

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
Other Names		0



GCSE

3310U60-1



**MATHEMATICS – NUMERACY
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER**

THURSDAY, 10 MAY 2018 – MORNING

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page at the back of the booklet. Question numbers must be given for the work written on the continuation page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 1(a), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

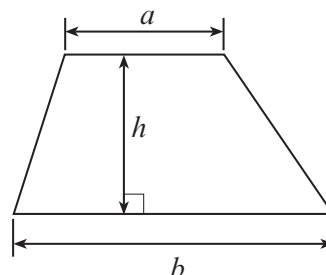
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	7	
2.	6	
3.	9	
4.	8	
5.	5	
6.	7	
7.	9	
8.	5	
9.	6	
10.	10	
11.	8	
Total	80	



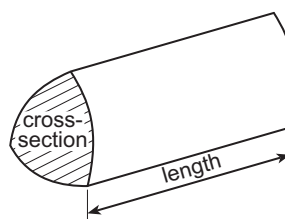
MAY183310U60101

Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

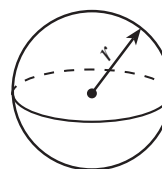


Volume of prism = area of cross-section $\times$ length



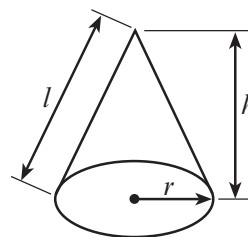
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

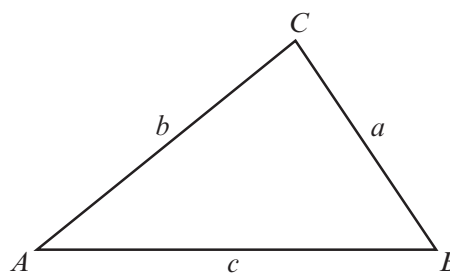


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

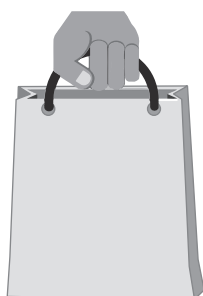


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1. In October 2011, a charge of 5p for a carrier bag was introduced in Wales. Money raised from this charge is given to charity.



For the period 1st October 2011 to 31st January 2015, it was estimated that a total of between £16.8 million and £21.9 million was donated to charity. This is as a result of people buying 5p carrier bags.

- (a) In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Calculate an estimate of how much per month was given to charity between 1st October 2011 and 31st January 2015.

You must show all your working.

[4 + 2 OCW]

Period of time: 1st October 2011 to 31st January 2015

Oct 2011 to Oct 2014 = 3 years = 36 months

Oct 2014 to end of Jan 2015 = 4 months

Total time = 36 + 4 = 40 months

Amount donated: I will use the mean of the two estimates.

(£16.8 million + £21.9 million) ÷ 2 = £19.35 million

Amount given per month = total amount ÷ number of months

= £19.35 million ÷ 40

= £0.48375 million

= £483 750

So an estimate of the amount given to charity each month

is about £484 000 (£0.48 million) per month.



- (b) Over time, there has been a reduction in the use of 5p carrier bags. This is because more people are using their own bags.

What impact might this have had on the amount given to charity for the month of September 2014 when compared with September 2012? [1]

Less money would have been given to charity in September 2014

than in September 2012, because fewer 5p bags were bought.



2. (a) Megan and Rhodri both set out at the same time from home to go to the swimming pool. Rhodri travels by car. Megan cycles straight through the park.

