

Surname <i>StemUp Model Solution</i>
First name(s)

Centre Number

Candidate Number
0



GCSE

3310U50-1



FRIDAY, 20 MAY 2022 – MORNING

**MATHEMATICS – NUMERACY
UNIT 1: NON-CALCULATOR
HIGHER TIER**

1 hour 35 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for the work written on the additional page.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 4, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

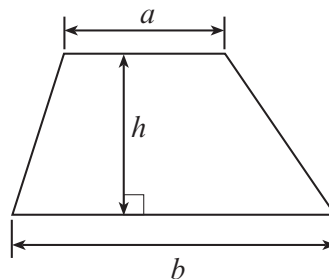
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	6	
2.	5	
3.	6	
4.	6	
5.	10	
6.	2	
7.	6	
8.	9	
9.	7	
10.	13	
Total	70	



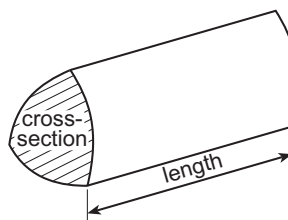
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

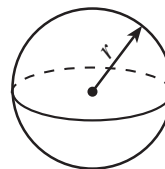


Volume of prism = area of cross-section $\times$ length



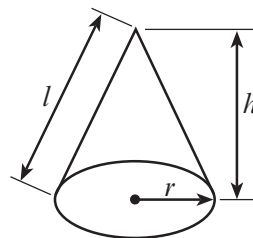
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

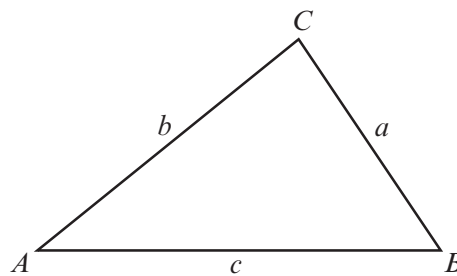


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



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1. (a) People travel by bus or by train from Hiraddug Station.
On Tuesday, 420 people travelled by bus from the station.
The ratio of the number of people travelling by bus to the number of people travelling by train was 20 : 17.

Calculate the number of people who travelled by train from the station on Tuesday. [3]

$$\text{Bus : Train} = 20 : 17$$

$$20 \text{ parts} = 420 \text{ people}$$

$$1 \text{ part} = 420 \div 20 = 21 \text{ people}$$

$$\text{Train} = 17 \text{ parts} = 17 \times 21 = 357$$

357 people travelled by train on Tuesday.

- (b) The price of a ticket to travel by bus to Glaswen Station has increased by 5% in each of the last 2 years.
Two years ago, the price of a ticket was £4.
Calculate the current price of a ticket. [3]

$$\text{After 1 year: } 5\% \text{ of } £4 = £0.20$$

$$\text{Price last year} = £4 + £0.20 = £4.20$$

$$\text{After 2 years: } 5\% \text{ of } £4.20 = £0.21$$

$$\text{Current price} = £4.20 + £0.21 = £4.41$$

The current price of a ticket is £4.41.



2. (a) Gwyn is making some packs.
Each pack contains one nut, one bolt and one washer.

To make up these packs, Gwyn buys:

- some boxes that contain 30 nuts each
- some boxes that contain 5 bolts each
- some boxes that contain 25 washers each.



Gwyn wants to buy the **least possible number of boxes** so that, in making up the packs, he uses **all** of the nuts, bolts and washers he has bought.

Complete the table below to show the number of boxes of each item that Gwyn needs to buy.

You must show all your working.

[3]

$$30 = 2 \times 3 \times 5, \quad 5 = 5, \quad 25 = 5 \times 5$$

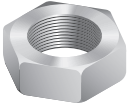
$$\text{LCM of } 30, 5 \text{ and } 25 = 2 \times 3 \times 5 \times 5 = 150$$

So Gwyn needs 150 of each item.

$$\text{Nuts: } 150 \div 30 = 5 \text{ boxes}$$

$$\text{Bolts: } 150 \div 5 = 30 \text{ boxes}$$

$$\text{Washers: } 150 \div 25 = 6 \text{ boxes}$$

		Number of boxes needed
	Nuts (30 in each box)	5..... boxes
	Bolts (5 in each box)	30..... boxes
	Washers (25 in each box)	6..... boxes

- (b) Each washer has a thickness of 2 mm, correct to the nearest 0.5 mm.
Calculate the greatest possible thickness of a stack of 6 washers.

