

Surname <i>StemUp Model Solution</i>
First name(s)

Centre Number

Candidate Number
0



GCSE

3300U60-1



WEDNESDAY, 16 NOVEMBER 2022 – MORNING

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 4, the assessment will take into account the quality of your organisation, communication and accuracy in writing.

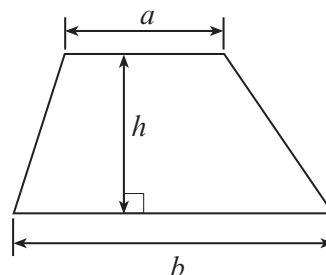
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	4	
3.	5	
4.	9	
5.	3	
6.	3	
7.	2	
8.	5	
9.	4	
10.	4	
11.	3	
12.	3	
13.	4	
14.	4	
15.	1	
16.	3	
17.	1	
18.	5	
19.	5	
20.	8	
Total	80	



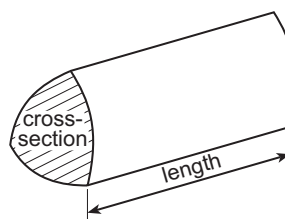
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

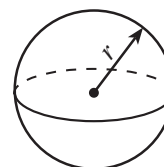


Volume of prism = area of cross-section $\times$ length



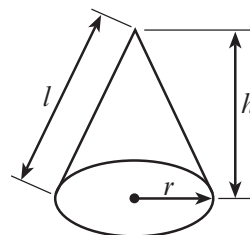
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

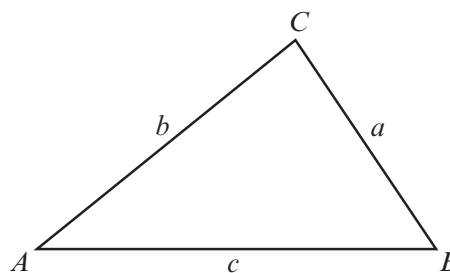


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. A solid metal cylinder has a radius of 2.3 cm and a height of 5 cm.

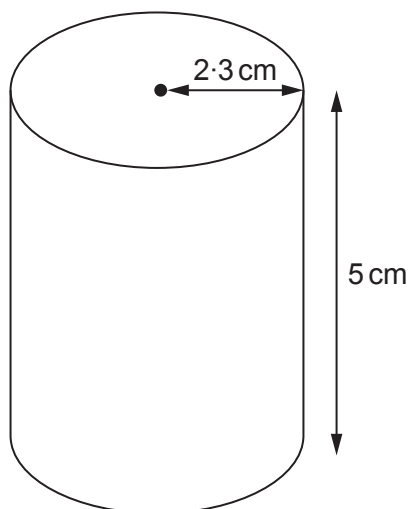


Diagram not drawn to scale

The mass of the cylinder is 423.1 g.

Find the density of the metal.

Give your answer in g/cm^3 .

[4]

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \pi \times 2.3^2 \times 5$$

$$= 26.45\pi = 83.09512569... \text{ cm}^3$$

$$\text{Density} = \text{mass} \div \text{volume}$$

$$= 423.1 \div 83.09512569...$$

$$= 5.091754739...$$

$$\text{Density} = 5.09 \text{ g/cm}^3$$



2. A solution to the equation

$$x^3 + 5x - 8 = 0$$

lies between 1 and 2.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 + 5x - 8$	
1	$1 + 5 - 8 = -2$	negative
2	$8 + 10 - 8 = 10$	positive
1.2	$1.728 + 6 - 8 = -0.272$	negative
1.3	$2.197 + 6.5 - 8 = 0.697$	positive
1.25	$1.953125 + 6.25 - 8 = 0.203125$	positive

Sign changes between 1.2 and 1.25, so the solution

lies between 1.2 and 1.25.

$x = 1.2$ (1 d.p.)



3. A rectangle has a width of $4x - 10$.

(a) Given that x is a whole number, explain why the value of x cannot be less than 3. [1]

If $x = 2$, the width would be $4 \times 2 - 10 = -2$ cm.

You cannot have a negative length for a side, so $4x - 10 > 0$,

which means $x > 2.5$. As x is a whole number, x cannot be less than 3.

(b) The perimeter of the rectangle is $14x - 4$.

