

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U50-1



A23-3300U50-1

MONDAY, 13 NOVEMBER 2023 – MORNING

MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet.
If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 1, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

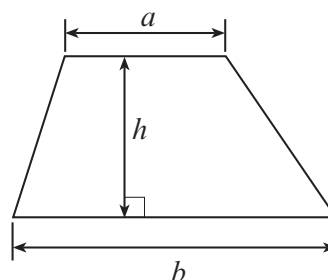
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	4	
3.	4	
4.	6	
5.	3	
6.	2	
7.	4	
8.	4	
9.	3	
10.	3	
11.	4	
12.	4	
13.	5	
14.	3	
15.	2	
16.	7	
17.	3	
18.	4	
19.	2	
20.	3	
21.	6	
Total	80	



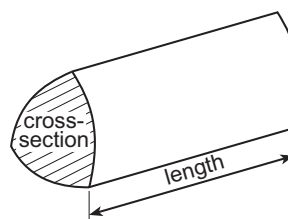
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

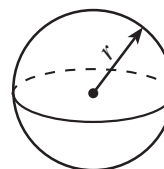


Volume of prism = area of cross-section $\times$ length



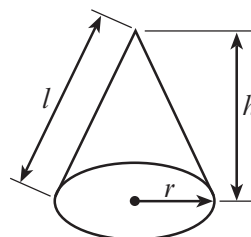
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

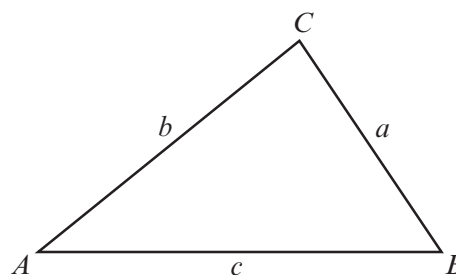


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. *In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

A cup contains some tea.

Elsie drinks $\frac{5}{7}$ of the tea.

There are 44 ml of tea left in the cup.

How much tea was in the cup before Elsie drank any?

[2 + 2 OCW]

Elsie drinks $\frac{5}{7}$ of the tea, so the fraction left is $1 - \frac{5}{7} = \frac{2}{7}$

$\frac{2}{7}$ of the tea = 44 ml

$\frac{1}{7}$ of the tea = $44 \div 2 = 22$ ml

Whole cup = $\frac{7}{7}$ of the tea = $22 \times 7 = 154$ ml

There was 154 ml of tea in the cup before Elsie drank any.



2. The Geometric Mean is a special type of average.

To find the Geometric Mean of three numbers, you must:

- multiply the three numbers together, and
- then find the cube root.

- (a) Find the Geometric Mean of 100, 0.3 and 0.9. [2]

$$100 \times 0.3 \times 0.9 = 30 \times 0.9 = 27$$

$$\sqrt[3]{27} = 3$$

$$\text{Geometric Mean} = 3$$

- (b) The Geometric Mean of three numbers is 10.
Two of the numbers are 8 and 25.
Find the third number. [2]

$$\text{Product of the three numbers} = 10^3 = 1000$$

$$8 \times 25 \times x = 1000$$

$$200x = 1000$$

$$x = 1000 \div 200 = 5$$

$$\text{The third number is } 5$$



3. (a) Write down an expression for the n th term of the following sequence. [2]

11, 15, 19, 23, ...

Common difference = +4, so the n th term starts $4n$

$n = 1: 4 \times 1 = 4, \text{ and } 11 - 4 = 7$

n th term = $4n + 7$

- (b) The n th term of a different sequence is given by $n^2 - 5$.
Write down the first three terms of this sequence. [2]

$n = 1: 1^2 - 5 = 1 - 5 = -4$

$n = 2: 2^2 - 5 = 4 - 5 = -1$

$n = 3: 3^2 - 5 = 9 - 5 = 4$

First three terms are -4 , -1 , 4



4. (a) Express 495 as a product of its prime factors in index form. [3]

$$495 \div 3 = 165$$

$$165 \div 3 = 55$$

$$55 \div 5 = 11$$

$$11 \div 11 = 1$$

$$495 = 3 \times 3 \times 5 \times 11$$

$$495 = 3^2 \times 5 \times 11$$

- (b) Explain how your answer to part (a) tells you that 495 is **not** a square number. [1]

For a square number, all the powers (indices) of the prime

factors must be even. Here the powers of 5 and 11 are 1,

so not all the powers are even. So 495 is not a square number.