[2]

Greatest thickness of one washer (upper bound)

$$= 2 + 0.25 = 2.25 \text{ mm}$$

$$\text{Stack of } 6 = 6 \times 2.25 = 13.5 \text{ mm}$$

$$\text{Greatest possible thickness} = 13.5 \text{ mm}$$



3. OrenVit is a company that produces bottles of orange juice. The company uses only bottles with a capacity of one litre.

Each day, the cost of producing bottles of orange juice is as follows:

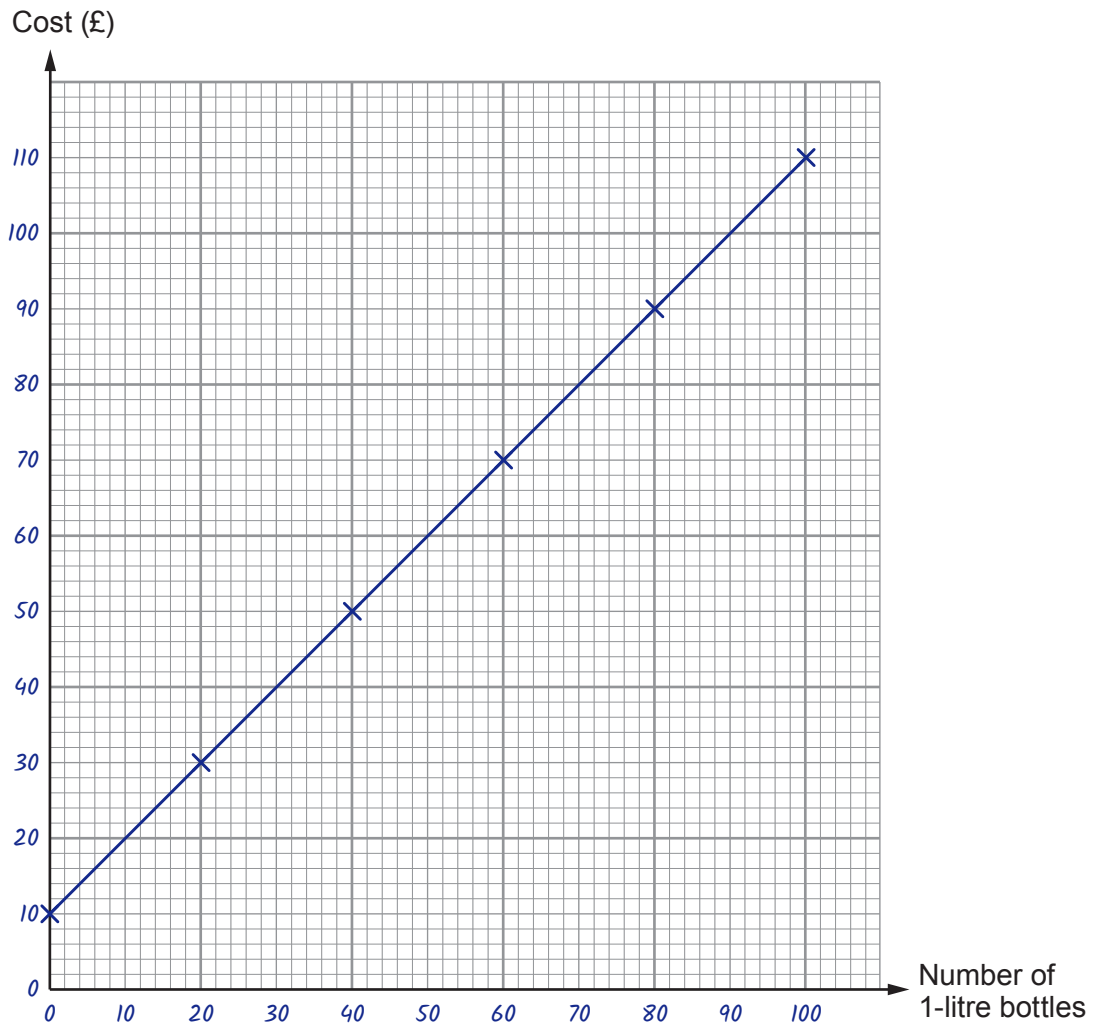
Fixed charge for use of equipment	£10
Cost of ingredients	80p per bottle
Cost of empty bottles with labels	20p per bottle

- (a) Draw a graph to show the total daily cost of producing between 0 and 100 bottles of orange juice.
Use the graph paper below.