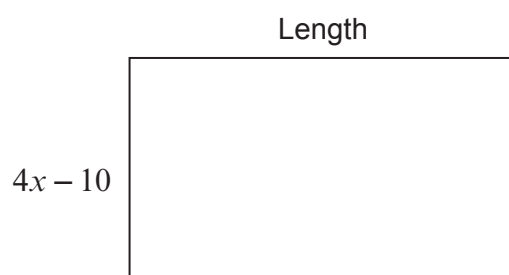


Diagram not drawn to scale

Find the length of the rectangle in **terms of x** .

[4]

$$\text{Perimeter} = 2 \times \text{length} + 2 \times \text{width}$$

$$2 \times \text{length} = 14x - 4 - 2(4x - 10)$$

$$= 14x - 4 - 8x + 20$$

$$= 6x + 16$$

$$\text{Length} = (6x + 16) \div 2$$

$$\text{Length} = 3x + 8$$



4. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

A, B and C are points on the circumference of a circle with centre O.
The length of BC is 10 cm.
The diameter of the circle is 18 cm.

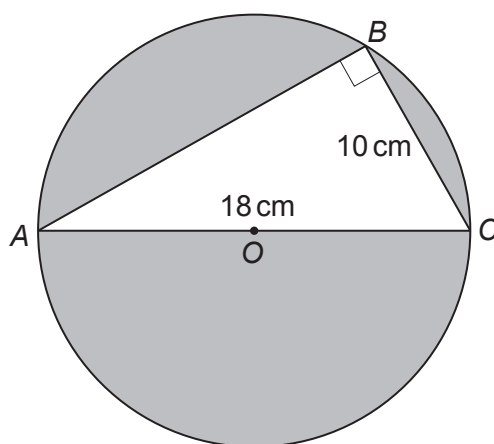


Diagram not drawn to scale

Calculate the shaded area.
You must show all your working.

[7 + 2 OCW]

AC is a diameter, so angle ABC = 90° (angle in a semicircle).

Using Pythagoras' theorem in triangle ABC to find AB:

$$AB^2 = AC^2 - BC^2 = 18^2 - 10^2 = 324 - 100 = 224$$

$$AB = \sqrt{224} = 14.96662955... \text{ cm}$$

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 10 \times 14.96662955...$$

$$= 74.83314774... \text{ cm}^2$$

$$\text{Radius} = 18 \div 2 = 9 \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = \pi \times 9^2 = 81\pi = 254.4690049... \text{ cm}^2$$

$$\text{Shaded area} = \text{area of circle} - \text{area of triangle}$$

$$= 254.4690049... - 74.83314774...$$

$$= 179.6358572... \text{ cm}^2$$

Therefore the shaded area is 179.6 cm² (1 d.p.).



5. Calculate the length of the side YZ in the triangle XYZ shown below.

[3]

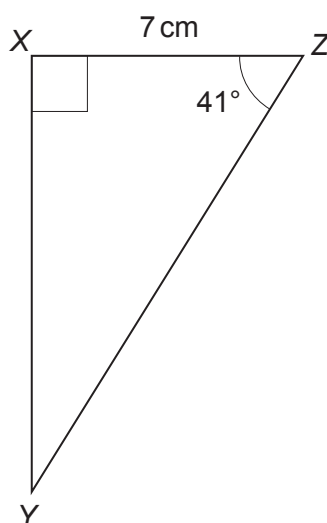


Diagram not drawn to scale

XZ is adjacent to the 41° angle and YZ is the hypotenuse.

$$\cos 41^\circ = \frac{7}{YZ}$$

$$YZ = \frac{7}{\cos 41^\circ}$$

$$YZ = 9.275090953...$$

$$YZ = 9.28 \text{ cm}$$



6. Two times are recorded correct to the **nearest 0.1 second**.

12.4 seconds
25.5 seconds

Calculate the greatest possible difference between these times.

