

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3310U60-1



THURSDAY, 7 NOVEMBER 2019 – MORNING

**MATHEMATICS – NUMERACY
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER**

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for the work written on the additional page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 4(a)(i), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

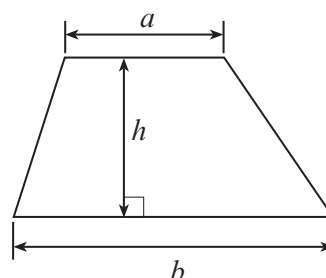
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	6	
2.	9	
3.	8	
4.	12	
5.	9	
6.	3	
7.	5	
8.	5	
9.	4	
10.	5	
11.	8	
12.	6	
Total	80	



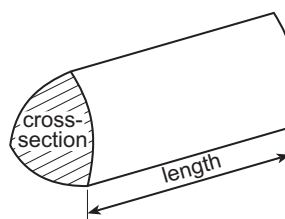
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Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

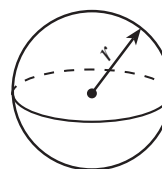


Volume of prism = area of cross-section $\times$ length



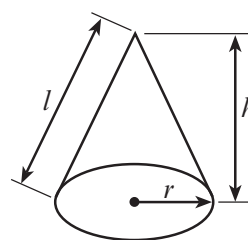
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

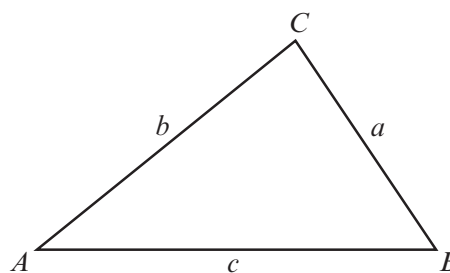


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. Rowan lives in New Zealand.
He is coming to Wales on holiday.
The currency used in New Zealand is the New Zealand dollar (\$).
1 New Zealand dollar = 100 cents.

The conversion rate at the exchange shop is $\$1 = \pounds 0.53$.
The exchange shop only has $\pounds 10$ and $\pounds 20$ notes.

Rowan only has $\$550$ saved.
He wants to exchange as close to $\$550$ as possible.
He asks for as few notes as possible.

Calculate:

- how many of each British note Rowan gets,
- how much he pays for his currency, correct to the nearest cent.

You must show all your working.

[6]

Change \$550 to pounds: $550 \times 0.53 = \pounds 291.50$

The shop only has $\pounds 10$ and $\pounds 20$ notes, so round down to a multiple of $\pounds 10$: Rowan can get $\pounds 290$ ($\pounds 1.50$ cannot be changed).

Fewest notes for $\pounds 290$: $14 \times \pounds 20 = \pounds 280$, plus $1 \times \pounds 10 = \pounds 290$

Rowan gets 14 $\pounds 20$ notes and 1 $\pounds 10$ note.

Cost in dollars of $\pounds 290 = 290 \div 0.53$

$= 547.169811...$

$= \$547.17$ (to the nearest cent)

Rowan gets fourteen $\pounds 20$ notes and one $\pounds 10$ note,

and he pays $\$547.17$ for his currency.



2. The picture shows a solid concrete step.

The step:

- stands on horizontal ground,
- has all of its edges vertical or horizontal,
- has a uniform cross-section.

