



GCSE MARKING SCHEME

AUTUMN 2023

**GCSE
MATHEMATICS
UNIT 2 – HIGHER TIER
3300U60-1**

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCSE MATHEMATICS
AUTUMN 2023 MARKING SCHEME

GCSE Mathematics Unit 2: Higher Tier	Mark	Comments
1.(a) 1 10	B2	B1 for each. Table takes precedence if conflicting values given.
1.(b) At least 4 correct plots and no incorrect plot. A smooth <u>curve</u> drawn through their plots.	P1 C1	FT 'their (-2,1)' and 'their (1,10)' OR (-2,1) and (1,10) plotted. Allow $\pm \frac{1}{2}$ a small square'. FT 'their 6 plots'. OR a curve through the 4 given points AND (-2,1) and (1,10) Allow intention to pass through their plots. (± 1 small square horizontally or vertically.)
2. (a) (Area =) $\frac{7.4 + 9.1}{2} \times 5.7$ or equivalent $\times 15.6$ Allow an answer from 733 to 734 (cm ³) inclusive.	M1 m1 A1	(= 47.025) May be seen in stages. Allow M1 for correct intent <u>seen</u> . e.g. $7.4 + 9.1 \times 5.7 \div 2$ CAO Note: 733.59 or 733.6 (cm ³)
2. (b) 733.59 $\times$ 19.3 14158(.287 (g)) Allow an answer from 14.1 to 14.2 (kg) inclusive.	M1 A1 B1	FT 'their volume from (a)' $\times$ 19.3 FT 'their 14158.287' $\div$ 1000 Allow 14 from correct working. Note: 14.158(287) or 14.16 or 14.2 (kg)
2.(b) <u>Alternative method (converting to g first)</u> $0.0193 \text{ (kg/cm}^3\text{)}$ 733.59×0.0193 Allow an answer from 14.1 to 14.2 (kg) inclusive	B1 M1 A1	FT 'their volume from (a)' and FT 'their 0.0193' provided a place value error has been made. Allow 14 from correct working. Note: 14.158(287) or 14.16 or 14.2 (kg)

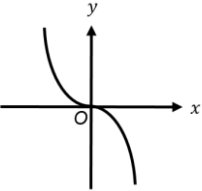
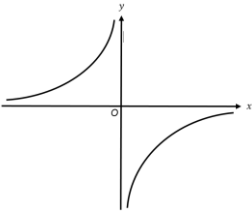
3. Identifying or implying that there are 16 possible correct combinations (e.g 2×6) or products (e.g.12) Identifies <u>all</u> possible combinations (e.g 2×6) or products (e.g 12) that are a factor of 36 $1 \times 6 = 6$, $1 \times 9 = 9$, $2 \times 6 = 12$ $2 \times 9 = 18$, $3 \times 6 = 18$, $4 \times 9 = 36$ (Probability factor of 36 =) $\frac{6}{16}$ or equivalent. ISW	B1 B2 B1	Award B1 for • simply stating 16 • $(4 \times 4 =)16$ • completed sample space (need not be correct) • sight of $\frac{1}{4} \times \frac{1}{4}$ • sight of 16 in a denominator. FT 'their 16 possible correct products'. If products not used (e.g $2 + 6 = 8$), do not award B2 or B1. Award B2 for clearly identifying one of the following: • the 6 (and no more) combinations 1×6, 2×9, etc that form factors of 36 that can be achieved by the two spinners • the 6 (and no more) products of factors of 36 that can be achieved by the two spinners: 6, 9, 12, 18, 18, 36 • sight of $6 \times \frac{1}{4} \times \frac{1}{4}$ or equivalent. Award B1 for clearly identifying one of the following: • at least 4 combinations that are factors of 36 • at least 4 products of factors of 36 that can be achieved by the two spinners: 6, 9, 12, 18, 36 • all of the factors of 36 (1,2,3,4,6,9,12,18,36). FT 'their list' only if at least 12 combinations or products given with at least two factors of 36 that can be achieved by the two spinners clearly identified. Penalise, -1, any incorrect notation e.g. '6 out of 16'. Unsupported $\frac{6}{16}$ or $\frac{3}{8}$ or equivalent gains B1 B2 B1.
Organisation and Communication Accuracy of writing	OC1 W1	For OC1, candidates will be expected to: • present their response in a structured way • explain to the reader what they are doing at each step of their response • lay out their explanation and working in a way that is clear and logical • write a conclusion that draws together their results and explains what their answer means For W1, candidates will be expected to: • show all their working • make few, if any, errors in spelling, punctuation and grammar • use correct mathematical form in their working • use appropriate terminology, units, etc.