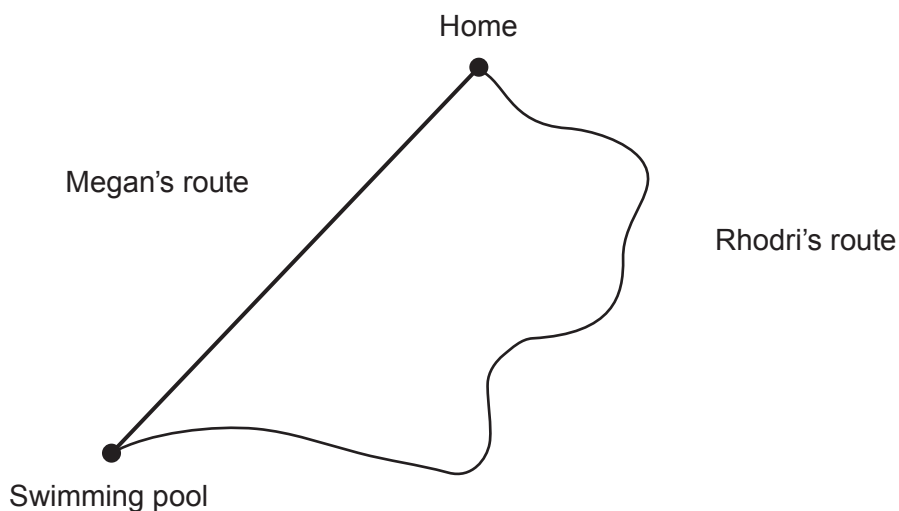


Diagram not drawn to scale

Rhodri's journey by car is 5.5 miles.
His average speed for the journey is 22 mph.

Megan's average speed on her bike is 12 mph.
Megan arrives at the swimming pool 5 minutes before Rhodri.

Calculate the distance Megan cycles.
Give your answer in miles.
You must show all your working.

[5]

$$\text{Rhodri's time} = \text{distance} \div \text{speed} = 5.5 \div 22 = 0.25 \text{ hours}$$

$$0.25 \text{ hours} = 0.25 \times 60 = 15 \text{ minutes}$$

$$\text{Megan's time} = 15 - 5 = 10 \text{ minutes}$$

$$10 \text{ minutes} = 10 \div 60 = 0.1666... \text{ hours}$$

$$\text{Megan's distance} = \text{speed} \times \text{time} = 12 \times 10 \div 60$$

$$= 2 \text{ miles}$$

Distance Megan cycles is 2 miles



- (b) Gary travelled a distance of 231 km in 3 hours and 30 minutes.
Calculate Gary's average speed in km/h.
Circle your answer.

[1]

0.015

1.1

66

70

77

3 hours 30 minutes = 3.5 hours, $231 \div 3.5 = 66$ km/h



3. Yared is going to make a door wedge.

- (a) The cross-section of the wedge is shown below.
The horizontal length is 12 cm and the vertical height is 3 cm.

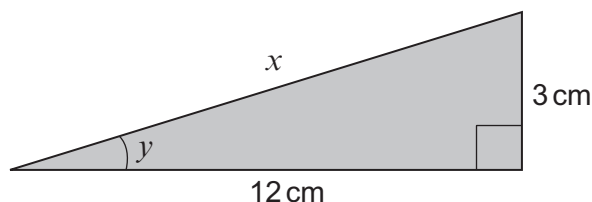


Diagram not drawn to scale

- (i) Calculate the length x .

Give your answer correct to 3 significant figures.

[4]

$$x^2 = 3^2 + 12^2$$

$$x^2 = 9 + 144 = 153$$

$$x = \sqrt{153} = 12.36931688\dots$$

$$x = 12.4 \text{ cm (3 s.f.)}$$

$$x = \dots\dots\dots 12.4 \dots\dots\dots \text{ cm}$$

- (ii) The wedge must fit under Yared's door.
The angle y must be less than 15° .
Show that this wedge will fit under Yared's door.
You must show all your working.

[3]

$$\tan y = \text{opposite} \div \text{adjacent} = 3 \div 12 = 0.25$$

$$y = \tan^{-1}(0.25)$$

$$y = 14.03624347\dots^\circ$$

$$y = 14.0^\circ$$

$$14.0^\circ < 15^\circ, \text{ so the angle } y \text{ is less than } 15^\circ$$

and the wedge will fit under Yared's door.



- (b) Yared decides to make a larger wedge that is mathematically **similar** to the one shown in part (a).
This wedge is to have a vertical height of 4.5 cm.

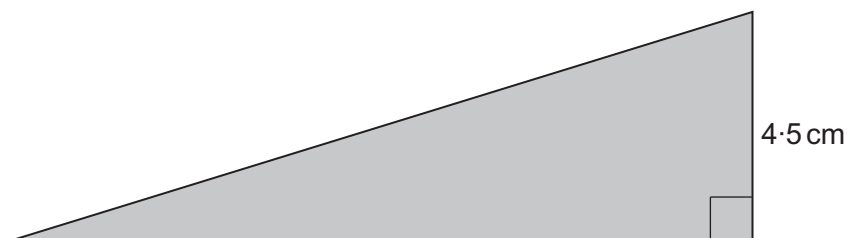


Diagram not drawn to scale

Calculate the horizontal length of this door wedge.