[3]

$$\text{Total cost} = \text{£}10 + (80\text{p} + 20\text{p}) \times \text{number of bottles}$$

$$\text{Cost (£)} = 10 + 1 \times \text{number of bottles, e.g. } 0 \rightarrow \text{£}10, 100 \rightarrow \text{£}110$$



- (b) One day, OrenVit produces 1750 **pints** of orange juice in one-litre bottles. Calculate the cost of producing this quantity of orange juice.

[3]

1 litre $\approx$ 1.75 pints, so $1750 \div 1.75 = 1000$ litres = 1000 bottles

Cost = $\pounds 10 + 1000 \times (80p + 20p) = \pounds 10 + 1000 \times \pounds 1 = \pounds 10 + \pounds 1000$

Cost of producing this orange juice = $\pounds 1010$

4. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Rectangular stickers with warnings written on them are often placed near water taps.

The rectangular sticker shown below warns of hot water. It has a length of 14 cm and an area of 42 cm^2 .



This water is hot

Diagram not
drawn to scale

The sticker below is mathematically similar to the first sticker. It is an enlargement of the first sticker, with scale factor 4.

This water is hot

Diagram not
drawn to scale

Calculate the length and the width of the larger sticker. You must show all your working.

[4 + 2 OCW]

Small sticker: area = length $\times$ width

Width of small sticker = $42 \div 14 = 3 \text{ cm}$

*The large sticker is an enlargement with scale factor 4,
so every length is multiplied by 4.*

Length of large sticker = $14 \times 4 = 56 \text{ cm}$

Width of large sticker = $3 \times 4 = 12 \text{ cm}$

Check: area = $56 \times 12 = 672 \text{ cm}^2 = 42 \times 4^2$

The larger sticker has a length of 56 cm

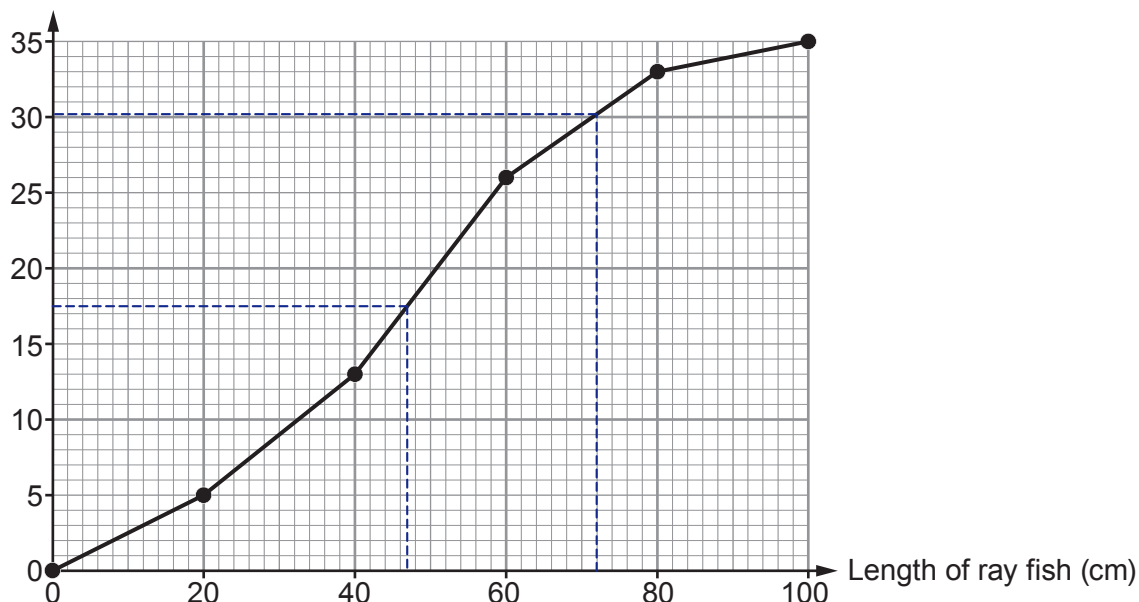
and a width of 12 cm.



5. (a) An aquarium has 35 ray fish.
The cumulative frequency graph shows information about the lengths of these ray fish.



Cumulative frequency



Use the cumulative frequency diagram to give the best estimates for the answers to the following questions.

