

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U60-1



A23-3300U60-1

WEDNESDAY, 15 NOVEMBER 2023 – MORNING

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** questions.
Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the questions correctly.
Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
In question 3, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

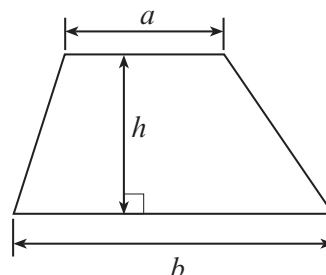
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	6	
3.	6	
4.	3	
5.	4	
6.	3	
7.	3	
8.	2	
9.	5	
10.	6	
11.	7	
12.	3	
13.	1	
14.	3	
15.	3	
16.	3	
17.	5	
18.	8	
19.	5	
Total	80	



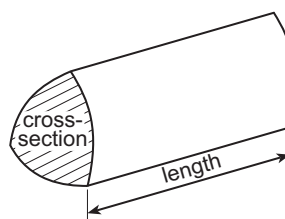
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

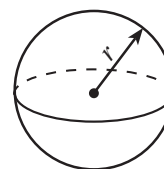


Volume of prism = area of cross-section $\times$ length



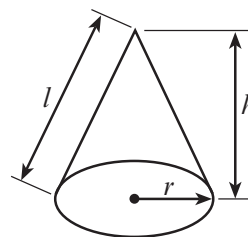
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

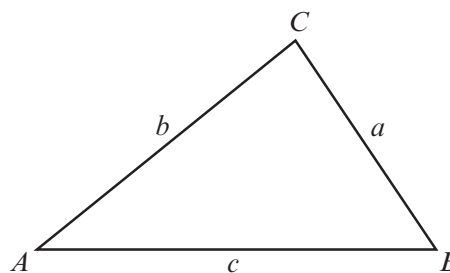


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. The table below shows some of the values of $y = x^2 + 4x + 5$ for values of x from -4 to 1 .

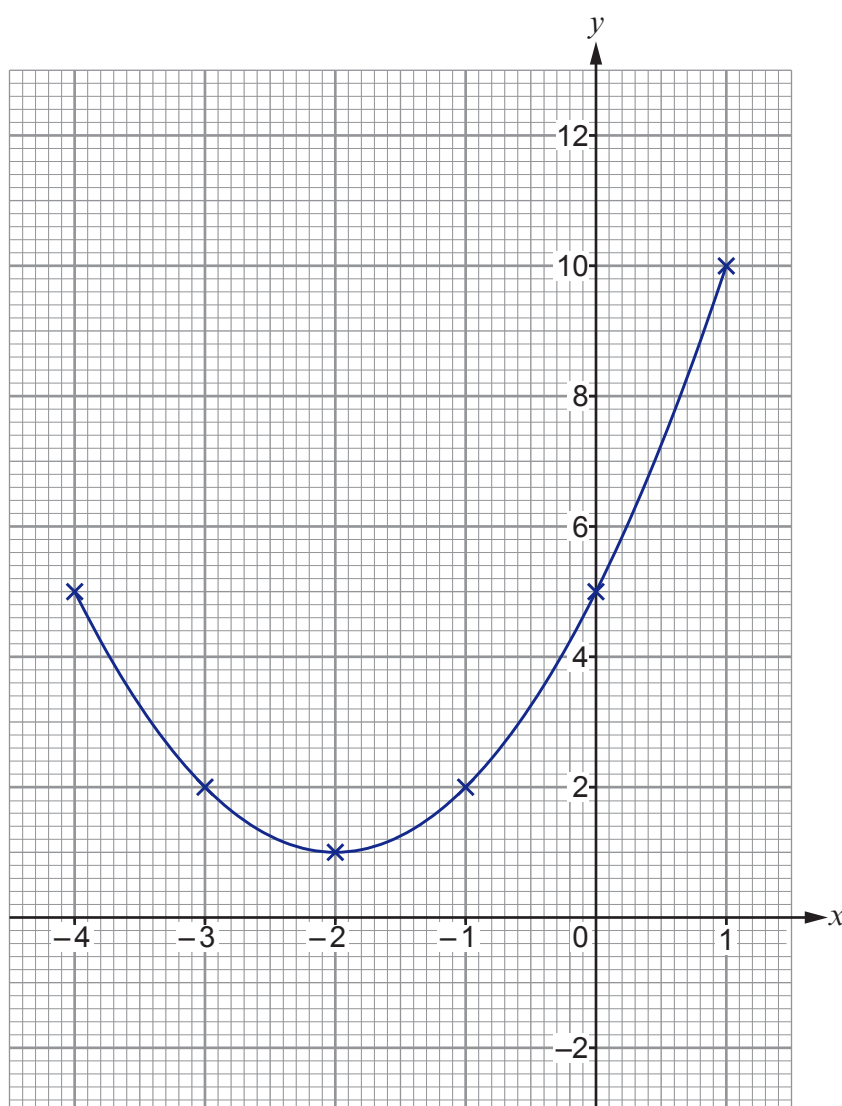
x	-4	-3	-2	-1	0	1
$y = x^2 + 4x + 5$	5	2	1	2	5	10