[3]

$$12.4 \text{ s: } 12.35 \leq t < 12.45$$

$$25.5 \text{ s: } 25.45 \leq t < 25.55$$

$$\text{Greatest difference} = \text{upper bound of } 25.5 - \text{lower bound of } 12.4$$

$$= 25.55 - 12.35$$

$$= 13.2 \text{ seconds}$$

7. A number has been increased by 60% to give an answer of 64.
What was the original number?

[2]

$$100\% + 60\% = 160\% = 1.6$$

$$\text{Original number} = 64 \div 1.6$$

$$= 40$$



8. Bag A and Bag B contain only red and blue balls.

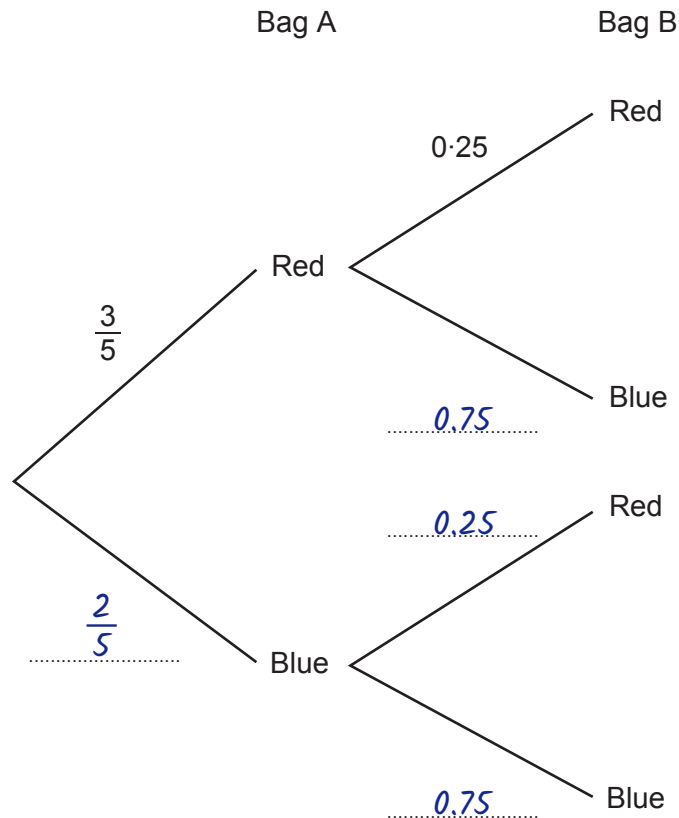
The probability of choosing a red ball from Bag A is $\frac{3}{5}$.

The probability of choosing a red ball from Bag B is 0.25.

A ball is chosen at random from each bag.

- (a) Complete the tree diagram below.

[2]



- (b) Find the probability that the two balls chosen are the same colour.

[3]

$$\frac{3}{5} = 0.6 \text{ and } \frac{2}{5} = 0.4$$

$$P(\text{red, red}) = 0.6 \times 0.25 = 0.15$$

$$P(\text{blue, blue}) = 0.4 \times 0.75 = 0.3$$

$$P(\text{same colour}) = 0.15 + 0.3$$

$$= 0.45$$



9. Solve the following simultaneous equations using an algebraic (not graphical) method.
Do not use a trial and improvement method.
You must show all your working.

[4]

$$3x + 5y = -2$$

$$5x + 4y = -12$$

$$(1) \quad 3x + 5y = -2 \quad \times 4: \quad 12x + 20y = -8$$

$$(2) \quad 5x + 4y = -12 \quad \times 5: \quad 25x + 20y = -60$$

$$\text{Subtract: } 25x - 12x = -60 - (-8)$$

$$13x = -52$$

$$x = -4$$

$$\text{Substitute } x = -4 \text{ into (1): } 3(-4) + 5y = -2$$

$$-12 + 5y = -2$$

$$5y = 10$$

$$y = 2$$

$$\text{Check in (2): } 5(-4) + 4(2) = -20 + 8 = -12 \quad \checkmark$$

$$x = -4, \quad y = 2$$



10. (a) Expand and simplify $(2h + 3t)(5h - 7t)$.

[3]

$$(2h + 3t)(5h - 7t) = 10h^2 - 14ht + 15ht - 21t^2$$
$$= 10h^2 + ht - 21t^2$$

- (b) Simplify $\frac{7(d+5)^8}{(d+5)^{-2}}$.

[1]

$$7(d+5)^{8-(-2)}$$
$$= 7(d+5)^{10}$$



11. A cone is joined to a hemisphere, as shown below.
The cone has a base radius of 8 cm and a slant height of 17 cm.
The hemisphere has the same radius as the cone.
Calculate the **surface area** of the composite solid.