- (i) What is the median length of these ray fish? [1]

Median is the $35 \div 2 = 17.5$ th value: read across from 17.5

47 cm

- (ii) How many of these ray fish have lengths greater than 72 cm? [1]

Up to 72 cm: cumulative frequency ≈ 30

Number longer than 72 cm = $35 - 30 = 5$ ray fish



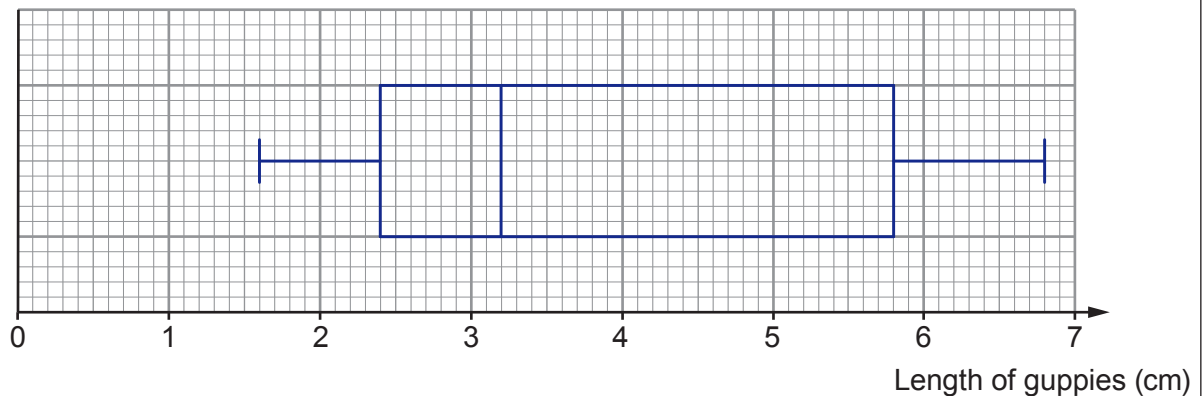
- (b) The aquarium also has guppies.
The table shows information about the length of the guppies in a sample of 60 guppies.



Minimum	Range	Median	Lower quartile	Interquartile range
1.6 cm	5.2 cm	3.2 cm	2.4 cm	3.4 cm

- (i) Draw a box-and-whisker plot of this data on the grid below.

[4]



$$\text{Max} = 1.6 + 5.2 = 6.8 \text{ cm} \quad \text{UQ} = 2.4 + 3.4 = 5.8 \text{ cm}$$

$$\text{Min} = 1.6, \text{ LQ} = 2.4, \text{ Median} = 3.2, \text{ UQ} = 5.8, \text{ Max} = 6.8$$

- (ii) How many of the sample of 60 guppies have a length greater than or equal to 2.4 cm?

[2]

25% of the guppies are shorter than the LQ (2.4 cm),

so 75% are 2.4 cm or longer.

$$75\% \text{ of } 60 = 3 \times 60 \div 4 = 45 \text{ guppies}$$

- (c) A carp was weighed in November 2021.
It was weighed again in April 2022.
The carp had a mass of 9.9 kg in April 2022.
Between these dates, the mass of the carp increased by 10%.



Calculate the mass of the carp in November 2021.

[2]

$$\text{April mass} = 110\% \text{ of November mass} = 9.9 \text{ kg}$$

$$1\% = 9.9 \div 110 = 0.09 \text{ kg}$$

$$\text{November mass} = 100\% = 0.09 \times 100 = 9 \text{ kg}$$

The mass of the carp in November 2021 was 9 kg.



6. The surface area of the Earth is 510 million km^2 .
Write this area in standard form.

[2]

$$510 \text{ million} = 510\,000\,000$$

$$= 5.1 \times 100\,000\,000 = 5.1 \times 10^8$$

$$5.1 \times 10^8 \text{ km}^2$$

7. Tregareth Zoo has a large glass enclosure for reptiles.
The enclosure consists of a hollow rectangular-based pyramid sitting on top of a cuboid, as shown below.