- (c) Find the Highest Common Factor (HCF) of 495 and 60. [2]

$$60 = 2 \times 2 \times 3 \times 5$$

$$495 = 3 \times 3 \times 5 \times 11$$

Common prime factors: 3 and 5

$$HCF = 3 \times 5 = 15$$

$$HCF = 15$$



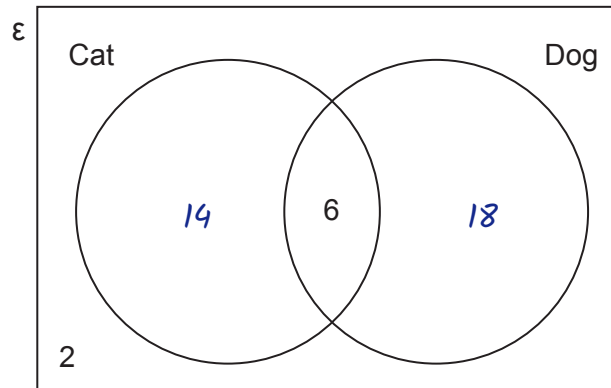
5. In a group of 40 people, some own a cat, some own a dog, and some own both a cat and a dog.
2 people in the group do not own a cat or a dog.

A person is chosen at random from the group.

The probability that the person owns a dog is $\frac{3}{5}$.

Complete the Venn diagram.

[3]



$$\text{Number who own a dog} = \frac{3}{5} \times 40 = 24$$

$$\text{Dog only} = 24 - 6 = 18$$

$$\text{Cat only} = 40 - 2 - 6 - 18 = 14$$

$$\text{Check: } 14 + 6 + 18 + 2 = 40$$



6. A number has been decreased by 10% to give an answer of 34.2.
What was the original number?

[2]

After a 10% decrease, 90% of the original number is left.

$$90\% = 34.2$$

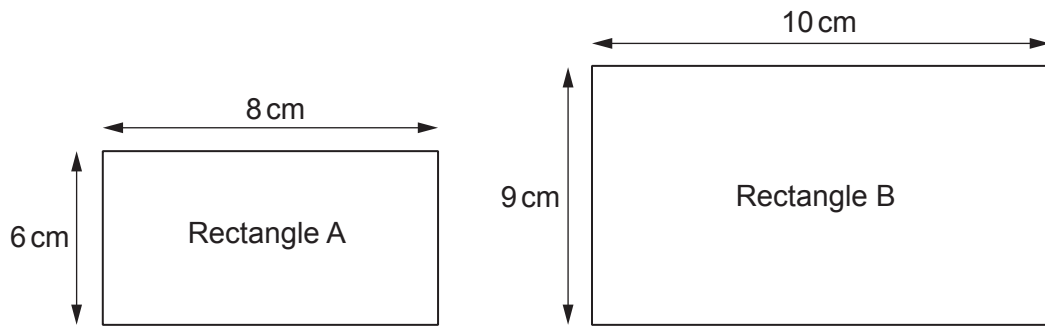
$$10\% = 34.2 \div 9 = 3.8$$

$$100\% = 3.8 \times 10 = 38$$

$$\text{Original number} = 38$$



7.



Diagrams not drawn to scale

- (a) Explain why Rectangle A is **not** mathematically similar to Rectangle B. [2]

Scale factor for the widths: $9 \div 6 = 1.5$

Scale factor for the lengths: $10 \div 8 = 1.25$

$1.5 \neq 1.25$, so the scale factors for the pairs of sides are not the same, so Rectangle A is not similar to Rectangle B.

- (b) Alun changes **only one** of the measurements of **Rectangle B** to make the two rectangles mathematically similar.