[3]

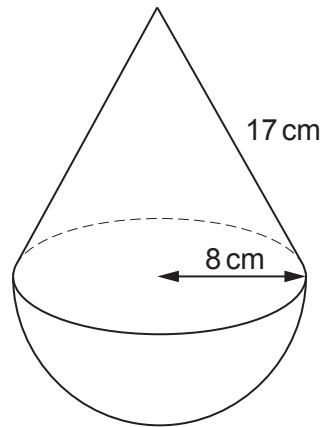


Diagram not drawn to scale

$$\text{Curved surface area of cone} = \pi r l = \pi \times 8 \times 17 = 136\pi$$

$$\text{Curved surface area of hemisphere} = \frac{1}{2} \times 4\pi r^2 = 2 \times \pi \times 8^2 = 128\pi$$

$$\text{Total surface area} = 136\pi + 128\pi = 264\pi$$

$$= 829.3804605\dots$$

$$\text{Surface area} = 829.4 \text{ cm}^2$$



12. Solve the equation $59x^2 - 7x - 13 = 0$.
Give your answers correct to 2 decimal places.
You must show all your working.

[3]

$$a = 59, b = -7, c = -13$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 59 \times (-13)}}{2 \times 59}$$

$$x = \frac{7 \pm \sqrt{3117}}{118} = \frac{7 \pm 55.83009941...}{118}$$

$$x = 62.83009941... \div 118 = 0.5324584696...$$

$$\text{or } x = -48.83009941... \div 118 = -0.4138144018...$$

$$x = 0.53 \text{ or } x = -0.41$$

13. Two similar shapes have perimeters of 241 cm and 719 cm.
The area of the smaller shape is 2063 cm^2 .

Calculate the area of the larger shape.
Give your answer in m^2 .

[4]

$$\text{Linear scale factor} = 719 \div 241 = 2.983402490...$$

$$\text{Area scale factor} = (719 \div 241)^2 = 8.900690415...$$

$$\text{Area of larger shape} = 2063 \times 8.900690415...$$

$$= 18362.12433 \text{ cm}^2$$

$$1 \text{ m}^2 = 100 \times 100 = 10000 \text{ cm}^2$$

$$18362.12433 \div 10000 = 1.836212433 \text{ m}^2$$

$$\text{Area of larger shape} = 1.84 \text{ m}^2$$



14. DE is the tangent to the circle at point A , as shown below.

$BC = 7\text{ cm}$ and $AC = 13\text{ cm}$.

$\hat{BAD} = 68^\circ$ and $\hat{CAE} = 80^\circ$.

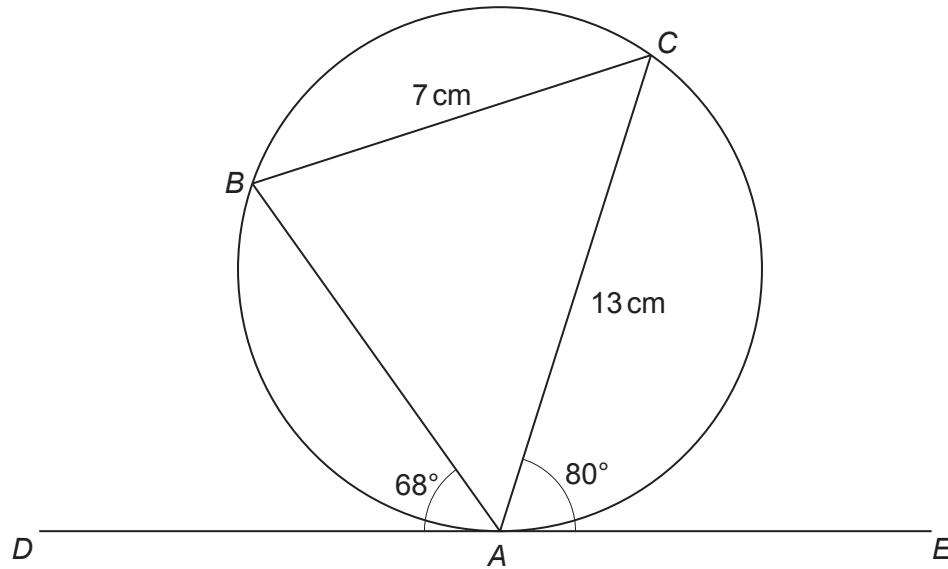


Diagram not drawn to scale

- (a) Find the size of $\hat{ACB}$.
State the angle property you have used to find your answer.