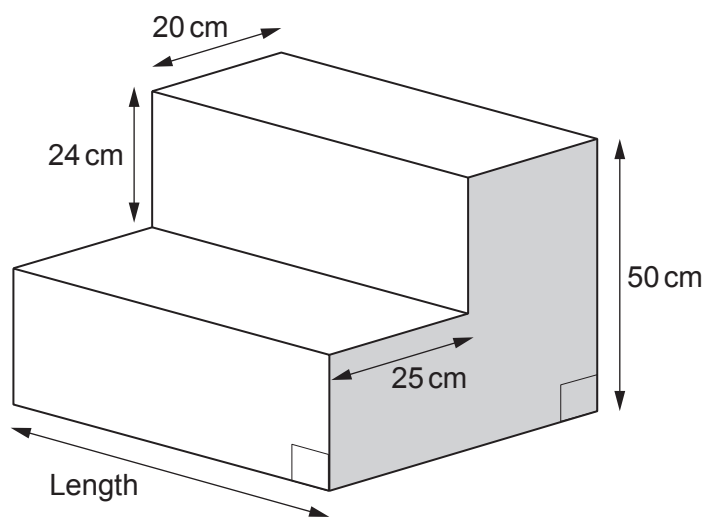
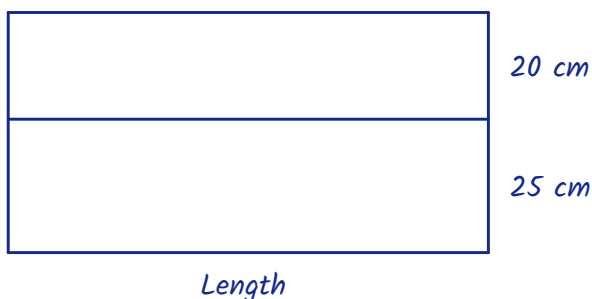


Diagram not drawn to scale

(a) Draw a sketch of the plan view of the concrete step.

[1]

Plan view



(b) The volume of concrete in the step is $66\,000\text{ cm}^3$.

(i) The concrete to make the step costs 39p per litre.

A builder charges a rate of £27 per hour.

Any fraction of an hour is charged as that fraction of his hourly rate.
(For example, half an hour is charged at half of £27.)

It takes him 1 hour 20 minutes to make the step.

There were no other costs.

Calculate the total cost of making the step.

[3]

$$\text{Volume} = 66\,000\text{ cm}^3 = 66\,000 \div 1000 = 66\text{ litres}$$

$$\text{Cost of concrete} = 66 \times 39\text{p} = 2574\text{p} = \text{£}25.74$$

$$\text{Builder: } 1\text{ hour } 20\text{ minutes} = 1\frac{1}{3}\text{ hours}$$



$$\text{Cost of builder} = 27 + \frac{1}{3} \times 27 = 27 + 9 = \text{£}36.00$$

$$\text{Total cost} = \text{£}25.74 + \text{£}36.00$$

$$= \text{£}61.74$$

The total cost of making the step is £61.74

- (ii) Calculate the length of the step.
Give your answer in cm.
You must show all your working.

[5]

Split the cross-section into two rectangles:

$$\text{Top step: } 20 \times 50 = 1000 \text{ cm}^2$$

$$\text{Bottom step: } 25 \times (50 - 24) = 25 \times 26 = 650 \text{ cm}^2$$

$$\text{Area of cross-section} = 1000 + 650 = 1650 \text{ cm}^2$$

$$\text{Length} = \text{volume} \div \text{area of cross-section}$$

$$= 66\,000 \div 1650$$

$$= 40 \text{ cm}$$

The length of the step is 40 cm.



3. (a) 40 people were asked how many mugs they have in their cupboards. The results are shown below.



Number of mugs	Frequency
1 to 5	3
6 to 10	7
11 to 15	12
16 to 20	18

- (i) From this data, which group contains the **median** number of mugs?
Circle your answer.

[1]

Can't tell 1 to 5 6 to 10 11 to 15 16 to 20

- (ii) Calculate an estimate of the mean number of mugs these people have in their cupboards.

[4]

Midpoints: 3, 8, 13, 18

$$3 \times 3 + 8 \times 7 + 13 \times 12 + 18 \times 18$$

$$= 9 + 56 + 156 + 324 = 545$$

$$\text{Mean} = 545 \div 40 = 13.625$$

Estimate of the mean = 13.625 mugs (about 14 mugs)



- (b) A cylindrical mug has an inner radius of 4.3 cm and an inner height of 11.8 cm.