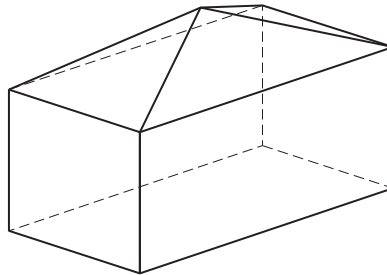
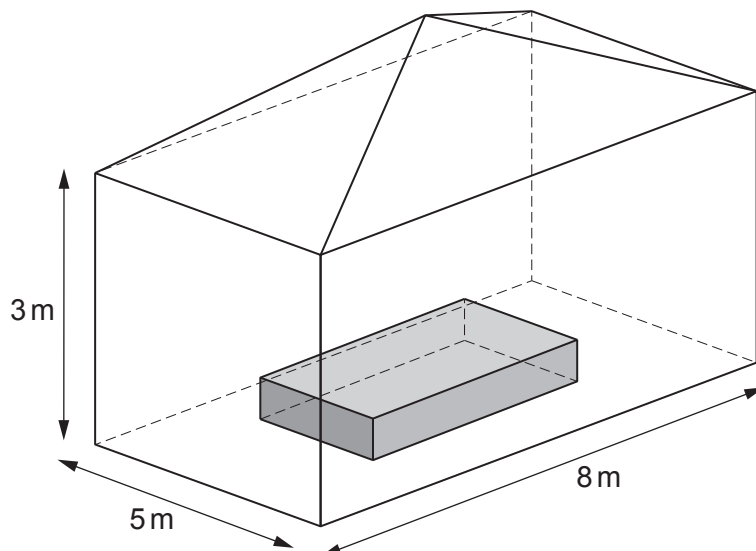
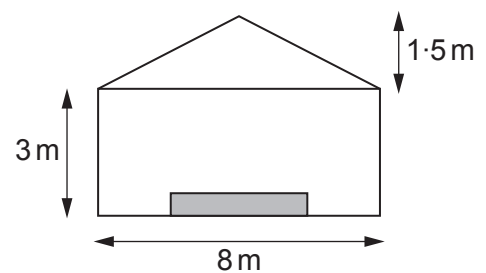


Diagram not
drawn to scale

The enclosure is to be modified by placing a wooden cuboid onto the base.
This cuboid will reduce the capacity of the enclosure.



Front view



Diagrams not
drawn to scale



The pyramid has a vertical height of 1.5 m.

The inserted wooden cuboid has dimensions 4 m by 3.5 m by 0.5 m.

Calculate the capacity of this modified enclosure as a percentage of the capacity of the original enclosure.

[6]

Original enclosure:

$$\text{Volume of cuboid} = 5 \times 8 \times 3 = 120 \text{ m}^3$$

$$\begin{aligned} \text{Volume of pyramid} &= \frac{1}{3} \times \text{base area} \times \text{height} = \frac{1}{3} \times 5 \times 8 \times 1.5 \\ &= \frac{1}{3} \times 40 \times 1.5 = \frac{1}{3} \times 60 = 20 \text{ m}^3 \end{aligned}$$

$$\text{Capacity of original enclosure} = 120 + 20 = 140 \text{ m}^3$$

Modified enclosure:

$$\text{Volume of wooden cuboid} = 4 \times 3.5 \times 0.5 = 14 \times 0.5 = 7 \text{ m}^3$$

$$\text{Capacity of modified enclosure} = 140 - 7 = 133 \text{ m}^3$$

$$\text{Percentage} = \frac{133}{140} \times 100$$

$$= \frac{133 \div 7}{140 \div 7} \times 100 = \frac{19}{20} \times 100 = 95\%$$

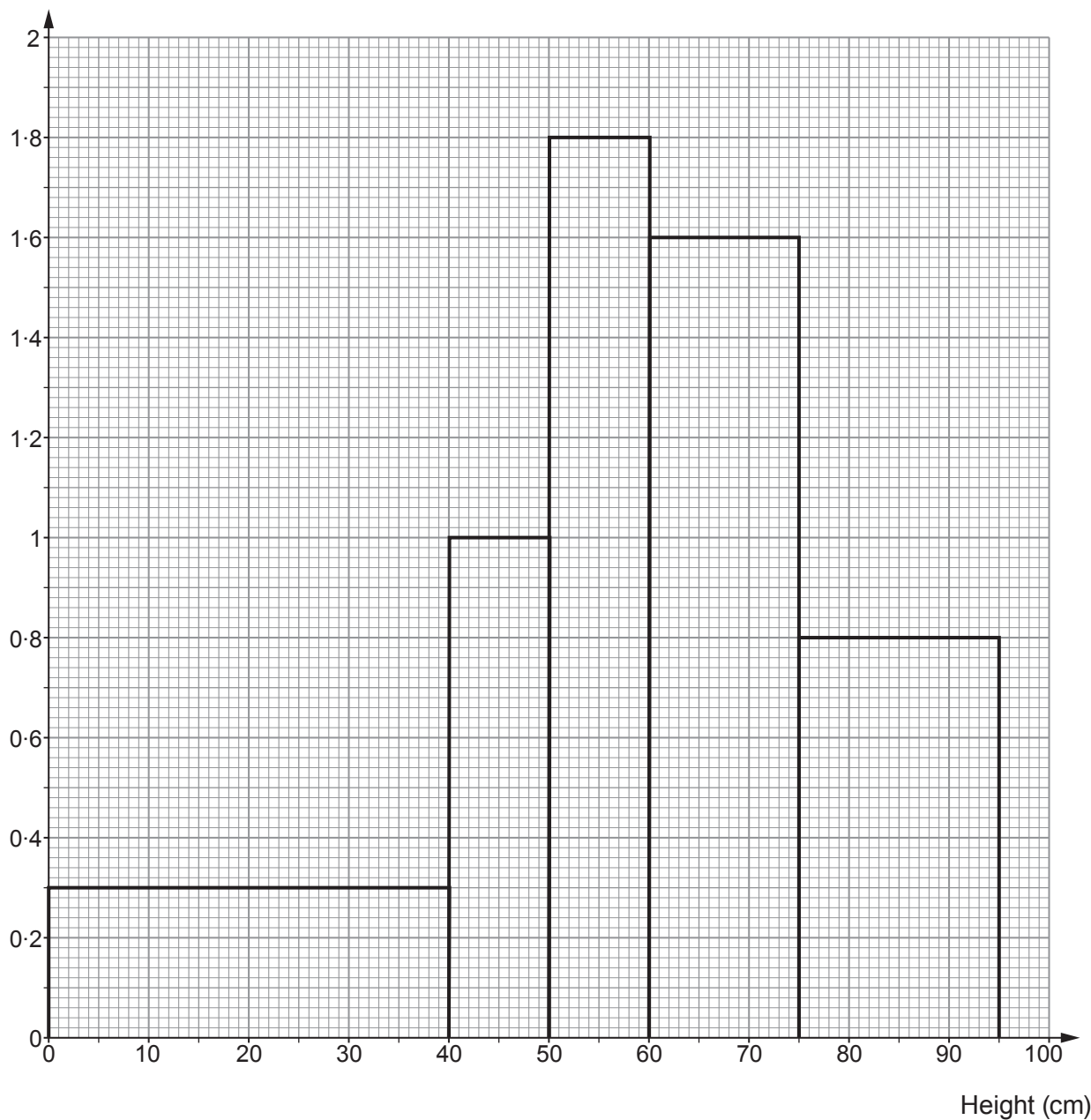
The modified enclosure holds 95% of the original capacity.