[2]

$$\text{Scale factor} = 4.5 \div 3 = 1.5$$

$$\text{Horizontal length} = 12 \times 1.5$$

$$= 18 \text{ cm}$$

The wedge will be 18 cm long



4. A grass racetrack is shown in the diagram below.
 This is the region shaded in the diagram.
 Each end of the grass racetrack is created from semicircles.
 The inner semicircles have a radius of 15 m.
 The outer semicircles have a radius of 20 m.
 Each of the straight sections of the racetrack has a length of 65 metres.

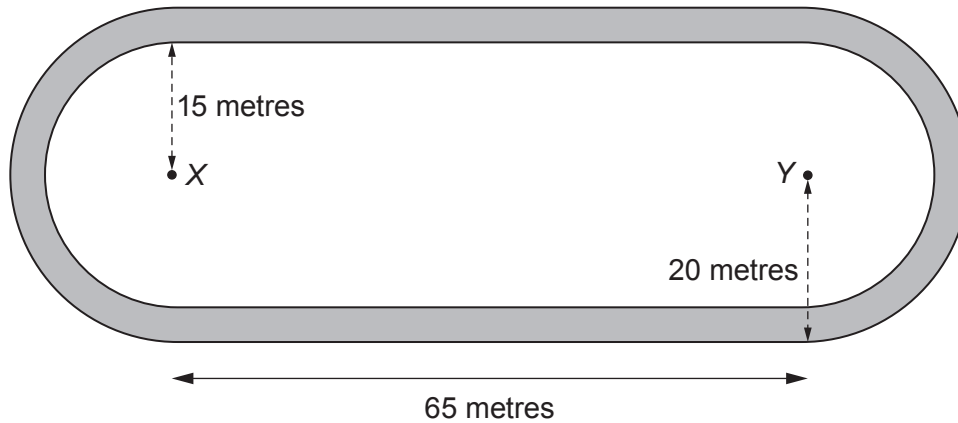


Diagram not drawn to scale

- (a) What is the total area of grass in the two **straight** sections of the racetrack?
 You must show all your working.

[2]

$$\text{Area of one straight section} = 65 \times 5 = 325 \text{ m}^2 \quad (\text{width} = 20 - 15 = 5 \text{ m})$$

$$\text{Total area of two straight sections} = 2 \times 325 = 650 \text{ m}^2$$



- (b) Calculate the area of the grass racetrack.
You must show all your working.

[4]

Both ends together make one full ring (two semicircles each end).

$$\text{Area of outer circle} = \pi \times 20^2 = 400\pi = 1256.637... \text{ m}^2$$

$$\text{Area of inner circle} = \pi \times 15^2 = 225\pi = 706.858... \text{ m}^2$$

$$\text{Area of both ends} = 1256.637... - 706.858...$$

$$= 175\pi = 549.778... \text{ m}^2$$

$$\text{Area of racetrack} = 549.778... + 650$$

$$= 1199.778... \text{ m}^2$$

$$\approx 1200 \text{ m}^2$$

- (c) The grass is to be treated with a fertiliser.
It costs 20p to treat each 3 m² of grass.
How much will it cost to treat the grass racetrack?
Give your answer correct to the nearest pound.
You must show all your working.

[2]

$$\text{Cost} = 1199.778... \div 3 \times 20\text{p} = 7998.52... \text{p}$$

$$= £79.985... = £80 \text{ to the nearest pound}$$

Cost is £ 80



5. Hot water is often stored in cylinders.
The water in the cylinder is heated for use in the shower.



A plumbing engineer wants to calculate how long a shower can be used continuously before the water runs cold. He uses the following formulae:

$$C = \frac{H(X - M)}{M - Y} \quad \text{and} \quad T = \frac{C + H}{F}$$

Where:

C is the additional volume of water that feeds into the cylinder, in litres.

H is the volume of hot water that the cylinder holds, in litres.

M is the temperature of the water in the shower, in °C.