Tea is poured into the mug.
The level of the tea is 2 cm below the top of the mug.



Calculate the volume of the tea in the mug.

[3]

$$\text{Height of tea} = 11.8 - 2 = 9.8 \text{ cm}$$

$$\text{Volume} = \pi r^2 h = \pi \times 4.3^2 \times 9.8$$

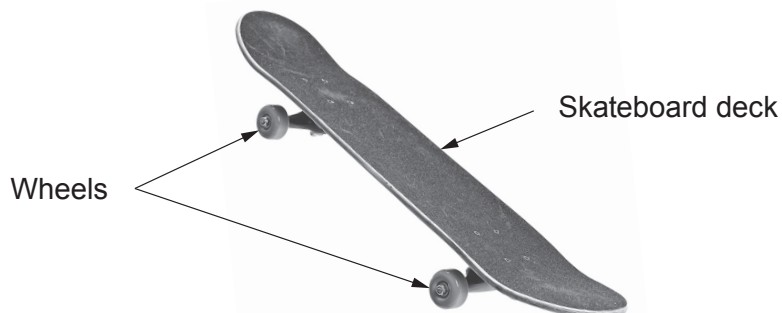
$$= \pi \times 18.49 \times 9.8$$

$$= 569.262872... \text{ cm}^3$$

$$\text{Volume of tea} = 569.3 \text{ cm}^3 \text{ (1 d.p.)}$$



4. Finbar's skateboard is shown below.



- (a) The diameter of each wheel on Finbar's skateboard is 6.4 cm.
He uses his skateboard to go to visit his friend Sab.
Sab lives 2340 metres from Finbar.

- (i) *In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

When Finbar visits Sab, how many times will each wheel on Finbar's skateboard rotate? [4 + 2 OCW]

$$\text{Circumference of a wheel} = \pi \times d = \pi \times 6.4 = 20.106192... \text{ cm}$$

One rotation moves the skateboard 20.106192... cm.

$$\text{Distance to Sab} = 2340 \text{ m} = 2340 \times 100 = 234\,000 \text{ cm}$$

$$\text{Number of rotations} = \text{distance} \div \text{circumference}$$

$$= 234\,000 \div (\pi \times 6.4)$$

$$= 234\,000 \div 20.106192...$$

$$= 11\,638.205...$$

So each wheel will rotate about 11 638 times when Finbar visits Sab.

- (ii) What assumption did you make in answering (a)(i)?

[1]

I assumed that Finbar rode his skateboard all the way, in a straight line directly to Sab's house, without carrying it.



- (b) A skateboard deck is usually made from one of maple wood, fibreglass or plastic. The density of these materials is given in the following table.

Skateboard deck material	Density (g/cm ³)
Maple wood	0.7
Fibreglass	2.6
Plastic	1.8

Finbar and Sab compare their skateboards.

	Finbar's skateboard	Sab's skateboard
Area of the skateboard deck	1800 cm ²	1600 cm ²
Thickness of the skateboard deck	1.2 cm	1.4 cm
Material used to make the deck	Fibreglass	Maple wood

The wheels and the fittings on both their skateboards are identical.

How much heavier is Finbar's skateboard than Sab's skateboard?

Give your answer in grams.

You must show all your working.

[5]

$$\text{Mass} = \text{density} \times \text{volume}$$

$$\text{Finbar's deck: volume} = 1800 \times 1.2 = 2160 \text{ cm}^3$$

$$\text{Mass} = 2.6 \times 2160 = 5616 \text{ g}$$

$$\text{Sab's deck: volume} = 1600 \times 1.4 = 2240 \text{ cm}^3$$

$$\text{Mass} = 0.7 \times 2240 = 1568 \text{ g}$$

The wheels and fittings are identical, so the difference is

$$\text{only in the decks: } 5616 - 1568 = 4048 \text{ g}$$

Finbar's skateboard is 4048 g heavier than Sab's.



