

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
Other Names		0



GCSE

3300U60-1



MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

WEDNESDAY, 14 NOVEMBER 2018 – MORNING

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** the questions in the spaces provided.
If you run out of space, use the continuation page at the back of the booklet. Question numbers must be given for all work written on the continuation page.
Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
In question 7, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	3	
3.	6	
4.	3	
5.	4	
6.	3	
7.	6	
8.	6	
9.	7	
10.	1	
11.	2	
12.	4	
13.	5	
14.	4	
15.	6	
16.	3	
17.	7	
18.	6	
Total	80	

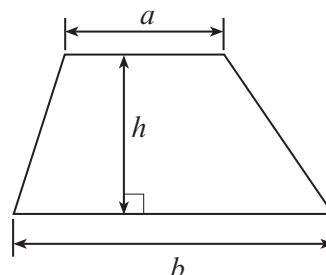
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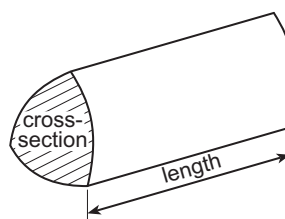
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Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

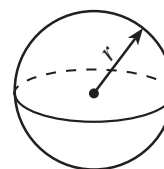


Volume of prism = area of cross-section $\times$ length



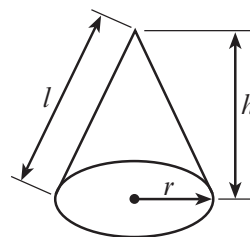
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

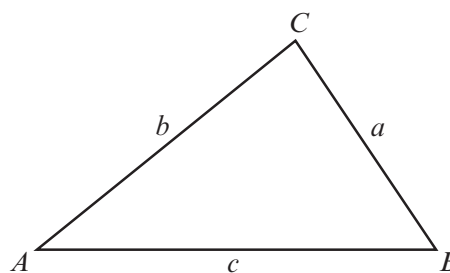


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) A number is decreased by 12% of its value.
This is done 3 times, each time decreasing the previous value by 12%.
Circle the multiplier that you would use to find the value after the 3 decreases. [1]

$\times 0.36$

$\times 0.88^3$

$\times 0.12^3$

$\times 0.3^{12}$

$\times 0.3^{88}$

- (b) Calculate the percentage change when 42.5 is increased to 45.9. [3]

$$\text{Change} = 45.9 - 42.5 = 3.4$$

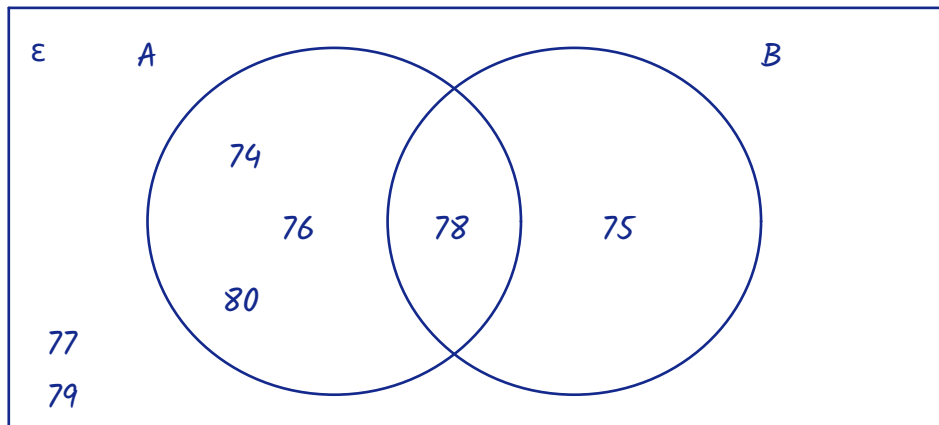
$$\text{Percentage change} = 3.4 \div 42.5 \times 100$$

$$= 0.08 \times 100$$

$$= 8\% \text{ increase}$$

2. Display the following information in a Venn diagram. [3]

- Universal Set (ϵ): Integers between 74 and 80 inclusive.
- Set A: Even numbers.
- Set B: Multiples of 3.