Capacity of modified enclosure is 95 % of the capacity of original enclosure



8. Robert's Plants is a company that grows ornamental trees to sell in its nursery. The histogram below shows the heights of the 80 trees in the nursery.

Frequency density



- (a) Trees that can be sold are at least 50 cm tall.
How many trees are **too short** to be sold?

[2]

Frequency = frequency density $\times$ class width

0 to 40 cm: $0.3 \times 40 = 12$ trees, 40 to 50 cm: $1 \times 10 = 10$ trees

Too short to be sold = $12 + 10 = 22$ trees



(b) Use the histogram to give the best estimates for the answers to the following questions.

- (i) What is the median height of the 80 trees?
Circle your answer.

[1]

47.5 cm

48 cm

50 cm

55 cm

60 cm

Median = 40.5th tree: 22 trees below 50 cm, 18 trees in 50 to 60 cm

- (ii) Calculate the inter-quartile range of the heights of the 80 trees.

[6]

Class frequencies: 0-40: 12, 40-50: 10, 50-60: $1.8 \times 10 = 18$,

60-75: $1.6 \times 15 = 24$, 75-95: $0.8 \times 20 = 16$ (total 80)

Lower quartile = $80 \div 4 = 20$ th tree

12 trees below 40 cm, so need $20 - 12 = 8$ more from 40-50 class

(frequency density 1, so 8 trees cover $8 \div 1 = 8$ cm)

Lower quartile = $40 + 8 = 48$ cm

Upper quartile = $3 \times 80 \div 4 = 60$ th tree

$12 + 10 + 18 = 40$ trees below 60 cm, so need $60 - 40 = 20$ more

from 60-75 class: $20 \div 1.6 = 12.5$ cm

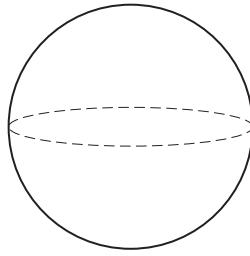
Upper quartile = $60 + 12.5 = 72.5$ cm

Inter-quartile range = $72.5 - 48$

= 24.5 cm



9. (a) Medi-Cap is a company that makes hollow spherical capsules for medicine.



One of its spherical capsules has a volume of $128\pi \text{ mm}^3$.

Calculate the radius of this capsule.

Give your answer in the form $a\sqrt[3]{12}$, where a is an integer.

You must show all your working.