- (a) Complete the table by finding the value of y for $x = -2$ and for $x = 1$. [2]

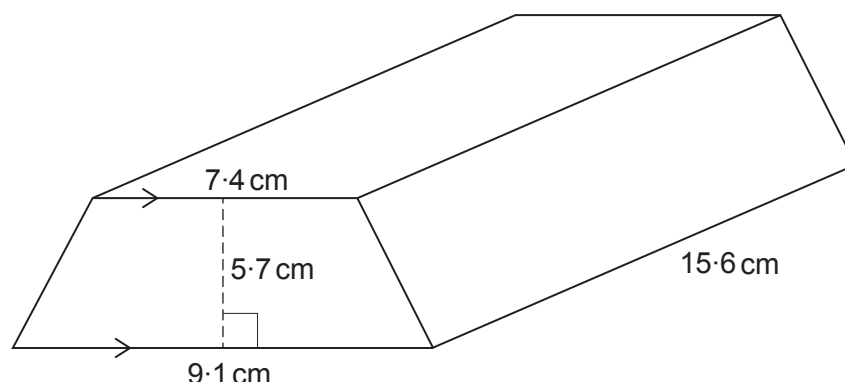
$$x = -2: y = (-2)^2 + 4 \times (-2) + 5 = 4 - 8 + 5 = 1$$

$$x = 1: y = 1^2 + 4 \times 1 + 5 = 1 + 4 + 5 = 10$$

- (b) On the graph paper below, draw the graph of $y = x^2 + 4x + 5$ for values of x from -4 to 1 . [2]



2.

*Diagram not drawn to scale*

- (a) Find the volume of the solid prism shown above.

[3]

$$\text{Area of cross-section} = \frac{7.4 + 9.1}{2} \times 5.7 = 47.025 \text{ cm}^2$$

$$\text{Volume} = \text{area of cross-section} \times \text{length}$$

$$\text{Volume} = 47.025 \times 15.6$$

$$= 733.59 \text{ cm}^3$$

- (b) The solid prism is made of gold.
Gold has a density of 19.3 g/cm^3 .

Calculate the mass of the prism.

Give your answer in **kilograms**.

[3]

$$\text{Mass} = \text{volume} \times \text{density}$$

$$\text{Mass} = 733.59 \times 19.3$$

$$= 14158.287 \text{ g}$$

$$1 \text{ kg} = 1000 \text{ g, so } 14158.287 \div 1000$$

$$= 14.158287 \text{ kg}$$

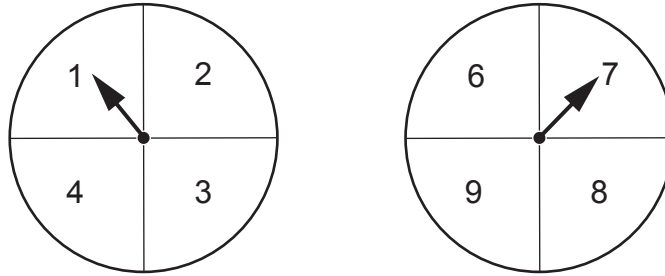
$$= 14.2 \text{ kg (to 3 s.f.)}$$

Mass of the prism = 14.2 kg



3. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Vera has two fair spinners.
Each spinner is divided into quarters.
One spinner shows the values 1, 2, 3 and 4.
The other spinner shows the values 6, 7, 8 and 9.