3. The table below shows some of the values of $y = x^2 + 4x - 1$ for values of x from -5 to 2 .

x	-5	-4	-3	-2	-1	0	1	2
$y = x^2 + 4x - 1$	4	-1	-4	-5	-4	-1	4	11

- (a) Complete the table by finding the value of y for $x = -2$ and for $x = 2$. [2]

$$x = -2: y = (-2)^2 + 4(-2) - 1 = 4 - 8 - 1 = -5$$

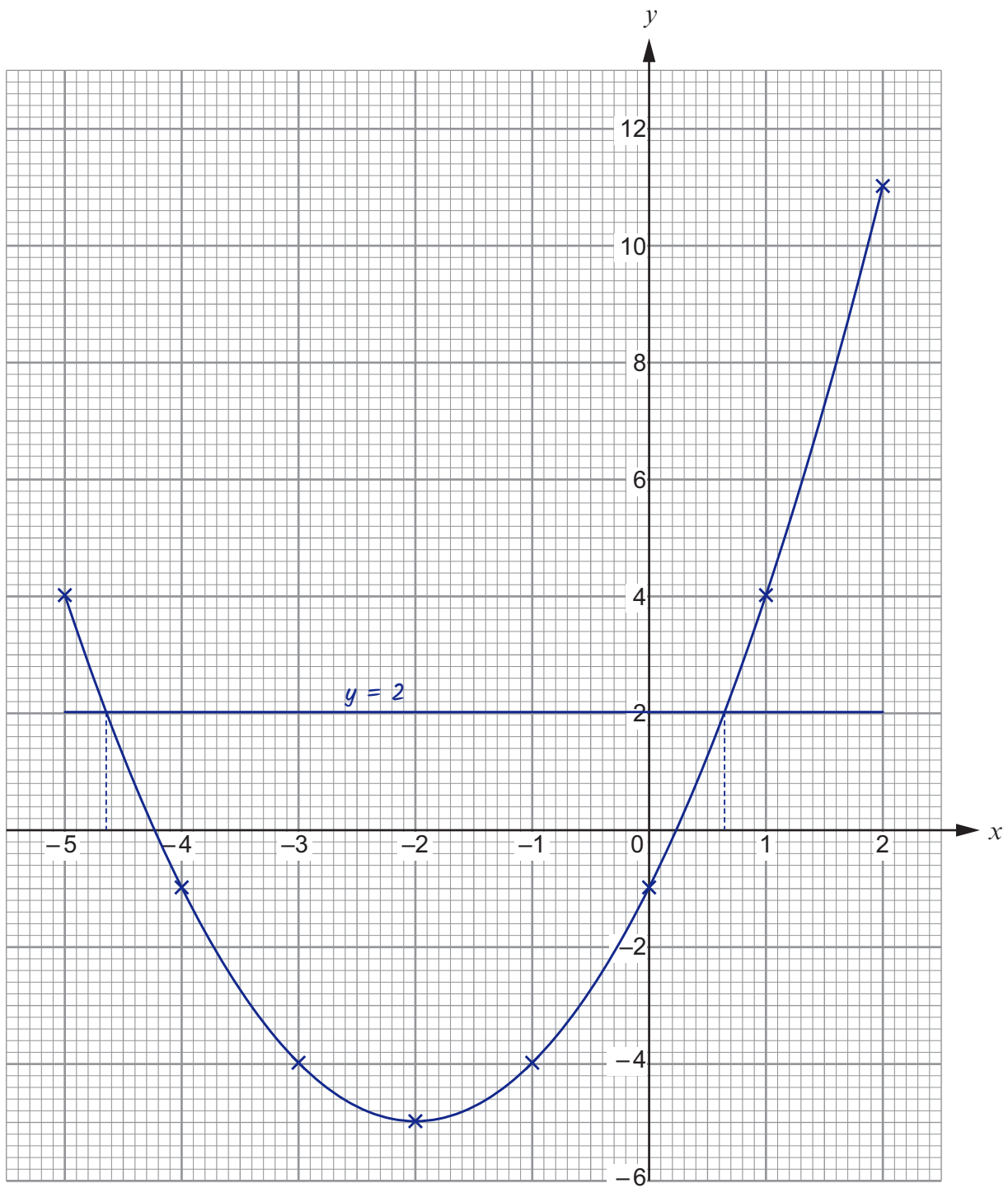
$$x = 2: y = 2^2 + 4(2) - 1 = 4 + 8 - 1 = 11$$