4.	$(AC^2 =) 8^2 + 4 \cdot 5^2$ or equivalent $(AC =) \sqrt{8^2 + 4 \cdot 5^2}$ or equivalent 9.1(7878...) (cm) ISW	M1	Check diagram. note: $(AC^2 =) 64 + 20 \cdot 25$.																																																
		m1	note: $(AC =) \sqrt{84 \cdot 25}$. FT $\sqrt{\text{'their } 84 \cdot 25 \text{'}}$ for m1 only provided M1 gained.																																																
		A1	Accept the answer rounded or truncated to at least one decimal place. Final answer of AC = 84.25 is M1m0A0.																																																
4. <u>Alternative method to find AC using Trig</u> A correct and complete method (using trigonometric relationships)		M2																																																	
	9.1(7878...) (cm) ISW	A1	Accept the answer rounded or truncated to at least one decimal place.																																																
5.	One correct evaluation $3 \leq x \leq 4$ 2 correct evaluations $3.75 \leq x \leq 3.95$, (one value < 80, one value > 80) 2 correct evaluations $3.75 \leq x \leq 3.85$, (one value < 80, one value > 80) $x = 3.8$	B1	Correct evaluation regarded as enough to identify if < 80 or > 80.																																																
		B1	If evaluations not seen accept 'too high' or 'too low'. Look out for $x^3 + 6x - 80 = 0$																																																
		M1	<table><tr><td>x</td><td>$x^3 + 6x$</td><td></td><td></td></tr><tr><td>3</td><td>45</td><td></td><td></td></tr><tr><td>3.1</td><td>48.391</td><td></td><td></td></tr><tr><td>3.2</td><td>51.968</td><td></td><td></td></tr><tr><td>3.3</td><td>55.737</td><td></td><td></td></tr><tr><td>3.4</td><td>59.704</td><td></td><td></td></tr><tr><td>3.5</td><td>63.875</td><td></td><td></td></tr><tr><td>3.6</td><td>68.256</td><td>3.75</td><td>75.2343..</td></tr><tr><td>3.7</td><td>72.853</td><td>3.84</td><td>79.6631..</td></tr><tr><td>3.8</td><td>77.672</td><td>3.85</td><td>80.1666..</td></tr><tr><td>3.9</td><td>82.719</td><td>3.95</td><td>85.3298..</td></tr><tr><td>4</td><td>88</td><td></td><td></td></tr></table>	x	$x^3 + 6x$			3	45			3.1	48.391			3.2	51.968			3.3	55.737			3.4	59.704			3.5	63.875			3.6	68.256	3.75	75.2343..	3.7	72.853	3.84	79.6631..	3.8	77.672	3.85	80.1666..	3.9	82.719	3.95	85.3298..	4	88		
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		A1	Unsupported $x = 3.8$ is awarded B0B0M0A0. An answer of $x = 3.8$ can only be awarded M1A1, following sight of 2 correct evaluations $3.75 \leq x \leq 3.85$ (one evaluation < 80, one evaluation > 80). If 3.85 is given as 80 (truncated) award M0 A0 unless 'too high' or equivalent is indicated.																																																