Write down a possible length and width of Alun's new rectangle.
You must show all your working. [2]

Keep the width of B as 9 cm. Scale factor = $9 \div 6 = 1.5$

New length = $8 \times 1.5 = 12$ cm

Check: $12 \div 8 = 1.5$ and $9 \div 6 = 1.5$

Length = 12 cm

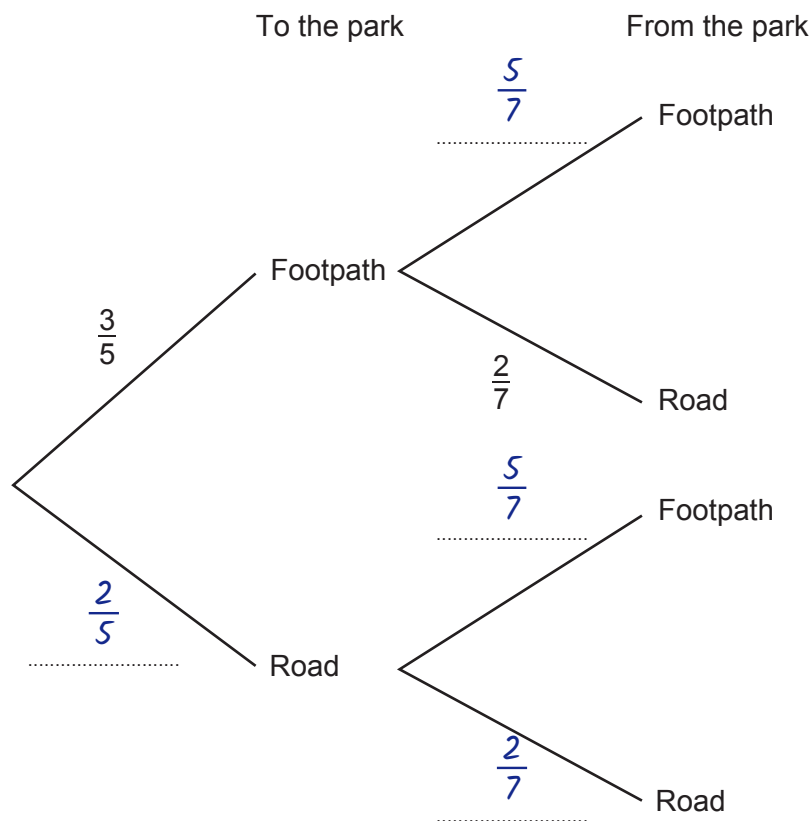
Width = 9 cm



8. Beti walks her dog to a local park and back home every day. She can either walk along a **footpath** or along a pavement at the side of a **road**. The probability that Beti walks to the park along the footpath is $\frac{3}{5}$. The probability that Beti walks home from the park along the road is $\frac{2}{7}$. Her decisions on which routes to walk to and from the park are independent of each other.

(a) Complete the tree diagram below.

[2]



- (b) Find the probability that Beti walks to and from the park along the footpath.

[2]

$$P(\text{footpath to park and footpath home}) = P(F \text{ to}) \times P(F \text{ from})$$

$$= \frac{3}{5} \times \frac{5}{7} = \frac{15}{35} = \frac{3}{7}$$



9. Factorise $x^2 - 8x - 20$, and hence solve $x^2 - 8x - 20 = 0$.

[3]

Two numbers that multiply to -20 and add to -8: -10 and 2

$$x^2 - 8x - 20 = (x - 10)(x + 2)$$

$$(x - 10)(x + 2) = 0$$

$$x - 10 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 10 \quad \text{or} \quad x = -2$$



10. The points B , C , D and E lie on the circumference of a circle, with centre O .
 AF is a tangent to the circle.
 AO is a straight line.

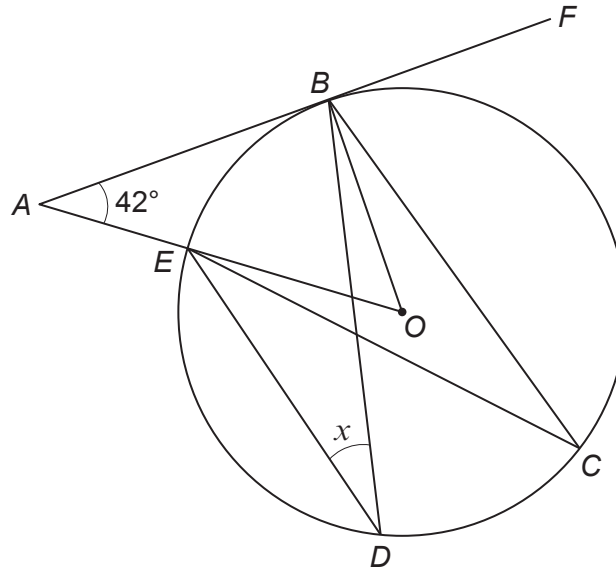


Diagram not drawn to scale

Calculate the size of angle x .
 You must show all your working.