X is the temperature of the hot water in the cylinder, in °C.

Y is the temperature of the cold water that feeds into the cylinder, in °C.

T is the time spent using the shower before the water runs cold, in minutes.

F is the rate of flow of water in the shower, in litres per minute.

Daisy's cylinder holds 300 litres of hot water.

The temperature of the hot water in her cylinder is 60°C.

The temperature of the cold water that feeds into Daisy's cylinder is 8°C.

The water in Daisy's shower is set at a temperature of 32°C.

Her shower has a rate of flow of 26 litres per minute.



Use the formulae to calculate

- the additional volume of water that feeds into Daisy's cylinder, in litres,
- the number of minutes Daisy's shower will run continuously before the water runs cold.

[5]

$$H = 300, X = 60, Y = 8, M = 32, F = 26$$

Additional volume of water:

$$C = H(X - M) \div (M - Y)$$

$$C = 300 \times (60 - 32) \div (32 - 8)$$

$$C = 300 \times 28 \div 24$$

$$C = 8400 \div 24$$

$$C = 350 \text{ litres}$$

Time before the water runs cold:

$$T = (C + H) \div F$$

$$T = (350 + 300) \div 26$$

$$T = 650 \div 26$$

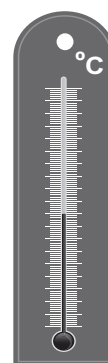
$$T = 25 \text{ minutes}$$

The additional volume of water feeding into Daisy's cylinder is 350 litres
and her shower will run for 25 minutes before the water runs cold.



6. Dr Khan and her daughter Faryl have different opinions about the mean temperature in their hallway.

Dr Khan and Faryl recorded the temperature in the hallway at 4 p.m. each day during the 30 days of April.



- (a) In her note pad, Dr Khan summarised the temperatures in a grouped frequency table.

Unfortunately, Dr Khan has torn the page containing the table from her note pad and has lost some of the original data.

Temperature, t ($^{\circ}\text{C}$)	Number of days
$20 \leq t < 21$	4
$21 \leq t < 22$	8
$22 \leq t < 23$	8
$23 \leq t < 24$	10

Calculate an estimate of the mean temperature at 4 p.m. for these 30 days in Dr Khan's hallway. [5]

$$\text{Missing number of days} = 30 - (4 + 8 + 8) = 30 - 20 = 10$$

$$\text{Midpoints: } 20.5, 21.5, 22.5, 23.5$$

$$20.5 \times 4 = 82 \quad 21.5 \times 8 = 172$$

$$22.5 \times 8 = 180 \quad 23.5 \times 10 = 235$$

$$\text{Total} = 82 + 172 + 180 + 235 = 669$$

$$\text{Mean} = 669 \div 30 = 22.3^{\circ}\text{C}$$

Estimate of the mean temperature at 4 p.m. for April in the hallway is 22.3 $^{\circ}\text{C}$



- (b) What assumption have you made in calculating an estimate of the mean temperature at 4 p.m. for April in Dr Khan's hallway? [1]

I assumed that the midpoint of each group represents all the temperatures in that group (e.g. 20.5 °C for $20 \leq t < 21$).

- (c) Faryl recorded the same temperatures as her mother at 4 p.m. each day during April. She found that the **actual** mean temperature in the hallway during April was **lower** than the correctly calculated estimate of the mean.

Explain how this can be true.