6. Sight of $(5x + 3)(2x - 1)$ or $2(5x + 3)(2x - 1)$ or equivalent (total area of both rectangles = $20x^2 + 2x - 6$ (cm²))	S1 B2	Intention to $(2 \times)$ width $\times$ length. Allow $4 \times$ width $\times$ length or equivalent for S1. May be implied in later working if B2 or B1 awarded. Mark final answer for B2. Allow $20x^2 + 2x - 6$ for B2. Award B1 for sight of one of the following: $20x^2 + 12x - 10x - 6$ with at least three terms out of the four correct (must have x^2 term) $10x^2 + 6x - 5x - 3$ $2(10x^2 + x - 3)$ $10x^2 + x - 3$ $40x^2 + 4x - 12$. If no marks, award SC1 for one of the following: $40x^2 + 24x - 20x - 12$. $20x^2 + 22x + 6$ from $2(5x + 3)(2x + 1)$ $20x^2 - 2x - 6$ from $2(5x - 3)(2x + 1)$ $20x^2 - 22x + 6$ from $2(5x - 3)(2x - 1)$.						
7. <u>Method using angle XYZ</u> $YZ = \frac{18.6}{\tan 40^\circ}$ or $\frac{18.6 \times \sin 50}{\sin 40}$ or equivalent = 22(.166..)(cm)	M2 A1	Check diagram for answer. Award M1 for one of the following $\tan 40^\circ = \frac{18.6}{YZ}$ $\frac{YZ}{\sin 50} = \frac{18.6}{\sin 40}$ or equivalent Accept an answer rounded or truncated. Award M2A0 for any of the following unsupported answers: <table border="1" data-bbox="858 1211 1485 1305"> <thead> <tr> <th>Method</th><th>Radians</th><th>Gradians</th></tr> </thead> <tbody> <tr> <td>$\frac{18.6}{\tan 40}$</td><td>-16.648....</td><td>25.600...</td></tr> </tbody> </table>	Method	Radians	Gradians	$\frac{18.6}{\tan 40}$	-16.648....	25.600...
Method	Radians	Gradians						
$\frac{18.6}{\tan 40}$	-16.648....	25.600...						
7. <u>Alternative using angle YXZ</u> $YZ = 18.6 \times \tan 50^\circ$ = 22(.166..)(cm)	M2 A1	Award M1 for $\tan 50^\circ = \frac{YZ}{18.6}$ Accept an answer rounded or truncated Award M2A0 for any of the following unsupported answers: <table border="1" data-bbox="858 1615 1485 1693"> <thead> <tr> <th>Method</th><th>Radians</th><th>Gradians</th></tr> </thead> <tbody> <tr> <td>$18.6 \times \tan 50$</td><td>-5.057....</td><td>18.6</td></tr> </tbody> </table>	Method	Radians	Gradians	$18.6 \times \tan 50$	-5.057....	18.6
Method	Radians	Gradians						
$18.6 \times \tan 50$	-5.057....	18.6						
7. <u>Alternative method</u> Correct use of a 'two-step' method. 22(.166..)(cm) ISW	M2 A1	A partial trigonometric method is M0. Accept an answer rounded or truncated.						

8.	<table><tr><td>Working in mm</td><td>Working in cm</td></tr><tr><td>60.5×7</td><td>6.05×7</td></tr><tr><td>OR</td><td>OR</td></tr><tr><td>$420 + 0.5 \times 7$</td><td>$42 + 0.05 \times 7$</td></tr></table> 423.5 (mm) ISW	Working in mm	Working in cm	60.5×7	6.05×7	OR	OR	$420 + 0.5 \times 7$	$42 + 0.05 \times 7$	M1 A1	Allow $60 < \text{'their } 60.5' \leq 61$. Allow $6 \text{ cm} < \text{'their } 6.05' \text{ cm} \leq 6.1 \text{ cm}$. Allow 42.35 cm, provided units are given and correct. CAO. If no marks, award SC1 for sight of 60.5 OR 6.05.
Working in mm	Working in cm										
60.5×7	6.05×7										
OR	OR										
$420 + 0.5 \times 7$	$42 + 0.05 \times 7$										
9. Midpoints 25, 35, 45, (55), 65, 75	B1 B1 M1 m1 A1	May be implied in later working (i.e the correct products). FT 'their 10' provided $\neq 0$ or 1. Allow with consistent incorrect midpoints provided at least 5 within the correct interval including 'bounds' Allow use of a instead of 10 (sight of $1080 + 55a$). FT $22 + \text{'their } a'$ ($a \neq 0$). Allow use of a instead of 10. CAO. Must be derived from correct working. If no marks or first B1 only, award SC1 for one of the following: • $(1080 \div 22 =) 49(\cdot 09\dots)$ from use of $a = 0$ • $(1080 \div 32 =) 33.7(5)$ or 34 from use of $a = 0$ • $(1135 \div 23 =) 49(\cdot 3\dots)$ from use of $a = 1$ • $(1135 \div 32 =) 35(\cdot 46875)$ from use of $a = 1$. Award B1 B0 M1 m1 A0 for $\frac{1080 + 55a}{22 + a} \text{ or } \frac{1080 + 55a}{32}$ or equivalent expression involving a.									
10.	B2 M1 A1 m1 A1	x and y terms need to be collected for B2. If B2 not awarded, award B1 for one of the following: • $12x + 4y = 180$ or equivalent • $13x + 5x + 8x + 7y = 360$ • $26x + 7y = 360$ or equivalent FT 'their equations', provided of equivalent difficulty. Allow one error in one term (not the term with equal coefficients). CAO (for their equations). FT substitution of their '1st variable' if M1 gained. No marks for 'trial and improvement'. No marks for an unsupported answer.									