[3]

OB is a radius and AB is a tangent, so angle $ABO = 90^\circ$

Angle $AOB = 180^\circ - 90^\circ - 42^\circ = 48^\circ$ (angles in a triangle add to 180°)

E lies on AO , so angle $BOE = 48^\circ$

$x = 48^\circ \div 2$ (the angle at the centre is twice the angle
 at the circumference, both on arc BE)

$x = 24^\circ$



11. Solve the equation $\frac{10x+2}{3} - \frac{7x-3}{5} = 9$.

[4]

Multiply every term by 15:

$$5(10x + 2) - 3(7x - 3) = 9 \times 15$$

$$50x + 10 - 21x + 9 = 135$$

$$29x + 19 = 135$$

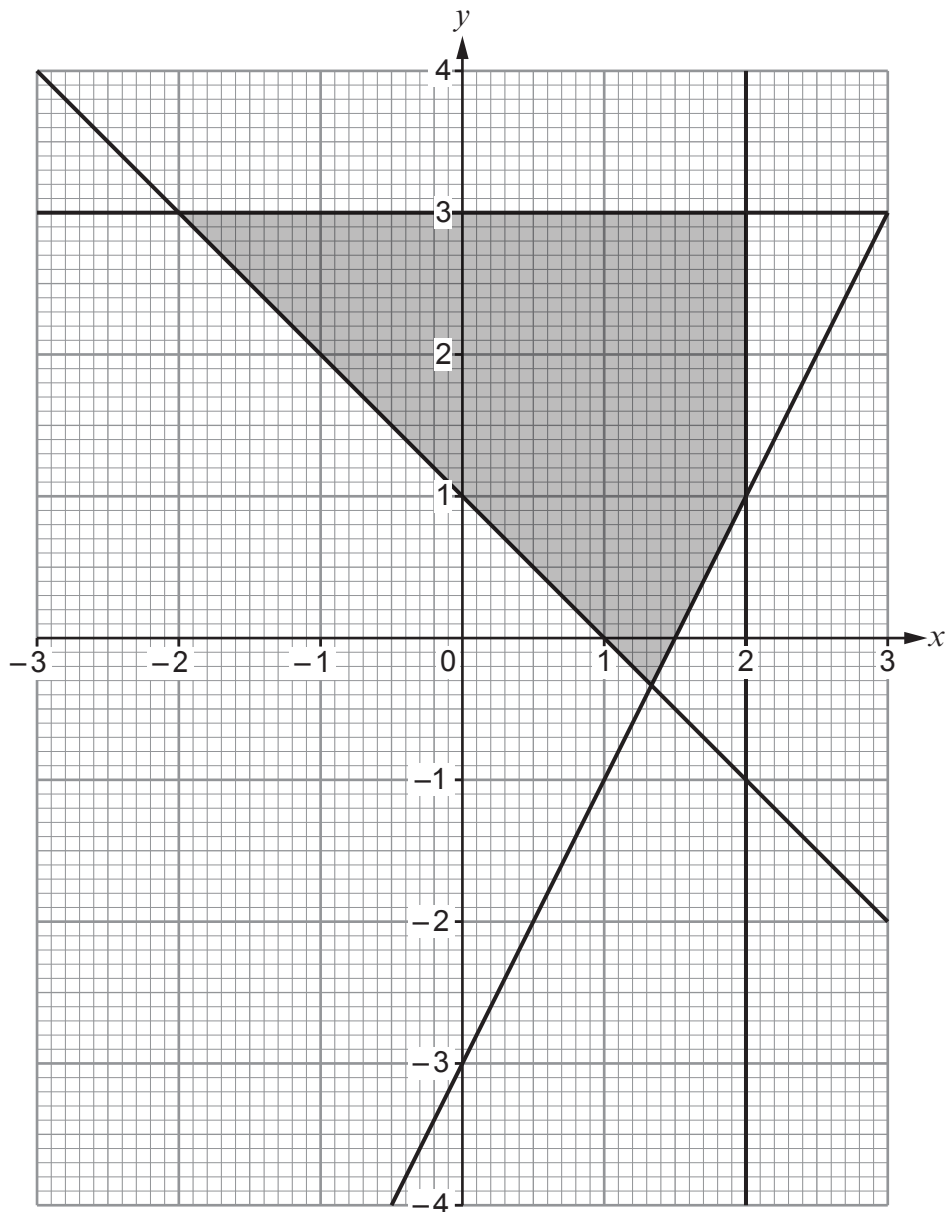
$$29x = 135 - 19 = 116$$

$$x = 116 \div 29$$

$$x = 4$$



12.