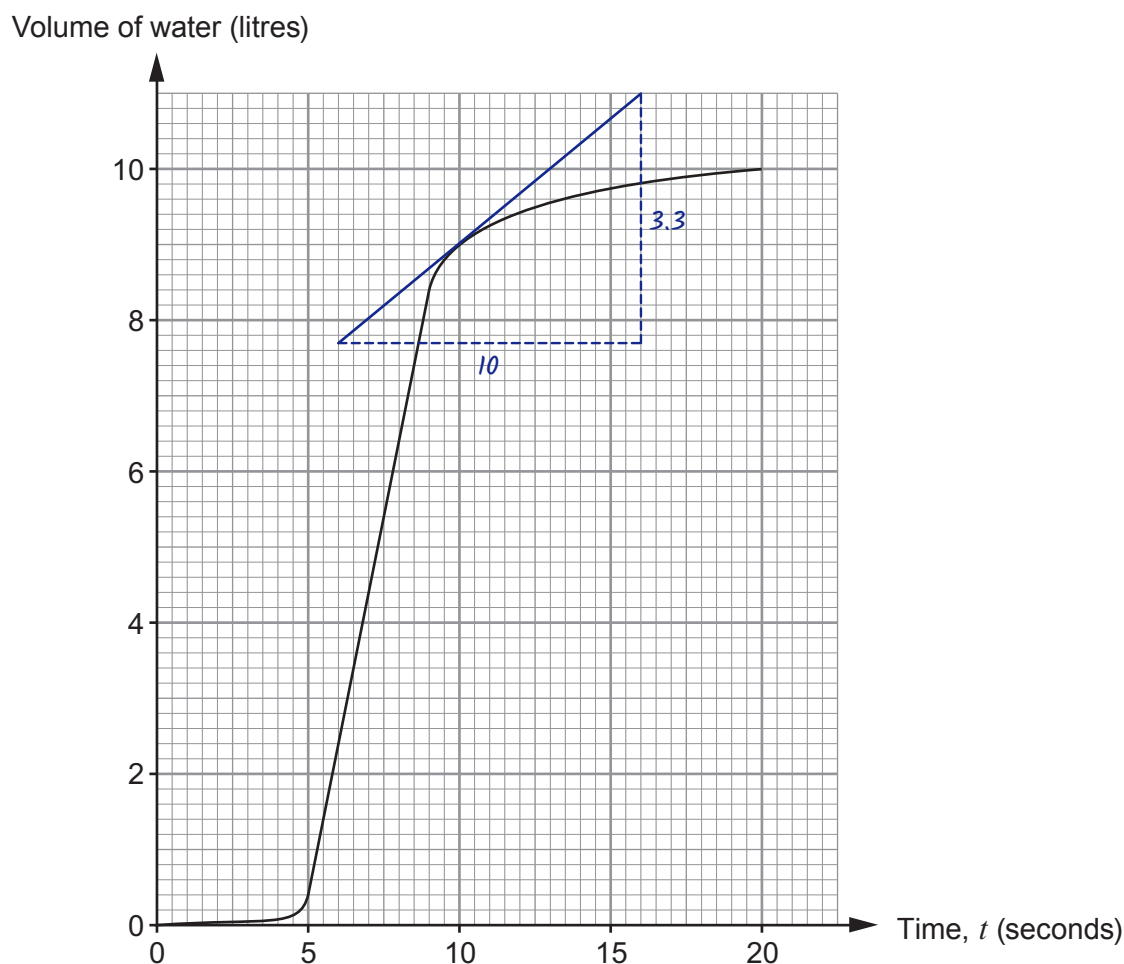
[1]

Many of the actual temperatures were less than the midpoint of the group they were in, i.e. they were often towards the lower end of each group, so the actual mean is lower than the estimate.



7. Gwen fills a 10-litre bucket with water from a tap. She turns the tap until it is fully open. The bucket fills up with water, and when Gwen thinks it is close to being full, she slowly closes the tap. The bucket is full after 20 seconds.

The graph below shows the volume of water in the bucket during the 20 seconds.



- (a) After how many seconds did Gwen start to close the tap?

[1]

9 seconds

- (b) Estimate at what rate water is entering the bucket at time $t = 10$ seconds. Give your answer in litres per second.

[3]

Tangent drawn at $t = 10$, passing through (6, 7.7) and (16, 11)

Gradient = increase in volume $\div$ increase in time

$$= (11 - 7.7) \div (16 - 6) = 3.3 \div 10$$

$$= 0.33 \text{ litres per second}$$



(c) When the tap is fully open, water flows out at 2 litres per second.

- (i) Express 2 litres per second in **gallons per minute**.
You must show all your working.

[3]

Remember
1 gallon = 8 pints

1 litre $\approx$ 1.75 pints

2 litres per second = $2 \times 60 = 120$ litres per minute

120 litres = $120 \times 1.75 = 210$ pints

210 pints = $210 \div 8 = 26.25$ gallons

So 2 litres per second = 26.25 gallons per minute

- (ii) Can a fully open tap fill a 90-gallon tank in under $3\frac{1}{2}$ minutes?
You must show all your working.