[illegible]

16.(a) 	B1	Third box
16.(b) Correct sketch of $y = -1/x$ in appropriate 2 quadrants with axes as asymptotes with no extra curves in the other quadrants. 	B2	Penalise -1 for the curling away from the asymptotes at the extremities only if B2 previously awarded. If not B2, award B1 for one of the following: • Correct sketch in 1 quadrant with axes as asymptotes with no more than 1 incorrect curve in another quadrant • Correct sketch in appropriate 2 quadrants with axes as asymptotes with extra incorrect curves in one or two of the other quadrants • for two curves sketched appropriately in both quadrants but not clearly with intention of axes as asymptotes • Correct sketch of $y = +1/x$ in appropriate 2 quadrants with axes as asymptotes.

17.(a)	$\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8}$ $= \frac{6}{280} \left(\frac{3}{140} \right) \text{ ISW}$	M1	
		A1	Accept a decimal answer of 0.021(4...). Allow an answer of 0.02 from correct working.
17.(b) [1–‘all pass to the left’–‘all pass to the right’] $1 - \left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8} \right) - \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8} \right)$	$= \frac{174}{280} \left(\frac{87}{140} \right) \text{ ISW}$	M2	Award only M1 if further incorrect work seen. FT from part (a) provided <1. M1 for any one of the following: $1 - \left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8} \right) \left[= \frac{137}{140} \right]$ $1 - \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8} \right) \left[= \frac{9}{14} \right]$ $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8} \right) \left[= \frac{53}{140} \right]$
		A1	CAO Accept a decimal answer of 0.62(1...)
17.(b) Alternative method 1 [‘2 pass left and 1 right’+‘2 pass right and 1 left’] $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8} \right)$ $+ \left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8} \right)$	$= \frac{174}{280} \left(\frac{87}{140} \right) \text{ ISW}$	M2	Award only M1 if further incorrect work seen. M1 for any one of the following: $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8} \right) \left[= \frac{7}{40} \right]$ $\left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8} \right) \left[= \frac{125}{280} \right]$
		A1	CAO Accept a decimal answer of 0.62(1...)
17.(b) Alternative method 2 [G no ball + D no ball + H no ball] $P(\text{G no ball}) + P(\text{D no ball}) + P(\text{H no ball}) =$ $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8} \right)$ $+ \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8} \right)$ $+ \left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8} \right)$	$= \frac{174}{280} \left(\frac{87}{140} \right) \text{ ISW}$	M2	Award only M1 if further incorrect work seen. M1 for any one of the following: $P(\text{G no ball}) = \left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8} \right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8} \right) \text{ or } \left(\frac{2}{7} \times \frac{5}{8} \right) \left[= \frac{5}{28} \right]$ $P(\text{D no ball}) = \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8} \right) + \left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8} \right) \text{ or } \left(\frac{4}{5} \times \frac{3}{8} \right) \left[= \frac{3}{10} \right]$ $P(\text{H no ball}) = \left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8} \right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8} \right) \text{ or } \left(\frac{1}{5} \times \frac{5}{7} \right) \left[= \frac{1}{7} \right]$
		A1	CAO Accept a decimal answer of 0.62(1...)
17.(b) Alternative method 3 [G receives 2 balls + D receives 2 balls + H receives 2 balls] $P(\text{H left, D right}) + P(\text{G left, H right}) + P(\text{D left, G right}) =$ $\left(\frac{3}{8} \times \frac{5}{7} \right) + \left(\frac{1}{5} \times \frac{5}{8} \right) + \left(\frac{2}{7} \times \frac{4}{5} \right)$	$= \frac{174}{280} \left(\frac{87}{140} \right) \text{ ISW}$	M2	Award only M1 if further incorrect work seen. M1 for any one of the following: $P(\text{H left, D right}) = \left(\frac{3}{8} \times \frac{5}{7} \right) \left[= \frac{15}{56} \right]$ $P(\text{G left, H right}) = \left(\frac{1}{5} \times \frac{5}{8} \right) \left[= \frac{1}{8} \right]$ $P(\text{D left, G right}) = \left(\frac{2}{7} \times \frac{4}{5} \right) \left[= \frac{8}{35} \right]$
		A1	CAO Accept a decimal answer of 0.62(1...)