Complete the following table to give the set of inequalities that describes the shaded region shown above.

One inequality has already been written for you.

[4]

Line through (0, 1) and (1, 0): gradient = -1, so $y = -x + 1$

Region is left of $x = 2$, below $y = 3$ and above $y = -x + 1$

$y \geq 2x - 3$
$x \leq 2$
$y \leq 3$
$y \geq -x + 1$



13. y is inversely proportional to the square of x .
 $y = 16$ when $x = 5$.

(a) Find an expression for y in terms of x .

[3]

$$y \propto \frac{1}{x^2}, \text{ so } y = \frac{k}{x^2}$$

$$16 = \frac{k}{5^2}$$

$$k = 16 \times 25 = 400$$

$$y = \frac{400}{x^2}$$

(b) Use the expression you found in part (a) to complete the following table.

[2]

x	5	0.1	2
y	16	40 000	100

$$x = 0.1: y = 400 \div 0.1^2 = 400 \div 0.01 = 40\,000$$

$$y = 100: 100 = 400 \div x^2$$

$$x^2 = 400 \div 100 = 4$$

$$x = 2 \text{ (or } -2)$$



14. In the diagram below, arc AD is part of a circle with centre O .
 Arc BC is part of a larger circle, also with centre O .
 OB and OC are straight lines.
 $OA = 14$ cm.
 $AB = 6$ cm.
 Reflex $\hat{AOD} = 300^\circ$.

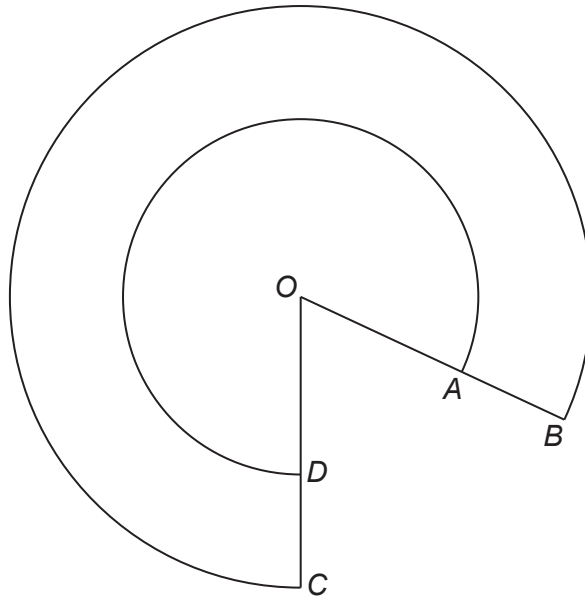


Diagram not drawn to scale

Calculate the difference between the length of the arc BC and the length of the arc AD .
 Give your answer as a multiple of π .

[3]

Radius of arc $AD = 14$ cm, radius of arc $BC = 14 + 6 = 20$ cm

$$\text{Arc } BC = \frac{300}{360} \times 2 \times \pi \times 20 = \frac{5}{6} \times 40\pi = \frac{100\pi}{3}$$

$$\text{Arc } AD = \frac{300}{360} \times 2 \times \pi \times 14 = \frac{5}{6} \times 28\pi = \frac{70\pi}{3}$$

$$\text{Difference} = \frac{100\pi}{3} - \frac{70\pi}{3} = \frac{30\pi}{3} = 10\pi$$

$$\text{Difference} = 10\pi \text{ cm}$$



15. In the following diagram:

- CB and DE are parallel
- CE and BD are straight lines
- $CA = AE$.

Prove that triangles ABC and ADE are congruent.

You must show all your working and explain your reasoning.