5. Robyn has 5 planks of wood each of length 2 m and width 10 cm.

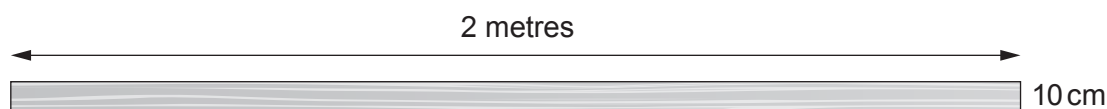


Diagram not drawn to scale

She lays the 5 planks horizontally on the floor. She leaves a **15 cm gap** between each plank, as shown below.



Diagram not drawn to scale

Robyn is planning to make a gate. She uses these 5 planks and one other plank that is to be placed diagonally, as shown below.

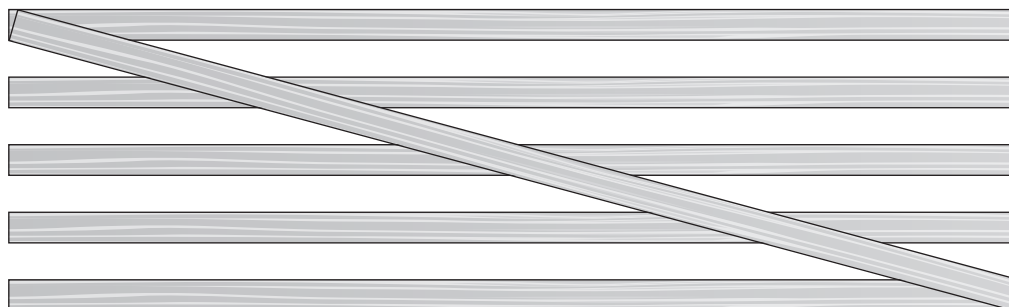


Diagram not drawn to scale

- (a) (i) Calculate an estimate of the length of the plank that is to be placed diagonally. Give your answer in metres. [4]

$$\text{Height of gate} = 5 \times 10 + 4 \times 15 = 50 + 60 = 110 \text{ cm} = 1.1 \text{ m}$$

$$\text{Diagonal}^2 = 2^2 + 1.1^2 = 4 + 1.21 = 5.21$$

$$\text{Diagonal} = \sqrt{5.21} = 2.282542\dots$$

$$\text{Length of the diagonal plank} \approx 2.28 \text{ m}$$



- (ii) What assumption did you make in calculating the length of the plank that is to be placed diagonally? [1]

I assumed the diagonal plank goes exactly from corner to corner, treating the planks as lines with no thickness.

- (b) Robyn finishes the gate with two end planks of wood.



Diagram not drawn to scale

The costs of the different sizes of planks of wood are in the following ratio:

cost of 1 horizontal plank : cost of 1 diagonal plank : cost of 1 end plank
= 3 : 4 : 5

An end plank costs £8.55.

Calculate the total cost of the planks needed to make the gate. [4]

One share = $8.55 \div 5 = £1.71$

Cost of 1 horizontal plank = $3 \times 1.71 = £5.13$

Cost of 1 diagonal plank = $4 \times 1.71 = £6.84$

Total = $5 \times 5.13 + 1 \times 6.84 + 2 \times 8.55$

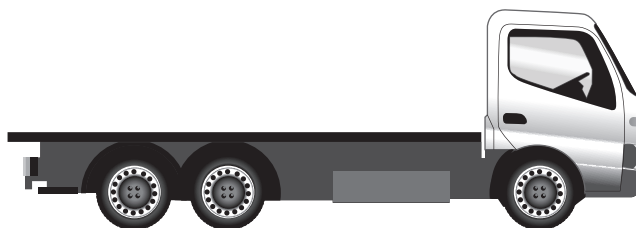
= $25.65 + 6.84 + 17.10$

= £49.59

The total cost of the planks for the gate is £49.59



6. Dafydd carries bags of gravel on the back of his lorry.



Each bag of gravel has a mass of 90 kg, correct to the nearest 5 kg.