[4]

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 = 128\pi$$

$$r^3 = 128 \times \frac{3}{4}$$

$$r^3 = 32 \times 3 = 96$$

$$r = \sqrt[3]{96} = \sqrt[3]{(8 \times 12)} = \sqrt[3]{8} \times \sqrt[3]{12}$$

$$r = 2\sqrt[3]{12} \text{ mm}$$

$$\text{Radius of the capsule} = 2\sqrt[3]{12} \text{ mm}$$



- (b) Medi-Cap also makes another shape of capsule, called a Bullet capsule.



These capsules are in the shape of two hollow hemispheres attached to each end of a hollow cylinder, as shown below.

The diameter of the cylinder, and of each hemisphere, is 8 mm.

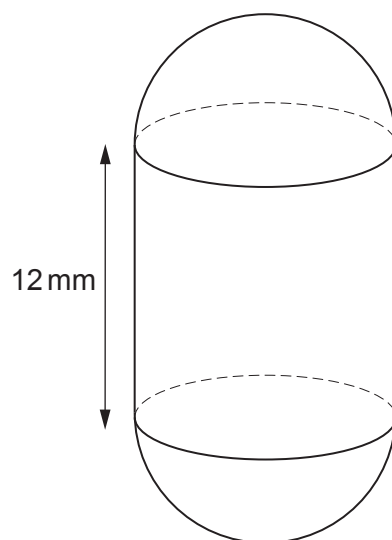


**Diagram not
drawn to scale**

To make a capsule easier to swallow, the exterior surface is coated with a glaze.
Calculate the surface area that is covered by the glaze.
Give your answer in terms of π in its simplest form.

[3]

$$\text{Radius} = 8 \div 2 = 4 \text{ mm}$$

$$\text{Curved surface of cylinder} = \pi dh = \pi \times 8 \times 12 = 96\pi \text{ mm}^2$$

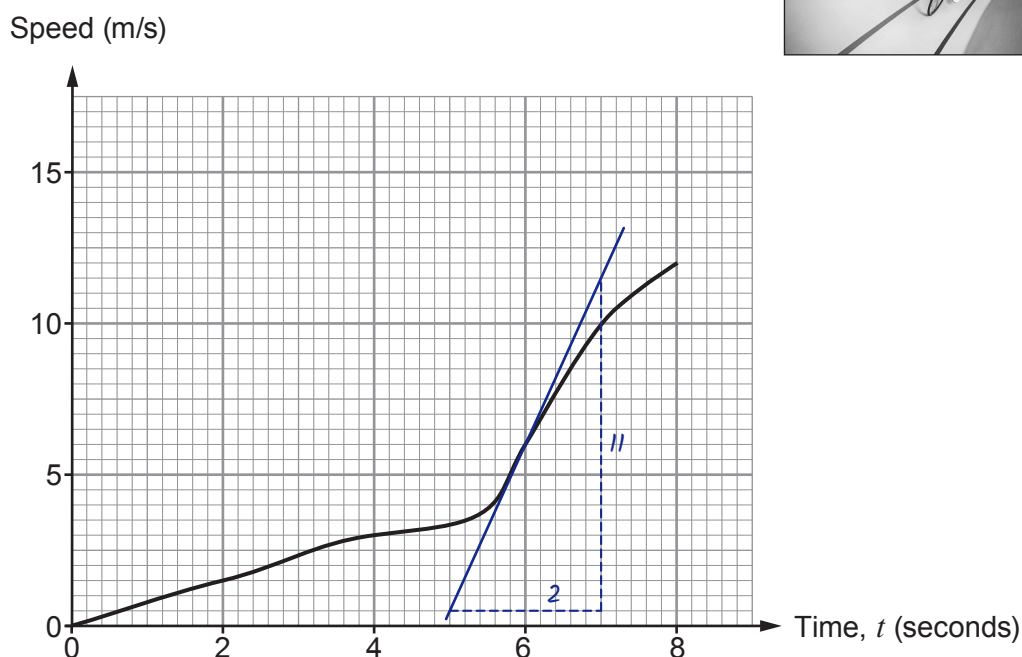
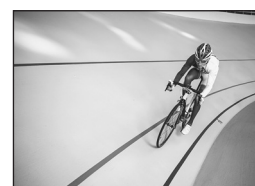
$$2 \text{ hemispheres} = 1 \text{ sphere: } 4\pi r^2 = 4 \times \pi \times 4^2 = 64\pi \text{ mm}^2$$

$$\text{Surface area covered by glaze} = 96\pi + 64\pi$$

$$= 160\pi \text{ mm}^2$$



10. Eliska competed in an indoor cycle race. The speed-time graph below shows Eliska's speed during the first 8 seconds of her race.