[2]

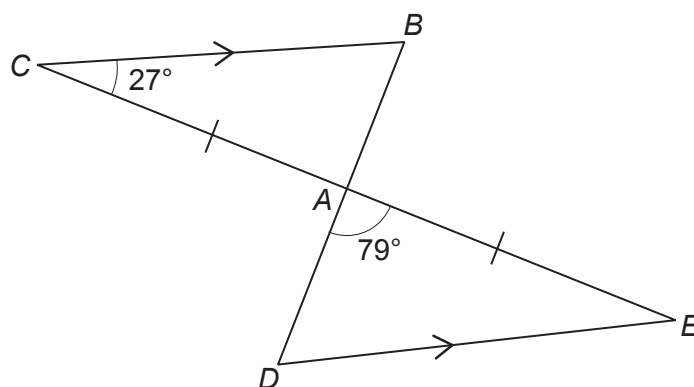


Diagram not drawn to scale

$\text{Angle } BAC = \text{angle } DAE = 79^\circ$ (vertically opposite angles)

$\text{Angle } BCA = \text{angle } DEA = 27^\circ$ (alternate angles, CB parallel to DE)

$\text{Angle } ABC = \text{angle } ADE = 180^\circ - 79^\circ - 27^\circ = 74^\circ$

(angles in a triangle add to 180°)

$CA = AE$ (given)

Two angles and the included side are equal (ASA), so

triangles ABC and ADE are congruent.



16. (a) Circle the correct answer for each of the following statements.

(i) $64^{\frac{2}{3}}$ is equal to

[1]

$$\frac{128}{3}$$

96

$$\frac{194}{3}$$

16

512

$$64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$$

(ii) $10000^{-\frac{1}{2}}$ is equal to

[1]

$$-\frac{1}{100}$$

$$\frac{1}{100}$$

- 5000

- 100

$$\frac{1}{5000}$$

$$10000^{-1/2} = \frac{1}{\sqrt{10000}} = \frac{1}{100}$$

(b) Express $0.07\dot{1}4$ as a fraction.

[2]

$$x = 0.0714141414...$$

$$100x = 7.1414141414...$$

$$10000x = 714.1414141414...$$

$$10000x - 100x = 714.1414141414... - 7.1414141414...$$

$$9900x = 707$$

$$x = \frac{707}{9900}$$



(c) Simplify $\sqrt{11\frac{1}{4}}$.

Give your answer in the form $\frac{a\sqrt{b}}{b}$, where a and b are integers. [2]

$$11\frac{1}{4} = \frac{44}{4} + \frac{1}{4} = \frac{45}{4}$$

$$\sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{\sqrt{4}} = \frac{\sqrt{9} \times \sqrt{5}}{2} = \frac{3\sqrt{5}}{2}$$

(d) Give an example of an irrational number that lies between 6 and 7. [1]

$$6^2 = 36 \text{ and } 7^2 = 49, \text{ and } 36 < 40 < 49, \text{ so } 6 < \sqrt{40} < 7$$

40 is not a square number, so $\sqrt{40}$ is irrational

My example of an irrational number is $\sqrt{40}$



17. If n is an integer, prove that $(2n-1)^2 + 7$ is always a multiple of 4. You must use an algebraic method. [3]

$$(2n-1)^2 + 7 = (2n-1)(2n-1) + 7$$

$$= 4n^2 - 2n - 2n + 1 + 7$$

$$= 4n^2 - 4n + 8$$

$$= 4(n^2 - n + 2)$$

$n^2 - n + 2$ is an integer, so 4 is a common factor:

$(2n-1)^2 + 7$ is always a multiple of 4.