Vera spins the two spinners. She then multiplies the two values together to form a product.
For example, the diagram above forms the product $1 \times 7 = 7$.

Consider the ways in which all the possible products can be formed.
Calculate the probability that the spinners will form a product that is a factor of 36.
You must show all your working to justify your answer. [4 + 2 OCW]

Each spinner has 4 equally likely values, so there are $4 \times 4 = 16$ products:

x	6	7	8	9
1	6	7	8	9
2	12	14	16	18
3	18	21	24	27
4	24	28	32	36

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Products that are factors of 36 (ringed in the table):

$1 \times 6 = 6$, $1 \times 9 = 9$, $2 \times 6 = 12$, $2 \times 9 = 18$, $3 \times 6 = 18$, $4 \times 9 = 36$

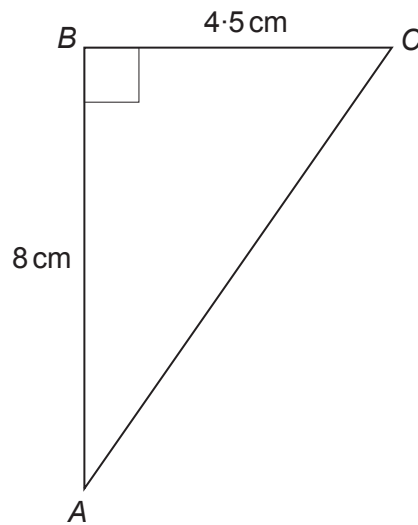
So 6 of the 16 equally likely products are factors of 36.

$$P(\text{product is a factor of 36}) = \frac{6}{16} = \frac{3}{8}$$

Therefore the probability that the spinners form a product that is a factor of 36 is $3/8$ ($= 0.375$).



4.

*Diagram not drawn to scale*

Calculate the length of the side AC.

[3]

By Pythagoras: $AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 4.5^2 = 64 + 20.25 = 84.25$

$AC = \sqrt{84.25} = 9.178779875...$

$AC = 9.18 \text{ cm (to 3 s.f.)}$



5. A solution of the equation

$$x^3 + 6x = 80$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 + 6x$	
3	$27 + 18 = 45$	too small
4	$64 + 24 = 88$	too big
3.5	$42.875 + 21 = 63.875$	too small
3.8	$54.872 + 22.8 = 77.672$	too small
3.9	$59.319 + 23.4 = 82.719$	too big
3.85	$57.066625 + 23.1 = 80.166625$	too big

3.8 gives less than 80 and 3.85 gives more than 80,

so the solution lies between 3.8 and 3.85.

$x = 3.8$ (to 1 d.p.)



6. The diagram below shows a shape made by joining two congruent rectangles together. The length of each rectangle is $(5x + 3)$ cm. The width of each rectangle is $(2x - 1)$ cm.