- (a) Estimate the greatest acceleration that Eliska achieved during the first 8 seconds of her race.
Give your answer in its simplest form. [4]

Steepest at $t \approx 5.8$ s: tangent through (5, 0.5) and (7, 11.5)

$$\text{Gradient} = \frac{\text{change in speed}}{\text{change in time}} = \frac{11.5 - 0.5}{7 - 5} = \frac{11}{2}$$

$$\text{Greatest acceleration} \approx \frac{11}{2} \text{ m/s}^2$$

- (b) Eliska's initial acceleration was $0.\dot{7}2 \text{ m/s}^2$.
Write $0.\dot{7}2$ as a fraction. [2]

$$x = 0.727272\ldots$$

$$100x = 72.727272\ldots$$

$$100x - x = 72.7272\ldots - 0.7272\ldots$$

$$99x = 72$$

$$x = \frac{72}{99} = \frac{8}{11}$$



- (c) Use the trapezium rule with exactly **4 strips** of equal width to estimate the distance Eliska travelled during the first 8 seconds of the race.

[3]

$$\text{Strip width } h = 8 \div 4 = 2 \text{ s}$$

$$\text{Speeds at } t = 0, 2, 4, 6, 8: 0, 1.5, 3, 6, 12 \text{ m/s}$$

$$\text{Distance} \approx \frac{h}{2} \times [\text{first} + \text{last} + 2 \times (\text{middle values})]$$

$$= \frac{1}{2} \times 2 \times (0 + 12 + 2 \times (1.5 + 3 + 6))$$

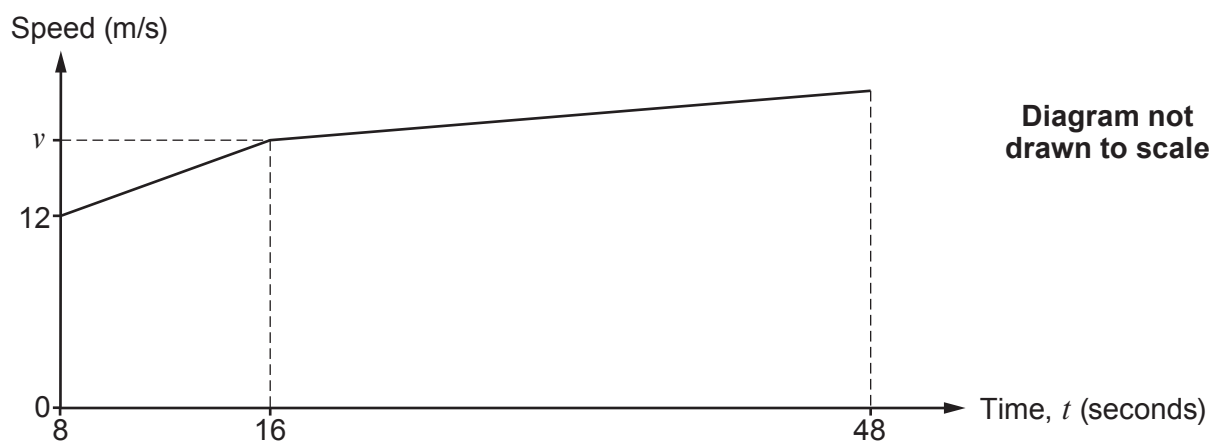
$$= 1 \times (12 + 2 \times 10.5) = 12 + 21$$

$$= 33 \text{ m}$$

Eliska travelled approximately 33 m.



- (d) The speed-time graph below shows Eliska's speed from $t = 8$ seconds to $t = 48$ seconds.
The graph shows that her speed increased at a constant rate from $t = 8$ to $t = 16$, and at a different constant rate from $t = 16$ to $t = 48$.



Eliska's speed at $t = 48$ was 1 m/s faster than her speed at $t = 16$.

From $t = 8$ to $t = 48$, Eliska travelled 550 m.

Form and solve an equation to calculate Eliska's speed at time $t = 16$ seconds.

[4]

Let v = speed at $t = 16$. Speed at $t = 48$ is $v + 1$. Distance = area.

$$t = 8 \text{ to } 16: \frac{1}{2} \times (12 + v) \times 8 = 4(12 + v) = 48 + 4v$$

$$t = 16 \text{ to } 48: \frac{1}{2} \times (v + v + 1) \times 32 = 16(2v + 1) = 32v + 16$$

$$48 + 4v + 32v + 16 = 550$$

$$36v + 64 = 550$$

$$36v = 486$$

$$v = \frac{486}{36} = \frac{27}{2} = 13.5$$

Eliska's speed at time $t = 16$ seconds is 13.5 m/s

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