18. Make t the subject of the following formula. [4]

$$\sqrt[3]{ct^3 - 9} = t$$

$$ct^3 - 9 = t^3 \quad (\text{cube both sides})$$

$$ct^3 - t^3 = 9$$

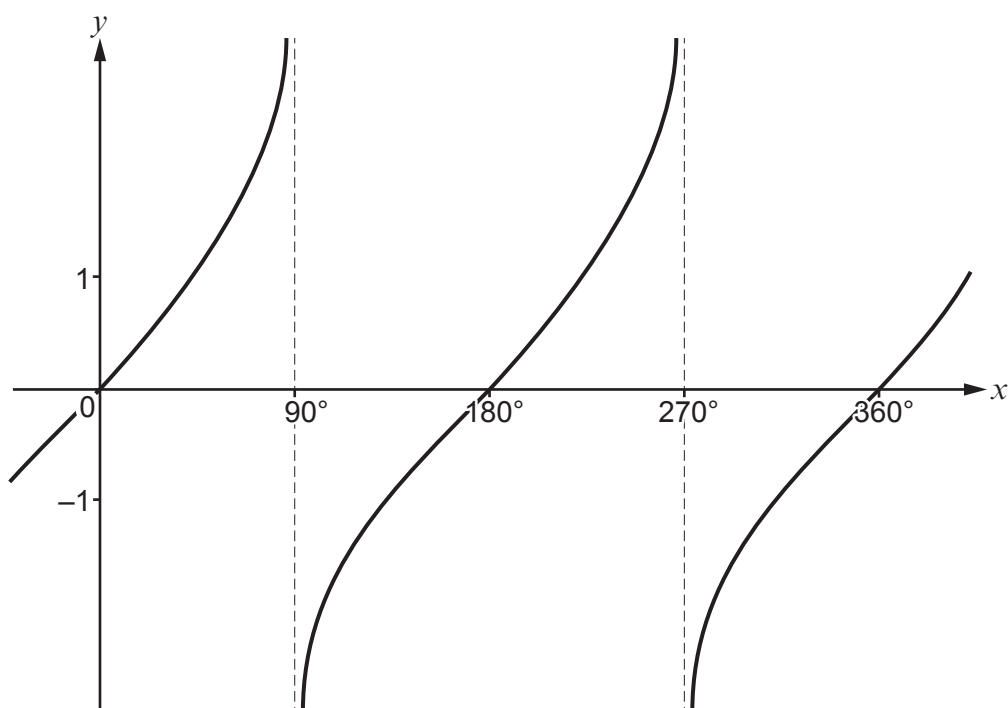
$$t^3(c-1) = 9$$

$$t^3 = \frac{9}{c-1}$$

$$t = \sqrt[3]{\frac{9}{c-1}}$$



19. The following diagram shows a sketch of $y = \tan x$ for values of x from 0° to 360° .



Given that $\tan 71^\circ = 2.9042$, correct to 4 decimal places, write down all the solutions of the equation

$$\tan x = -2.9042$$

for values of x from 0° to 360° .

[2]

$\tan x$ is negative between 90° and 180° , and between 270° and 360°

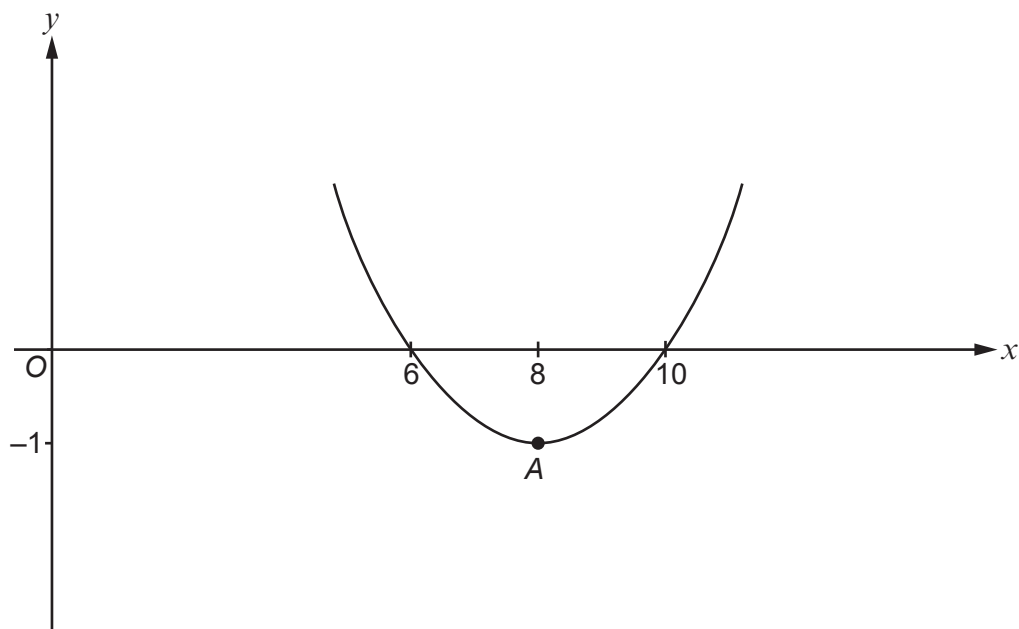
$$x = 180^\circ - 71^\circ = 109^\circ$$

$$x = 360^\circ - 71^\circ = 289^\circ$$

$$x = 109^\circ \text{ and } x = 289^\circ$$



20. The diagram shows a sketch of $y = f(x)$.
The point where the curve has a gradient of 0 is labelled A and has coordinates $(8, -1)$.