[2]

Angle $ACB = 68^\circ$

*Alternate segment theorem: the angle between a tangent and a chord
is equal to the angle in the alternate segment.*

- (b) Calculate the area of the triangle ABC .

[2]

Area = $\frac{1}{2} \times AC \times BC \times \sin ACB$

= $\frac{1}{2} \times 13 \times 7 \times \sin 68^\circ$

= 42.18686538...

Area of triangle $ABC = 42.2\text{ cm}^2$



15. Write down an **irrational** number whose value is between 9 and 10.
Write your answer in the box below.

[1]

$\sqrt{81} = 9$ and $\sqrt{100} = 10$, so $\sqrt{90}$ is between 9 and 10.

$\sqrt{90} = 9.486832981\dots$ and 90 is not a square number, so $\sqrt{90}$ is irrational.

Irrational number is:

$\sqrt{90}$

16. Fully factorise the expression $k^3p - kp^3$.

[3]

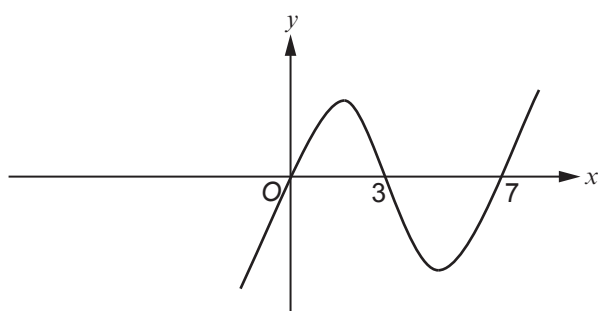
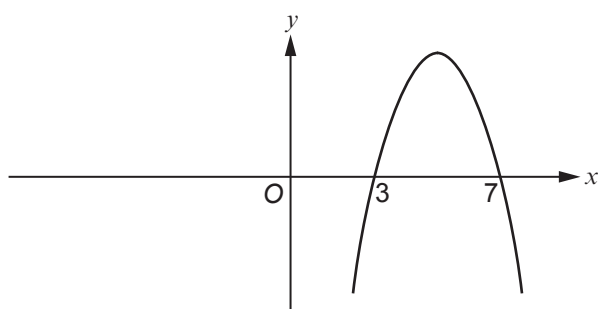
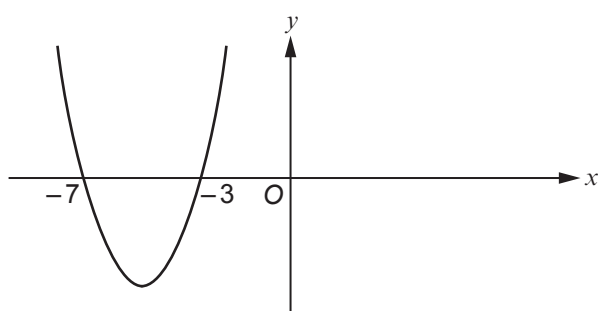
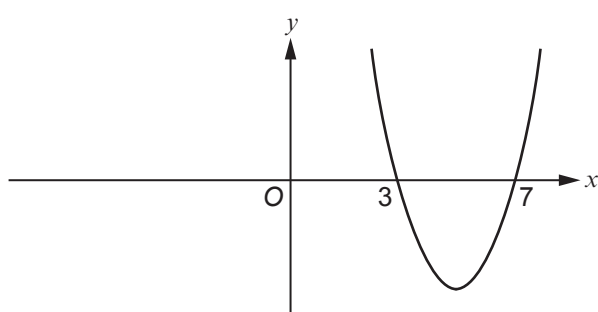
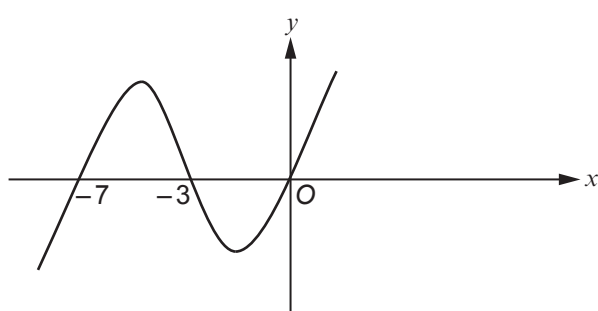
$$k^3p - kp^3 = kp(k^2 - p^2)$$

$$= kp(k + p)(k - p)$$



17. The equation $y = (x-3)(x-7)$ describes only **one** of the graphs below.
Put a tick (✓) in the box next to the graph that correctly shows this equation.