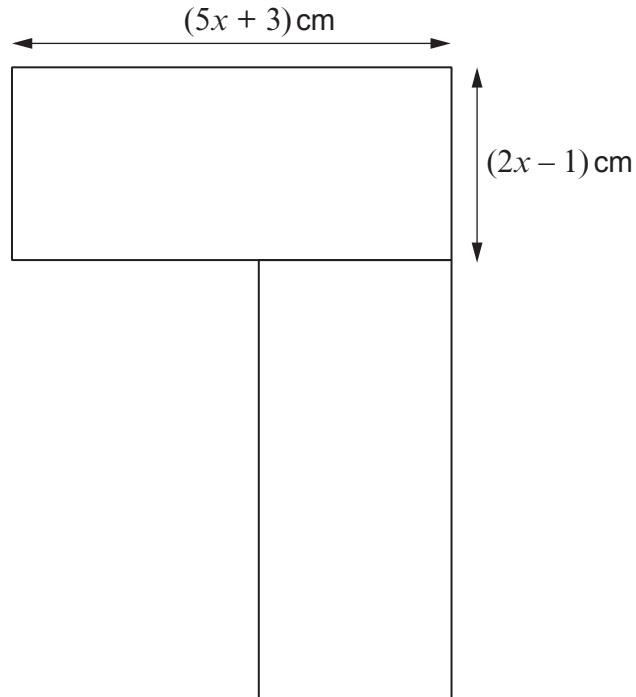


Diagram not drawn to scale

Write an expression for the total area of the shape in the form $ax^2 + bx + c$, where a , b and c are whole numbers. [3]

$$\text{Area of one rectangle} = (5x + 3)(2x - 1)$$

$$= 10x^2 - 5x + 6x - 3$$

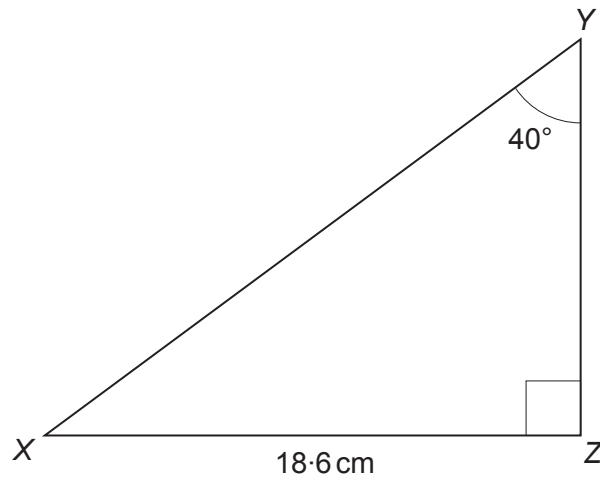
$$= 10x^2 + x - 3$$

$$\text{Total area} = 2 \times (10x^2 + x - 3)$$

$$\text{Total area} = 20x^2 + 2x - 6 \text{ (cm}^2\text{)}$$



7.

*Diagram not drawn to scale*

Calculate the length of the side YZ.

[3]

From angle Y: XZ is opposite and YZ is adjacent, so use tan.

$$\tan 40^\circ = \frac{\text{opp}}{\text{adj}} = \frac{18.6}{YZ}$$

$$YZ = 18.6 \div \tan 40^\circ = 18.6 \div 0.8390996312...$$

$$YZ = 22.16661682...$$

$$YZ = 22.2 \text{ cm (to 1 d.p.)}$$

8. 7 cubes are stacked on top of each other.

Each of these cubes has edges of length 60 mm, measured correct to the nearest millimetre.

Calculate the greatest possible height of this stack of 7 cubes.

[2]

$$\text{Upper bound of each edge} = 60 + 0.5 = 60.5 \text{ mm}$$

$$\text{Greatest height} = 7 \times 60.5$$

$$= 423.5 \text{ mm}$$



9. The grouped frequency table below shows data for 32 values of t . One of the frequencies in the table is represented by a .

t	Frequency
$20 \leq t < 30$	2
$30 \leq t < 40$	8
$40 \leq t < 50$	4
$50 \leq t < 60$	a
$60 \leq t < 70$	3
$70 \leq t < 80$	5

Find the estimated mean value of t for the 32 values.

[5]

$$\text{Total frequency: } 2 + 8 + 4 + a + 3 + 5 = 32$$

$$22 + a = 32, \text{ so } a = 10$$

$$\text{Midpoints: } 25, 35, 45, 55, 65, 75$$

$$\Sigma(f \times \text{midpoint}) = 25 \times 2 + 35 \times 8 + 45 \times 4 + 55 \times 10 + 65 \times 3 + 75 \times 5$$