The maximum mass the lorry can carry without overloading is 7500 kg.
However, this measurement is only correct to the nearest 100 kg.

Calculate the maximum number of bags that the lorry is **guaranteed** to be able to carry without overloading.

You must show all your working.

[3]

$$\text{Lower bound of maximum load} = 7500 - 50 = 7450 \text{ kg}$$

$$\text{Upper bound of mass of a bag} = 90 + 2.5 = 92.5 \text{ kg}$$

$$\text{Number of bags} = 7450 \div 92.5 = 80.54\dots$$

Round down: 80 bags (81 bags could overload the lorry)

Maximum number of bags that the lorry is guaranteed to be able to carry is 80



7. Llinos walks to the summit of Snowdon, passing the lake called Llyn Glaslyn. Her height above sea level increases by 485 m from Llyn Glaslyn to the summit.

From the summit, she sees two small boats on Llyn Glaslyn. Both boats are in the same direction from the summit. The angles of depression of the two boats are 41° and 27° , as shown in the diagram.

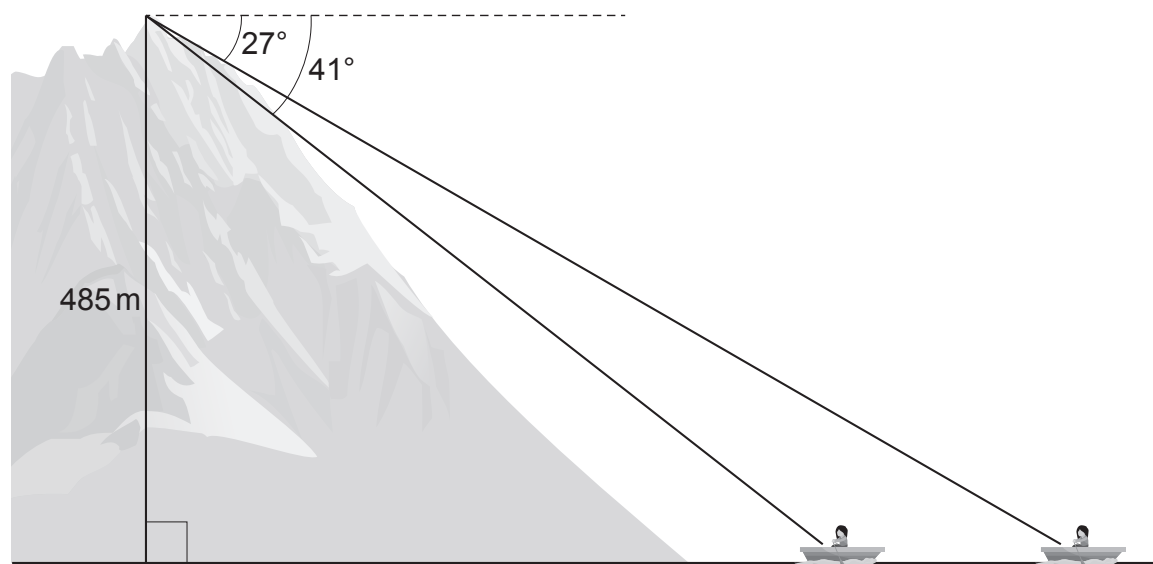


Diagram not drawn to scale

Calculate the distance between the boats.