[2]

Time = $90 \div 26.25 = 3.428571\dots$ minutes

*3.43 minutes < 3.5 minutes, so yes, a fully open tap can fill
the 90-gallon tank in under $3\frac{1}{2}$ minutes.*



8. The Headteacher of Ysgol Castell Gwyn wants to display pictures, drawn by pupils, along one side of a corridor. The pictures are to be in one row with no gaps between them, as shown in the diagram below.



Diagram not drawn to scale

The pictures are all square, with sides of length 15 cm, correct to the **nearest 0.5 cm**. The length of the corridor wall is 2010 cm, correct to the **nearest 10 cm**.

Calculate the smallest number of pictures and the greatest number of pictures that can be fitted in the row. [5]

Picture side: lower bound = 14.75 cm, upper bound = 15.25 cm

Wall length: lower bound = 2005 cm, upper bound = 2015 cm

Smallest number = shortest wall $\div$ largest picture

= $2005 \div 15.25 = 131.475...$ so 131 whole pictures

Greatest number = longest wall $\div$ smallest picture

= $2015 \div 14.75 = 136.610...$ so 136 whole pictures

Smallest number of pictures	Greatest number of pictures
<i>131</i>	<i>136</i>



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9. (a) Circle either TRUE or FALSE for each statement given below.

[2]

STATEMENT		
A nominal annual interest rate is not the same as an AER.	TRUE	FALSE
A savings account offers a nominal annual interest rate of 2%, with interest paid monthly. After a year, any investment will have increased in value by exactly 2%.	TRUE	FALSE
A savings account offers an AER of 2.4%, with interest paid monthly. The monthly interest rate the account offers will be exactly 0.2%.	TRUE	FALSE
£100 is invested in a savings account that pays monthly interest at a rate of 1%. There are no further transactions into or out of the account. The amount in the account after a year will be £112.	TRUE	FALSE

- (b) Benjamin invests £1000 into an account that pays interest every 6 months. He does not make any further payments into the account, and does not withdraw any money either.
After a year, there is £1036 in the account.

Calculate how much was in the account after 6 months.
Give your answer correct to the nearest penny.
You must show all your working.

[4]

Let the multiplier for each 6 months be m .

$$1000 \times m^2 = 1036$$

$$m^2 = 1036 \div 1000 = 1.036$$

$$m = \sqrt{1.036} = 1.017840852...$$

$$\text{Amount after 6 months} = 1000 \times 1.017840852...$$

$$= £1017.840852...$$

$$= £1017.84 \text{ (to the nearest penny)}$$



10. (a) A company makes plastic shelf supports for use in kitchen cupboards.
A shelf support is made by attaching a cylinder to a right-angled triangular prism.

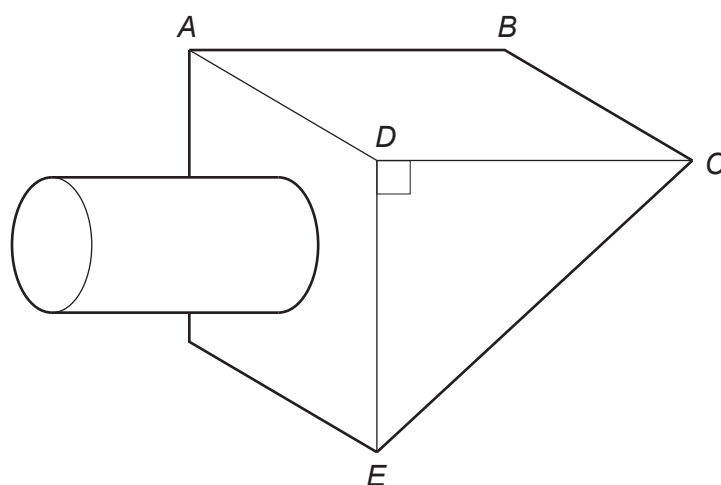


Diagram not drawn to scale

The cylinder has a diameter of 6 mm and a length of 9 mm.
The prism has dimensions $CD = 8$ mm, $DE = 8$ mm, $CE = 11.3$ mm and $BC = 10$ mm.

The company sells the shelf supports in packs of 500.
It needs to know the volume of plastic in 500 shelf supports.
Calculate the volume of 500 shelf supports.