$$= 50 + 280 + 180 + 550 + 195 + 375$$

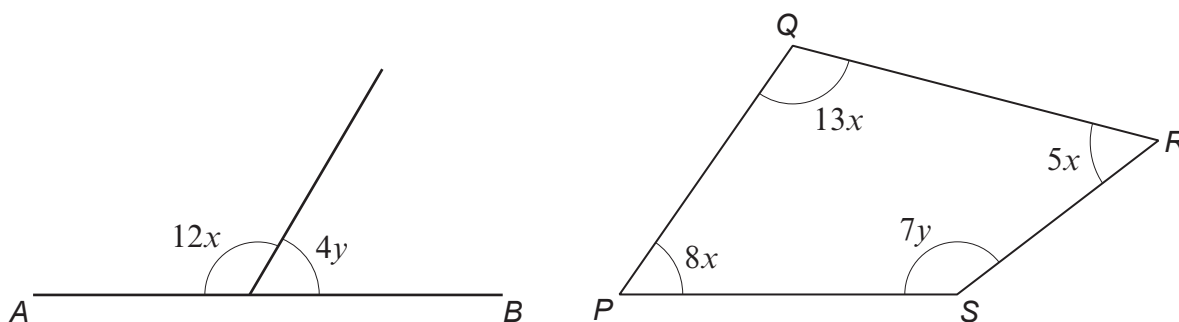
$$= 1630$$

$$\text{Estimated mean} = 1630 \div 32$$

$$\text{Estimated mean of } t = 50.9375$$



10. AB is a straight line.
 $PQRS$ is a quadrilateral.
 Each angle is given in terms of x or y .



Diagrams not drawn to scale

Use an algebraic method to find the value of x and the value of y .

[6]

Angles on a straight line add up to 180° :

$$12x + 4y = 180 \quad (1)$$

Angles in a quadrilateral add up to 360° :

$$13x + 5x + 8x + 7y = 360$$

$$26x + 7y = 360 \quad (2)$$

$$(1) \times 7: \quad 84x + 28y = 1260$$

$$(2) \times 4: \quad 104x + 28y = 1440$$

$$\text{Subtract:} \quad 20x = 180$$

$$x = 9$$

$$\text{Substitute } x = 9 \text{ into (1): } 12 \times 9 + 4y = 180$$

$$108 + 4y = 180$$

$$4y = 72, \text{ so } y = 18$$

$$\text{Check in (2): } 26 \times 9 + 7 \times 18 = 234 + 126 = 360 \checkmark$$

$$x = 9 \quad \text{and} \quad y = 18$$



11. (a) Factorise $6x^2 + 19x + 10$.

[2]

$$6 \times 10 = 60; 15 \times 4 = 60 \text{ and } 15 + 4 = 19$$

$$6x^2 + 15x + 4x + 10$$

$$= 3x(2x + 5) + 2(2x + 5)$$

$$= (2x + 5)(3x + 2)$$

- (b) Fully factorise the expression $m^3 - 25m$.

[3]

$$m^3 - 25m = m(m^2 - 25)$$

$$m^2 - 25 = m^2 - 5^2, \text{ a difference of two squares}$$

$$= m(m + 5)(m - 5)$$

- (c) Factorise $(p + 7)(p + 29) + 2(p + 7)$.

[2]

$$(p + 7) \text{ is a common factor:}$$

$$(p + 7)(p + 29) + 2(p + 7) = (p + 7)[(p + 29) + 2]$$

$$= (p + 7)(p + 31)$$



12. Triangle ABC has sides $AB = 36.1$ cm and $AC = 13.8$ cm, as shown below.
 $\angle BAC = 29^\circ$.

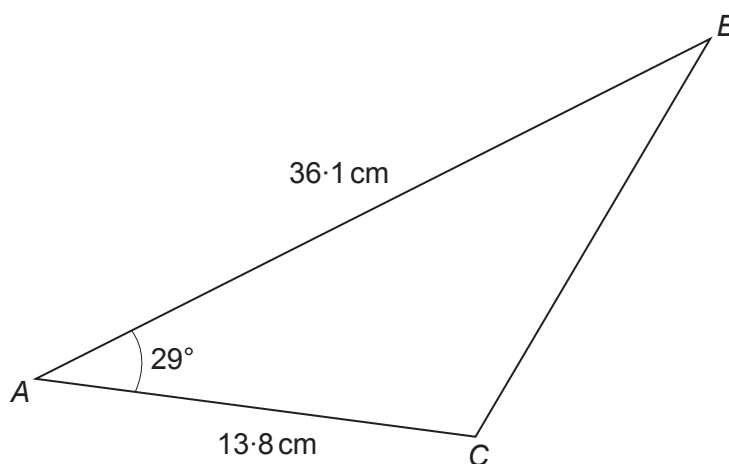


Diagram not drawn to scale

Calculate the length of the side BC.

