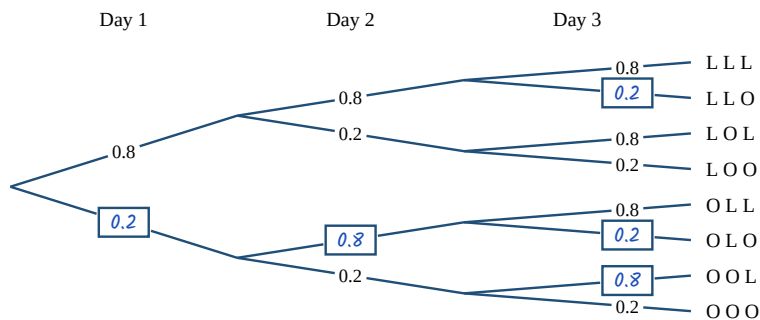
$$(a) \frac{1}{8} \quad (b) \frac{7}{8}$$

4 / 4

38 The probability that Dan's bus is late is 0.8 on each of three days.

(a) Complete the tree diagram. The upper branch of each pair is for "late" (L).

(b) Work out the probability that the bus is on time on all three days.



(a) the two branches from each point add up to 1, so 0.8 and 0.2

$$(b) P(\text{not}) = 0.2 \\ 0.2 \times 0.2 \times 0.2 = 0.008$$

(b) 0.008

3 / 3

39 The probability that it rains on any day is 0.4. Whether it rains on one day does not depend on the other days.

Iwan says, "The probability that it rains on at least one of the next three days is $0.4 + 0.4 + 0.4 = 1.2$."

Is Iwan correct? You must show your working.

$$P(\text{no rain on a day}) = 0.6$$

$$P(\text{no rain on all three days}) = 0.6 \times 0.6 \times 0.6$$

$$= 0.216$$

$$P(\text{at least one rainy day}) = 1 - 0.216 = 0.784$$

this is not 1.2

No. The probability is 0.784, not 1.2

3 / 3

- 40 In Cardiff, the probability of rain on any day in winter is $\frac{2}{5}$.

Whether it rains on one day does not depend on the other days.

- (a) Work out the probability that it rains on all three days, Thursday, Friday and Saturday.
 (b) Work out the probability that it rains on at least one of the three days, Thursday, Friday and Saturday.

$$(a) \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$$

$$(b) P(\text{none}) = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$$

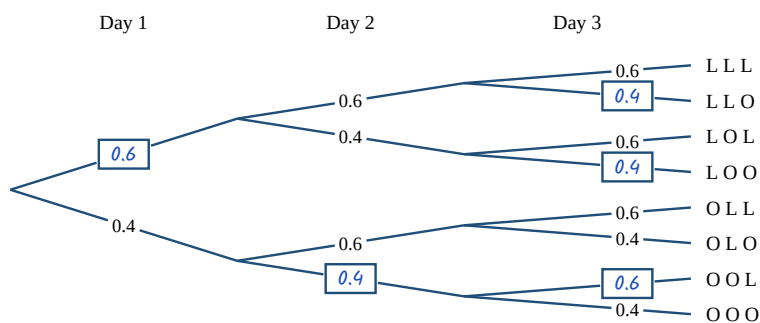
$$1 - \frac{27}{125} = \frac{98}{125}$$

$$(a) \frac{8}{125} \quad (b) \frac{98}{125}$$

4 / 4

- 41 The probability that Dan's bus is late is 0.6 on each of three days.

- (a) Complete the tree diagram. The upper branch of each pair is for "late" (L).
 (b) Work out the probability that the bus is late on all three days.



(a) the two branches from each point add up to 1, so 0.6 and 0.4
 (b) $0.6 \times 0.6 \times 0.6 = 0.216$

$$(b) 0.216$$

3 / 3

- 42 A fair coin is thrown three times.

- (a) Calculate the probability of getting exactly two heads.
 (b) Work out the probability of getting not three heads.

(a) HHT, HTH, THH are 3 ways each $\frac{1}{8}$

$$3 \times \frac{1}{8} = \frac{3}{8}$$

$$(b) P(\text{three heads}) = \frac{1}{8}$$

$$1 - \frac{1}{8} = \frac{7}{8}$$

$$(a) \frac{3}{8} \quad (b) \frac{7}{8}$$

4 / 4

- 43 A bag contains 4 green counters and 5 yellow counters.