[5]

Angle of elevation from a boat = angle of depression (alternate angles)

Nearer boat: $\tan 41^\circ = 485 \div d_1$, so $d_1 = 485 \div \tan 41^\circ = 557.928... \text{ m}$

Further boat: $\tan 27^\circ = 485 \div d_2$, so $d_2 = 485 \div \tan 27^\circ = 951.866... \text{ m}$

Distance between the boats = $951.866... - 557.928...$

= $393.937... \text{ m}$

The distance between the boats is 393.9 m (1 d.p.)



8. Gary and Carys are fire officers.

Last week, they recorded that 5 engines were able to pump 26 000 gallons of water onto a fire in 3 minutes.



- (a) Show that 9 engines would be able to pump 143 000 gallons of water in under 9 minutes 15 seconds.

[4]

5 engines pump 26 000 gallons in 3 minutes.

9 engines pump 26 000 gallons in $3 \times 5 \div 9 = 1.666...$ minutes.

$143\ 000 \div 26\ 000 = 5.5$

9 engines pump 143 000 gallons in $1.666... \times 5.5$ minutes

$(3 \times 5/9 \times 143\ 000/26\ 000) = 9.1666... \text{ minutes}$

$0.1666... \times 60 = 10 \text{ seconds}$, so the time is 9 minutes 10 seconds.

9 minutes 10 seconds is less than 9 minutes 15 seconds,

so 9 engines would pump 143 000 gallons in under 9 min 15 s.

- (b) Give one possible reason why the 9 engines **may not** be able to pump 143 000 gallons of water in under 9 minutes 15 seconds.

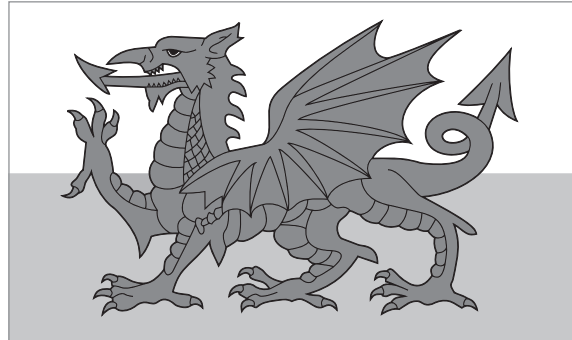
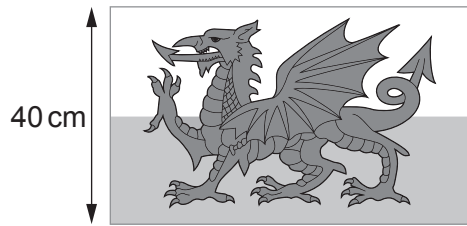
[1]

One of the engines may be faulty, or there may not be enough

water available to pump at the same rate.



9. A company makes Welsh flags in mathematically similar sizes. Two of their similar flags are shown.



Diagrams not drawn to scale

The area of the larger flag is 96% greater than the area of the smaller flag.
The height of the smaller flag is 40 cm.

Calculate the height of the larger flag.

[4]

$$\text{Area scale factor} = 100\% + 96\% = 196\% = 1.96$$

$$\text{Length scale factor} = \sqrt{1.96} = 1.4$$

$$\text{Height of larger flag} = 40 \times 1.4$$

$$= 56 \text{ cm}$$



10. On 1st January 2017, Samantha and Dyfan invested money into different savings accounts. They did not make any further payments into their accounts or withdraw any money from their accounts.

- (a) Samantha invested £2000 in a savings account that paid interest at a rate of 0.95% every 3 months.

Show that Samantha would have £2038.18 in the account after 6 months. [1]

$$2000 \times 1.0095^2 = 2038.1805 = \text{£}2038.18 \text{ (after two 3-month periods)}$$

- (b) Dyfan invested £3000 in a savings account that paid interest at a rate of 1.02% every 3 months.
Interest is paid on the last day of each 3-month period.