[3]

Cosine rule: $BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos A$

$$BC^2 = 36.1^2 + 13.8^2 - 2 \times 36.1 \times 13.8 \times \cos 29^\circ$$

$$= 1303.21 + 190.44 - 871.4360914...$$

$$= 622.2139086...$$

$$BC = \sqrt{622.2139086...} = 24.94421593...$$

$$BC = 24.9 \text{ cm (to 1 d.p.)}$$

13. Calculate the cube root of 8×10^{216} .
 Circle the correct answer.

[1]

2×10^6

2×10^{72}

2×10^{216}

8×10^6

8×10^{72}

$$\sqrt[3]{8} = 2 \text{ and } (10^{216})^{1/3} = 10^{216 \div 3} = 10^{72}$$



14. A solid has a height of 225 cm.
A larger, mathematically **similar** solid has a height of 855 cm.
The surface area of the smaller solid is 5300 cm^2 .

Calculate the surface area of the larger solid.

[3]

$$\text{Length scale factor} = 855 \div 225 = 3.8$$

$$\text{Area scale factor} = 3.8^2 = 14.44$$

$$\text{Surface area of larger solid} = 5300 \times 14.44$$

$$= 76532 \text{ cm}^2$$



15. The values $b = 23$, $c = 0.73$ and $d = 8.3$ are each given correct to 2 significant figures.

Use the formula

$$a = b - \frac{c}{d}$$

to calculate the greatest possible value of a .
Give your answer correct to 2 decimal places.
You must show all your working.

[3]

Greatest a : use the greatest b and the smallest value of $c \div d$,

so use the lower bound of c and the upper bound of d .

Upper bound $b = 23.5$, lower bound $c = 0.725$, upper bound $d = 8.35$

$$a = 23.5 - \frac{0.725}{8.35}$$

$$= 23.5 - 0.08682634731...$$

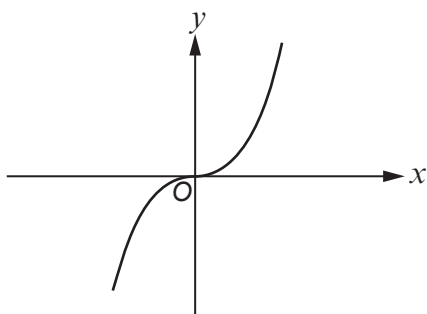
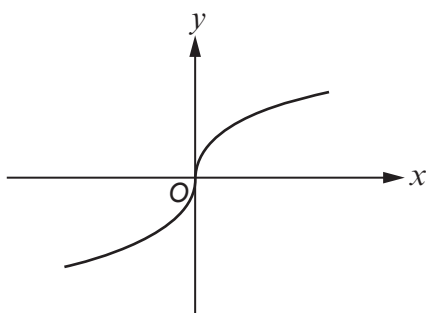
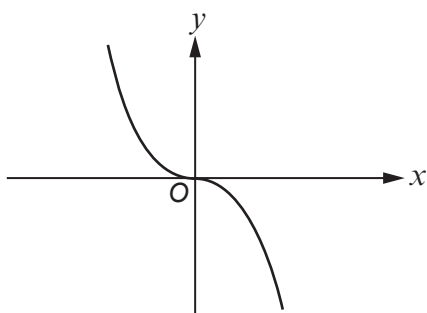
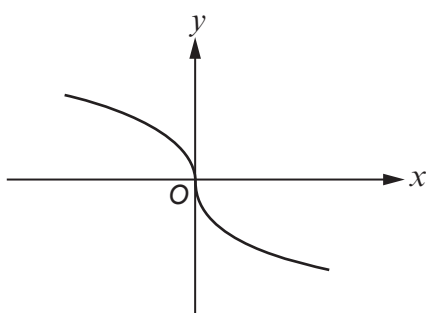
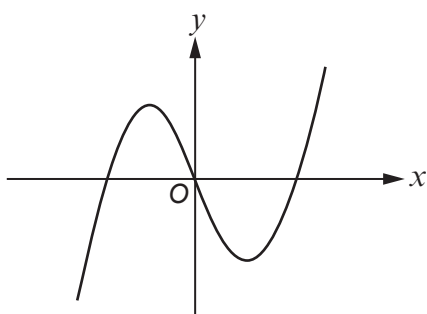
$$= 23.41317365...$$