A counter is taken at random, its colour is noted and it is put back. This is done three times.

(a) Work out the probability that at least one of the three counters is green.

(b) Work out the probability that none of the three counters is green.

$$(a) P(\text{none}) = \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$$

$$1 - \frac{125}{729} = \frac{604}{729}$$

$$(b) P(\text{not}) = \frac{5}{9}$$

$$\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$$

$$(a) \frac{604}{729} \quad (b) \frac{125}{729}$$

4 / 4

- 44 Bag A contains 3 red counters and 2 blue counters. Bag B contains 2 red counters and 6 blue counters.

Bag C contains 1 red counter and 5 blue counters.

One counter is taken at random from each bag.

(a) Work out the probability that at least one of the three counters is red.

(b) Work out the probability that all three counters are red.

$$(a) P(\text{none}) = \frac{2}{5} \times \frac{3}{4} \times \frac{5}{6} = \frac{1}{4}$$

$$1 - \frac{1}{4} = \frac{3}{4}$$

$$(b) \frac{3}{5} \times \frac{1}{4} \times \frac{1}{6} = \frac{1}{40}$$

$$(a) \frac{3}{4} \quad (b) \frac{1}{40}$$

4 / 4

- 45 A spinner has three colours, red, white and yellow.

The probability of landing on red is 0.2 and the probability of landing on white is 0.3.

The spinner is spun three times. Each spin is independent of the others.

(a) Work out the probability that the spinner lands on red exactly once.

(b) Work out the probability that the spinner lands on red all three times.

(a) 3 ways

$$3 \times 0.2 \times 0.8 \times 0.8 = 0.384$$

$$(b) 0.2 \times 0.2 \times 0.2 = 0.008$$

$$(a) 0.384 \quad (b) 0.008$$

4 / 4

- 46 A machine makes three parts. Each part is perfect with the same probability, independent of the others.

The probability that all three parts are perfect is 0.064.

Work out the probability that a part is perfect.

$$p \times p \times p = p^3 = 0.064$$

$$p \text{ is the cube root of } 0.064 = 0.4$$

0.4

5 / 5

- 47 A bag contains 5 red counters and 6 blue counters.

A counter is taken at random, its colour is noted and it is put back. This is done three times.

(a) Work out the probability that exactly one of the three counters is red.

(b) Work out the probability that all three counters are red.

(a) 3 ways

$$3 \times \frac{5}{11} \times \frac{6}{11} \times \frac{6}{11} = \frac{540}{1331}$$

$$(b) \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} = \frac{125}{1331}$$

(a) $\frac{540}{1331}$ (b) $\frac{125}{1331}$

5 / 5

- 48 A spinner has three colours, green, yellow and red.

The probability of landing on green is 0.3 and the probability of landing on yellow is 0.3.

The spinner is spun three times. Each spin is independent of the others.

(a) Work out the probability that the spinner lands on green all three times.

(b) Work out the probability that the spinner lands on green exactly twice.

$$(a) 0.3 \times 0.3 \times 0.3 = 0.027$$

(b) 3 ways

$$3 \times 0.3 \times 0.3 \times 0.7 = 0.189$$

(a) 0.027 (b) 0.189

5 / 5

- 49 Ffion takes three penalty kicks. The probability that she scores is the same for each kick.

The probability that Ffion scores with all three kicks is $\frac{1}{64}$.

Work out the probability that she scores with one kick.

$$p \times p \times p = p^3 = \frac{1}{64}$$

$$p \text{ is the cube root of } \frac{1}{64} = \frac{1}{4}$$

$$\frac{1}{4}$$

5 / 5

- 50 A factory uses three machines, A, B and C. They work independently of each other. On any day, the probability that A works is 0.9, the probability that B works is 0.95 and the probability that C works is 0.92.

(a) Calculate the probability that exactly one of the machines works on a day.

(b) Calculate the probability that all three machines work on a day.

$$(a) 0.9 \times 0.05 \times 0.08 + 0.1 \times 0.95 \times 0.08 + 0.1 \times 0.05 \times 0.92$$

$$\text{total} = 0.0158$$

$$(b) 0.9 \times 0.95 \times 0.92 = 0.7866$$

$$(a) 0.0158 \quad (b) 0.7866$$

5 / 5