[1]

☐☐☐☒☐

18. The following nine cards are placed in a box.



Catherine chooses **three** cards at random from the box, without replacement.

- (a) Calculate the probability that the three cards drawn show the letters 'A', 'F' and 'O' in that order. [2]

There is 1 A, 1 F and 2 O's among the 9 cards.

$$P(A, F, O) = \frac{1}{9} \times \frac{1}{8} \times \frac{2}{7}$$

$$= \frac{2}{504} = \frac{1}{252}$$

- (b) Calculate the probability that two of the three cards show the same letter. [3]

Two the same means two O's or two N's (there are 2 of each).

$$P(O, O, \text{other}) = \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} = \frac{14}{504}$$

The other card can come 1st, 2nd or 3rd, so there are 3 orders.

$$P(\text{two O's}) = 3 \times \frac{14}{504} = \frac{42}{504}$$

$$P(\text{two N's}) = 3 \times \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} = \frac{42}{504}$$

$$P(\text{two the same}) = \frac{42}{504} + \frac{42}{504} = \frac{84}{504} = \frac{1}{6}$$



19. Make c the subject of the following formula.

[5]

$$a(b + c^2) + d(e - c^2) = f$$

$$ab + ac^2 + de - dc^2 = f$$

$$ac^2 - dc^2 = f - ab - de$$

$$c^2(a - d) = f - ab - de$$

$$c^2 = \frac{f - ab - de}{a - d}$$

$$c = \pm \sqrt{\frac{f - ab - de}{a - d}}$$

20. The diagram shows a triangle ABC and a circle with centre C .
The points B and D lie on the circumference of the circle.

The length of the line AB is 19 cm.
The length of the line AC is 29 cm.
The radius of the circle is x cm.

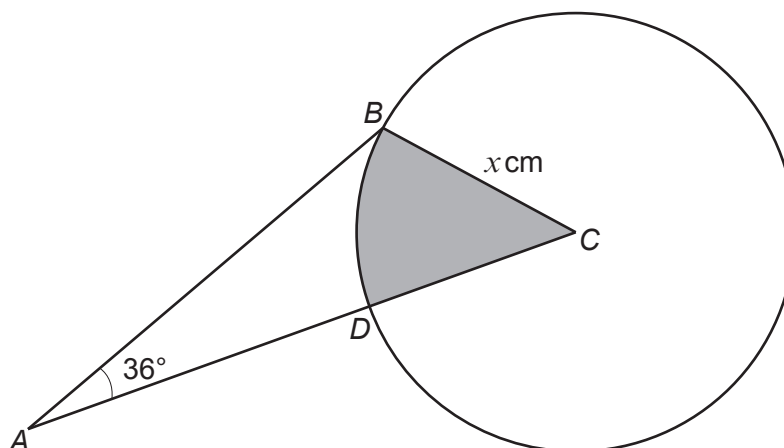


Diagram not drawn to scale



Calculate the area of the shaded sector BCD .

[8]

Using the cosine rule in triangle ABC to find $BC = x$:

$$x^2 = 19^2 + 29^2 - 2 \times 19 \times 29 \times \cos 36^\circ$$

$$x^2 = 361 + 841 - 1102 \times \cos 36^\circ$$

$$x^2 = 310.4632722...$$

$$x = \sqrt{310.4632722...} = 17.61996800... \text{ cm}$$

Using the sine rule to find angle BCD (= angle BCA):

$$\frac{\sin BCA}{19} = \frac{\sin 36^\circ}{17.61996800...}$$

$$\sin BCA = \frac{19 \times \sin 36^\circ}{17.61996800...} = 0.6338217978...$$

$$\text{Angle } BCA = \sin^{-1}(0.6338217978...) = 39.33265409...^\circ$$

$$\text{So angle } BCD = 39.33265409...^\circ$$

$$\text{Area of sector } BCD = \theta \div 360 \times \pi r^2$$

$$= \frac{39.33265409...}{360} \times \pi \times 17.61996800...^2$$

$$= 106.5640838...$$

$$\text{Area of sector } BCD = 106.6 \text{ cm}^2$$

END OF PAPER



[illegible]