[4]

$$\text{Cylinder: radius} = 6 \div 2 = 3 \text{ mm}$$

$$\text{Volume of cylinder} = \pi \times 3^2 \times 9 = 81\pi = 254.469... \text{ mm}^3$$

$$\text{Triangle CDE area} = \frac{1}{2} \times 8 \times 8 = 32 \text{ mm}^2$$

$$\text{Volume of prism} = 32 \times 10 = 320 \text{ mm}^3$$

$$\text{Volume of one support} = 254.469... + 320 = 574.469... \text{ mm}^3$$

$$\text{Volume of 500 supports} = 574.469... \times 500$$

$$= 287\,234.5... \text{ mm}^3$$

$$\approx 287\,235 \text{ mm}^3$$

$$\text{Volume of 500 shelf supports} = \underline{\underline{287\,235 \text{ mm}^3}}$$



(b)



The company also makes metal door handles for kitchen cupboards. One of the door handles it makes is shown below. It is formed by joining two cylinders. One of the cylinders has a diameter of 4 cm and a length of 1.2 cm. The other cylinder has a diameter of 1.8 cm and a length of 3 cm.

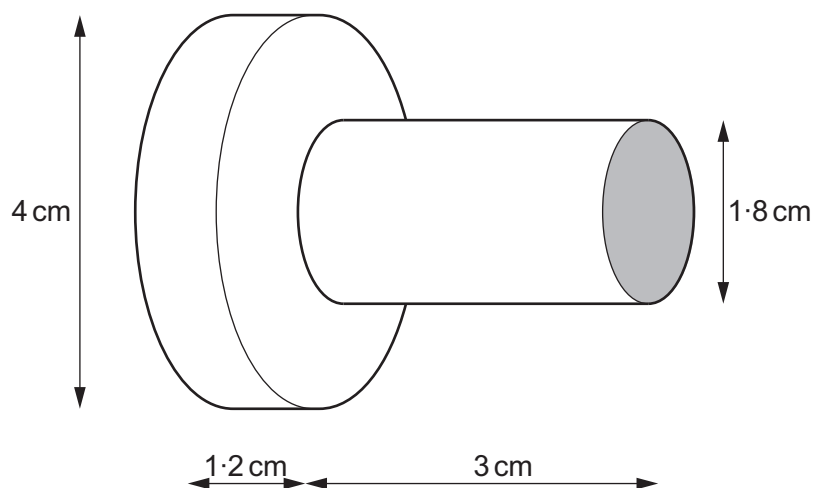


Diagram not drawn to scale

At present, the company paints **all** the surfaces of the handle with a protective finish **after** the two cylinders have been joined together.

The shaded circular face is pressed against a cupboard door when fitted. In future, the company is not going to paint this shaded circular face. This is to reduce costs.

Calculate the percentage reduction in the area that is painted.

[6]

Large cylinder: radius 2 cm, length 1.2 cm

Small cylinder: radius 0.9 cm, length 3 cm

Two circular faces of large cylinder = $2 \times \pi \times 2^2 = 8\pi = 25.132... \text{ cm}^2$

(the joined part of the face is replaced by the small end face)

Curved surface of large cylinder = $\pi \times 4 \times 1.2 = 4.8\pi = 15.079... \text{ cm}^2$



$$\text{Curved surface of small cylinder} = \pi \times 1.8 \times 3 = 5.4\pi = 16.964... \text{ cm}^2$$

$$\begin{aligned} \text{Total area painted at present} &= 8\pi + 4.8\pi + 5.4\pi \\ &= 18.2\pi = 57.176... \text{ cm}^2 \end{aligned}$$

$$\text{Area of shaded circular face} = \pi \times 0.9^2 = 0.81\pi = 2.544... \text{ cm}^2$$

$$\begin{aligned} \text{Percentage reduction} &= 2.544... \div 57.176... \times 100 \\ &= 0.81\pi \div 18.2\pi \times 100 \\ &= 4.450549... \% \\ &\approx 4.45\% \end{aligned}$$

The area that is painted is reduced by about 4.45%.