Calculate the date when Dyfan will first have over £3600 in his account. [4]

$$\text{Value after } n \text{ periods of 3 months} = 3000 \times 1.0102^n$$

$$n = 17: 3000 \times 1.0102^{17} = 3564.89... \text{ (not over £3600)}$$

$$n = 18: 3000 \times 1.0102^{18} = 3601.25... \text{ (over £3600)}$$

$$18 \text{ periods} \times 3 \text{ months} = 54 \text{ months} = 4 \text{ years } 6 \text{ months}$$

Interest is paid on the last day of each 3-month period,

so 4 years 6 months after 1st January 2017 is 30th June 2021.

Date when Dyfan will first have over £3600 in his account is 30th June 2021



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11. Two farmers have bought some farmland between them.
The farmland is in the shape of a quadrilateral $ABCD$, as shown below.

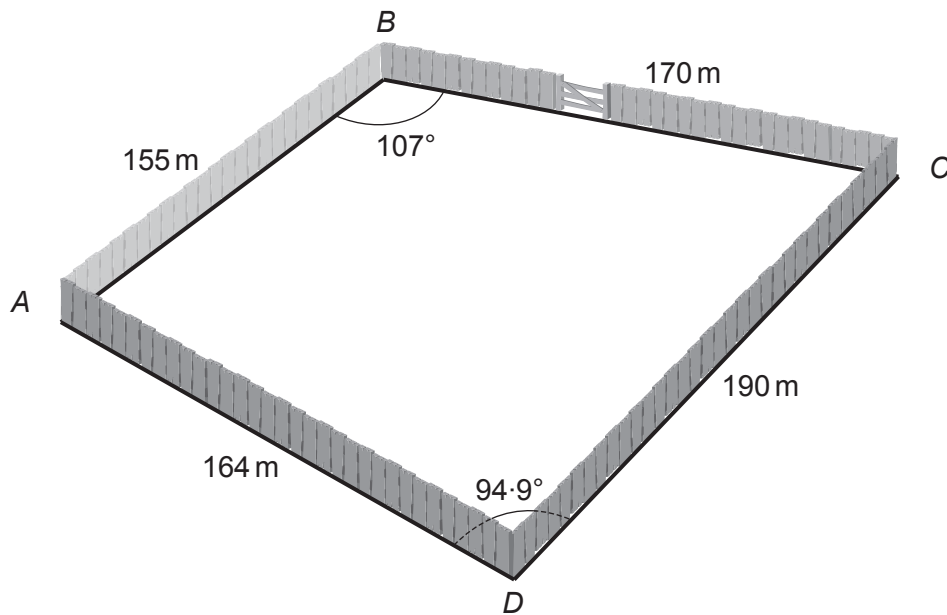


Diagram not drawn to scale

The farmers want to divide the farmland equally by building a straight fence.

- (a) One of the farmers has suggested building the fence from A to C .
Show that this does not divide the farmland equally.

[3]

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area } ABC = \frac{1}{2} \times 155 \times 170 \times \sin 107^\circ = 12\,599.3... \text{ m}^2$$

$$\text{Area } ACD = \frac{1}{2} \times 164 \times 190 \times \sin 94.9^\circ = 15\,523.0... \text{ m}^2$$

$$12\,599.3 \text{ m}^2 \neq 15\,523.0 \text{ m}^2, \text{ so the two parts are not equal.}$$

The fence from A to C does not divide the farmland equally.



- (b) To divide the land equally, the fence is built from A to F where $CF = 17.9$ m. To construct the fence, posts are placed at A and F . Other posts are then located along AF , so that the posts are no more than 3 m apart.

Calculate the smallest number of posts needed, including the posts at A and F .