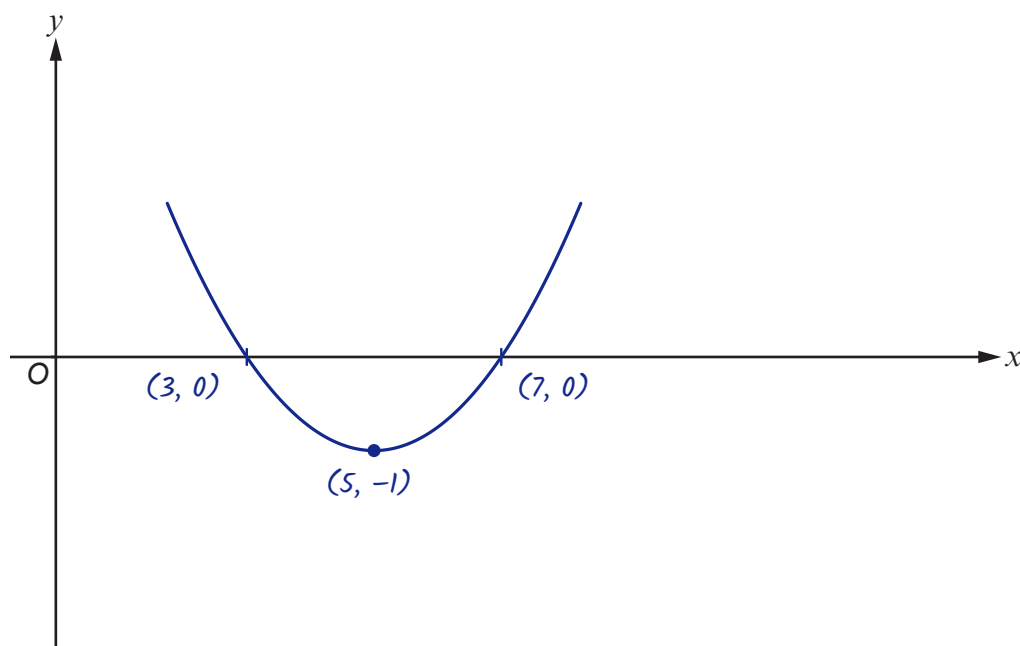


On the axes below, sketch the curve $y = f(x+3)$.

You must:

- indicate the coordinates of any point where the new curve crosses an axis
- complete the coordinates of the point on the curve where the gradient is 0.

[3]



The coordinates of the point on the curve where the gradient is 0 are (5 , -1).



21. (a) A box contains seven black counters, three white counters and one red counter. Aled takes two counters at random from the box. These counters are not replaced.

Calculate the probability that the two counters that Aled chose are both the same colour. [3]

$$P(\text{both black}) = \frac{7}{11} \times \frac{6}{10} = \frac{42}{110}$$

$$P(\text{both white}) = \frac{3}{11} \times \frac{2}{10} = \frac{6}{110}$$

$$P(\text{both red}) = 0 \text{ (there is only one red counter)}$$

$$P(\text{same colour}) = \frac{42}{110} + \frac{6}{110} + 0 = \frac{48}{110} = \frac{24}{55}$$

- (b) A second box contains n yellow cards and $(n+1)$ red cards. Delyth takes two cards at random from the second box. These cards are not replaced.

What is the probability that the two cards that Delyth chose are both yellow? Give your answer as an algebraic fraction in its simplest form. [3]

$$\text{Total number of cards} = n + (n + 1) = 2n + 1$$

$$P(\text{both yellow}) = \frac{n}{2n+1} \times \frac{n-1}{2n}$$

$$= \frac{n(n-1)}{2n(2n+1)}$$

$$= \frac{n-1}{2(2n+1)} = \frac{n-1}{4n+2}$$

END OF PAPER