18.(a) $E\hat{C}G = 30(^{\circ})$ Sight of $C\hat{E}G = 45(^{\circ})$ AND $C\hat{G}E = 105(^{\circ})$ $CE = \frac{5 \times \sin 105}{\sin 45}$ $= 6.8(3\dots\text{cm})$ or $\frac{5+5\sqrt{3}}{2} \text{ (cm)}$	B1 B2 M2 A1	Angles must be clearly stated or seen on the diagram. B1 for $C\hat{E}G = 45(^{\circ})$ OR $C\hat{G}E = 105(^{\circ})$ Sight of $C\hat{E}G = 45(^{\circ})$-implies previous B1. FT 'their $E\hat{C}G = 30'$ to calculate: $C\hat{G}E = \text{'their } 30' / 2 + 90$ $C\hat{E}G = 90 - \text{'their } 30' \times 3/2$ Use of correct angles implies B1B2. FT 'their $C\hat{G}E = 105'$ AND 'their $C\hat{E}G = 45'$. M1 for $\frac{CE}{\sin 105} = \frac{5}{\sin 45}$ Award M2 for a correct use of 'two-step' trigonometric relationship. M0 otherwise. Allow 7(cm) from correct working.
<u>18(a) Alternative version to find OC first and then use triangle OCE</u> $C\hat{O}E = 30(^{\circ})$ Sight of $C\hat{E}G = 45(^{\circ})$ $\left(OC = \frac{2.5}{\sin(15)} = \right)$ $9.6(5\dots\text{cm})$ or $9.7(\text{cm})$ or $\frac{5\sqrt{6} + 5\sqrt{2}}{2} \text{ (cm)}$ $CE = \frac{9.6(5\dots) \times \sin 30}{\sin 45}$ $= 6.8(3\dots\text{cm})$ or $\frac{5+5\sqrt{3}}{2} \text{ (cm)}$	B1 B1 B1 M2 A1	Angles must be clearly stated or seen on the diagram. Sight of $C\hat{E}G = 45(^{\circ})$ implies previous B1 FT 'their $E\hat{C}G = 30'$ to calculate: $C\hat{E}G = 90 - \text{'their } 30' \times 3/2$ Or equivalent e.g. using the sine rule in triangle OCG FT 'sin ($\frac{1}{2}$ of 'their $C\hat{O}E = 30'$) Use of correct side and angles implies B1B1B1. FT 'their derived OC' AND 'their $C\hat{O}E = 30'$ AND 'their $C\hat{E}G = 45'$. M1 for $\frac{CE}{\sin 30} = \frac{9.6(5\dots)}{\sin 45}$ Award M2 for a correct use of 'two-step' trigonometric relationship. M0 otherwise. Allow an answer of 7(cm) from correct working.
18.(b) $\frac{1}{2} \times 5 \times 6.8(3\dots) \times \sin 30$ $= 8.5(\dots\text{cm}^2)$ or $\frac{25+25\sqrt{3}}{8} \text{ (cm}^2\text{)}$	M1 A1	FT their solution in (a), provided it does not refer to a side other than CE AND 'their $E\hat{C}G = 30(^{\circ})$'.