Greatest possible value of $a = 23.41$ (to 2 d.p.)



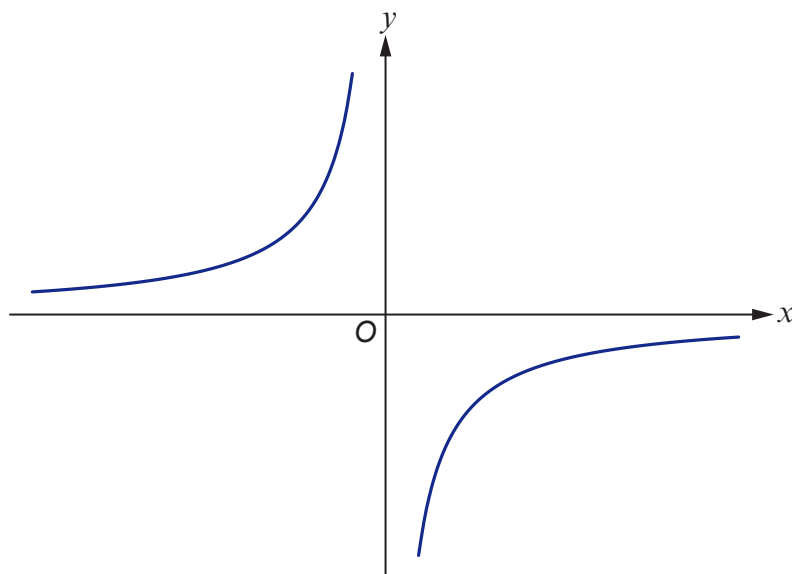
16. (a) Put a tick next to the graph that represents the equation $y = -x^3$.

[1]

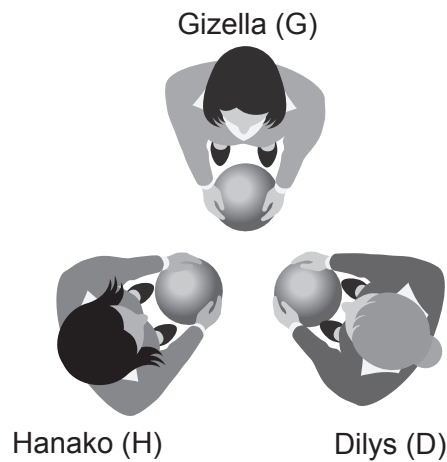
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- (b) Use the axes below to sketch the graph of $y = -\frac{1}{x}$.

[2]



17. Three children, Gizella (G), Dilys (D) and Hanako (H), stand in a triangle, as shown below.



Each child holds a ball.

They must choose to pass their ball to the child on their left or the child on their right.

All 3 children pass their ball at the same time.

The probability that they pass it to their left is shown in the table below.

Name of child	Probability of passing to the left
Gizella	$\frac{1}{5}$
Dilys	$\frac{2}{7}$
Hanako	$\frac{3}{8}$

- (a) Calculate the probability that every child passes their ball to the left.