[5]

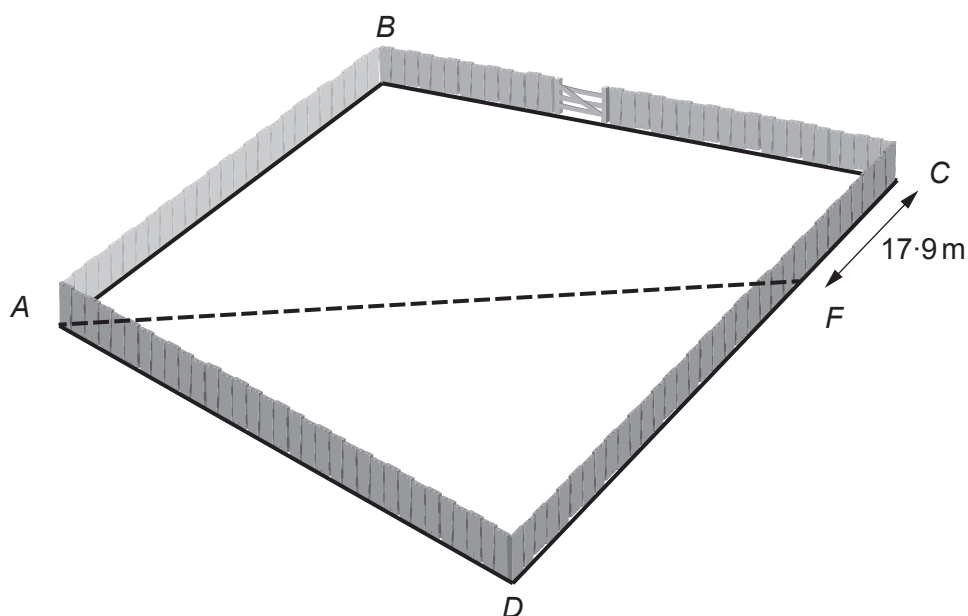


Diagram not drawn to scale

$$DF = 190 - 17.9 = 172.1 \text{ m}$$

Cosine rule in triangle ADF:

$$AF^2 = 164^2 + 172.1^2 - 2 \times 164 \times 172.1 \times \cos 94.9^\circ$$

$$AF^2 = 61\,336.09\dots$$

$$AF = \sqrt{61\,336.09\dots} = 247.661\dots \text{ m}$$

$$\text{Number of 3 m gaps} = 247.661\dots \div 3 = 82.55\dots, \text{ round up to 83 gaps}$$

$$\text{Number of posts} = 83 + 1 = 84$$

The smallest number of posts needed is 84.



12. Laura and Matthew are buying a house priced at £150 000.
In order to buy the house, they will need to have a mortgage.

A mortgage is a loan that is paid back over a number of years.

They have saved a deposit of £15 000.
They need a mortgage of £135 000.

A bank has offered them a mortgage of £135 000 at an interest rate of 2.4% per annum, with interest added monthly.

To calculate the monthly payments needed in order to pay back the mortgage, they use the following formula:

$$M = \frac{r \times P}{1 - (1 + r)^{-12n}}$$

where:

M is the amount of each monthly payment,

P is the mortgage needed,

r is the **monthly** interest rate as a decimal,

n is the number of years taken to pay back the mortgage.

- (a) The annual interest rate is 2.4%.
What is the monthly rate, as a decimal?
Circle your answer.

[1]

0.24

0.024

0.00002

0.002

0.2

- (b) Laura and Matthew are considering whether to take out a mortgage over 25 years or 30 years.

They have correctly calculated their monthly payments to be £598.86 when paying back the mortgage over 25 years.

How much more will it cost **in total** to pay back the mortgage over 30 years than over 25 years?

[5]

$$30 \text{ years: } M = (0.002 \times 135\,000) \div (1 - 1.002^{-360}) = 526.4208... = \pounds 526.42$$

$$\text{Total over 30 years} = 526.42 \times 360 = \pounds 189\,511.20$$

$$\text{Total over 25 years} = 598.86 \times 300 = \pounds 179\,658.00$$

$$\text{Difference} = 189\,511.20 - 179\,658.00 = \pounds 9853.20$$

It will cost £9853.20 more over 30 years than over 25 years.



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