11. A sensor can detect any movement up to a distance of 6.5 m.

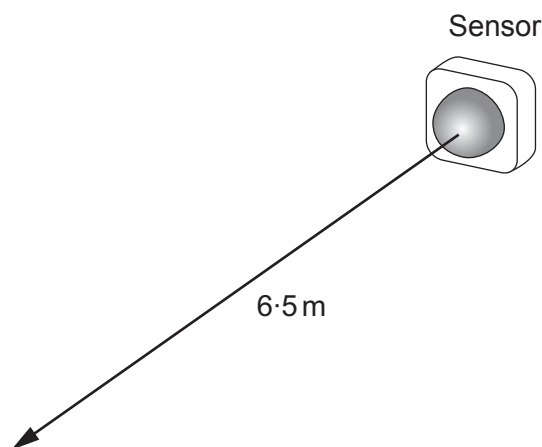


Diagram not drawn to scale

- (a) A storeroom is in the shape of a cuboid, as shown below.
The sensor is placed at A, so that
- it is aimed directly at B, where $BD = 2$ m,
 - the front of the sensor is 20 cm from A along the line AB.

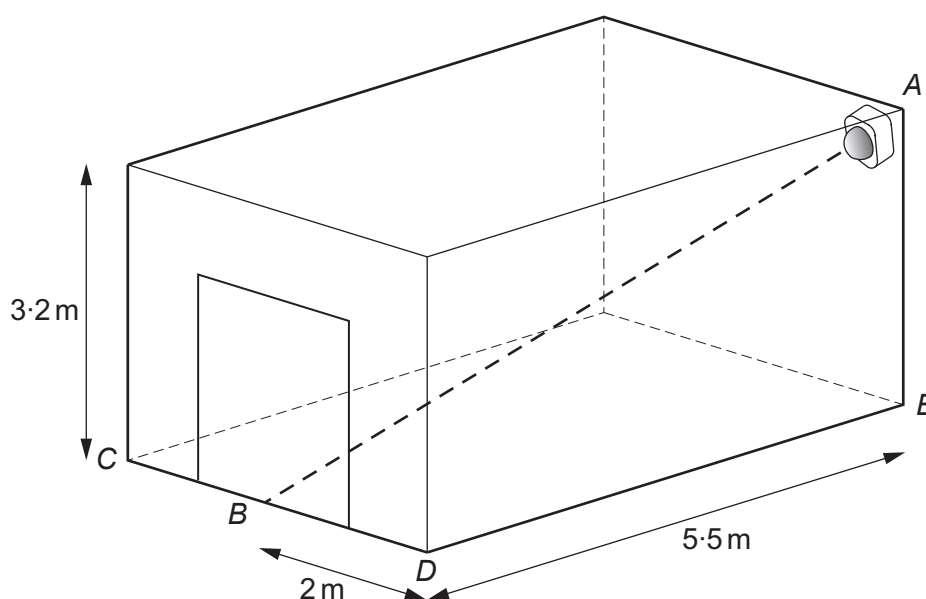


Diagram not drawn to scale

Will the sensor be able to detect movement at B?
You must show all your working.

[5]

$$BE^2 = BD^2 + DE^2 = 2^2 + 5.5^2 = 4 + 30.25 = 34.25$$

$$BE = \sqrt{34.25} = 5.852349\dots\text{m}$$

$$AB^2 = BE^2 + AE^2 = 34.25 + 3.2^2 = 34.25 + 10.24 = 44.49$$



$$AB = \sqrt{44.49} = 6.670082\dots\text{m} \approx 6.67\text{ m}$$

The front of the sensor is $20\text{ cm} = 0.2\text{ m}$ from A, so the
distance from the sensor to B $= 6.670082\dots - 0.2 = 6.470082\dots\text{m}$

$6.47\text{ m} < 6.5\text{ m}$, so yes, the sensor will be able to detect
movement at B.

(b) Show that $\hat{BAE} = 61.3^\circ$, correct to 1 decimal place.

[3]

Triangle ABE is right-angled at E.

$$\cos \hat{BAE} = AE \div AB = 3.2 \div 6.670082\dots$$

$$\cos \hat{BAE} = 0.479754\dots$$

$$\hat{BAE} = \cos^{-1}(0.479754\dots)$$

$$\hat{BAE} = 61.3306\dots^\circ$$

$$\hat{BAE} = 61.3^\circ \text{ (correct to 1 decimal place)}$$

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