[2]

$$P(\text{all pass left}) = \frac{1}{5} \times \frac{2}{7} \times \frac{3}{8} = \frac{6}{280} = \frac{3}{140}$$



- (b) Calculate the probability that one of the three children does not receive a ball. [3]

Every child receives a ball only if all pass left or all pass right.

$$P(\text{all pass right}) = \frac{4}{5} \times \frac{5}{7} \times \frac{5}{8} = \frac{100}{280}$$

$$P(\text{one child gets no ball}) = 1 - \frac{6}{280} - \frac{100}{280}$$

$$= \frac{174}{280} = \frac{87}{140}$$



18. The diagram below is a sketch of a **regular** 12-sided polygon, with centre O. The polygon has sides of length 5 cm. AE and OE are straight lines.

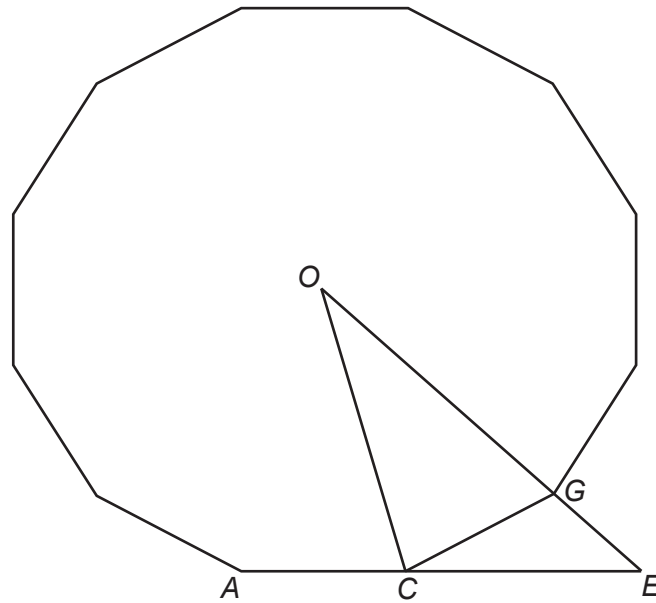


Diagram not drawn to scale

- (a) Calculate the length of CE.
You must show all your working.

[6]

Exterior angle of a regular 12-sided polygon = $360^\circ \div 12 = 30^\circ$

ACE is a straight line, so angle ECG = 30° (exterior angle).

Angle COG = $360^\circ \div 12 = 30^\circ$ and $OC = OG$ (isosceles triangle),

so angle OGC = $(180^\circ - 30^\circ) \div 2 = 75^\circ$.

Angle CGE = $180^\circ - 75^\circ = 105^\circ$ (angles on a straight line OGE)

Angle CEG = $180^\circ - 30^\circ - 105^\circ = 45^\circ$ (angles in a triangle add to 180°)

CG = 5 cm (a side of the polygon). Sine rule in triangle CGE:

$$\frac{CE}{\sin 105^\circ} = \frac{5}{\sin 45^\circ}$$

$$CE = 5 \times \sin 105^\circ \div \sin 45^\circ$$

$$= 6.830127019...$$

$$CE = 6.83 \text{ cm (to 3 s.f.)}$$



(b) Hence, calculate the area of triangle CGE .

[2]

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times CG \times CE \times \sin ECG = \frac{1}{2} \times 5 \times 6.830127019... \times \sin 30^\circ \\ &= 8.537658774...\end{aligned}$$

$$\text{Area of triangle } CGE = 8.54 \text{ cm}^2 \text{ (to 3 s.f.)}$$



19. Use the quadratic formula to solve the equation $\frac{x+3}{x} + 5 = 7x$.

Give your answers as decimal numbers.
You **must** show all your working.

[5]

Multiply every term by x:

$$x + 3 + 5x = 7x^2$$

$$7x^2 - 6x - 3 = 0$$

$$a = 7, b = -6, c = -3$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 7 \times (-3)}}{2 \times 7}$$

$$x = \frac{6 \pm \sqrt{120}}{14}$$

$$\sqrt{120} = 10.95445115...$$

$$x = (6 + 10.95445115...) \div 14 = 1.211032225...$$

$$x = (6 - 10.95445115...) \div 14 = -0.3538893679...$$

$$x = 1.21 \text{ or } x = -0.354 \text{ (to 3 s.f.)}$$

END OF PAPER



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