- (b) On the graph paper opposite, draw the graph of $y = x^2 + 4x - 1$ for values of x from -5 to 2 . [2]

- (c) Draw the line $y = 2$ on the graph paper.
Write down the values of x where the line $y = 2$ cuts the curve $y = x^2 + 4x - 1$. [2]

Values of x are -4.65 and 0.65





4. Jonathan writes a whole number on a card.

- The number is between 55 and 83 inclusive.
- The square root of the number is greater than 8 but less than 9.
- The highest common factor (HCF) of 49 and the number is 7.
- $\frac{3}{5}$ of the number is a multiple of 6.

What number is on Jonathan's card?

[3]

$$8^2 = 64 \text{ and } 9^2 = 81, \text{ so } 64 < \text{number} < 81$$

Possible numbers: 65 to 80

HCF of 49 and the number is 7, so the number is a multiple of 7 but not a multiple of 49 ($49 = 7 \times 7$).

Multiples of 7 from 65 to 80: 70, 77

Neither is a multiple of 49, so check the last condition:

$$\frac{3}{5} \times 70 = 42 = 6 \times 7, \text{ a multiple of 6 } \checkmark$$

$$\frac{3}{5} \times 77 = 46.2, \text{ not a multiple of 6 } \times$$

So the number on the card is 70.

The number on Jonathan's card is 70.....



5. A solution of the equation

$$x^3 - 13x = 107$$

lies between 5 and 6.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 - 13x$	
5	$125 - 65 = 60$	too low
6	$216 - 78 = 138$	too high
5.5	$166.375 - 71.5 = 94.875$	too low
5.7	$185.193 - 74.1 = 111.093$	too high
5.6	$175.616 - 72.8 = 102.816$	too low
5.65	$180.362125 - 73.45 = 106.912125$	too low

At $x = 5.65$, $x^3 - 13x$ is less than 107 (too low) and

at $x = 5.7$ it is greater than 107 (too high), so the solution

lies between 5.65 and 5.7, which rounds to 5.7.

$x = 5.7$ (correct to 1 decimal place)



6. (a) What is the gradient of the straight line with equation $6y = 3x + 7$?
Circle the correct answer.

[1]

$$\left(\frac{1}{2}\right)$$

6

2

3

$$\frac{7}{6}$$

$$y = \frac{1}{2}x + \frac{7}{6}, \text{ so the gradient is } \frac{1}{2}$$

- (b) What is the value of y at the point where the line $5x + y + 3 = 0$ crosses the y -axis?
Circle the correct answer.

[1]

0

-5

3

$$(-3)$$

$$\frac{5}{3}$$

$$\text{At the } y\text{-axis } x = 0: y + 3 = 0, \text{ so } y = -3$$

- (c) What are the coordinates of the point where the lines with equations $x + y = 7$ and $x - y = 3$ intersect?
Circle the correct answer.

[1]

(4, 3)

(7, 4)

$$(5, 2)$$

(3, 7)

(-5, 2)

$$\text{Add: } 2x = 10, \text{ so } x = 5$$

$$y = 7 - 5 = 2, \text{ so the point is } (5, 2)$$



7. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

The perimeter of a square is 76.4 m.

Calculate the length of its diagonal.
You must show all your working.

[4 + 2 OCW]

A square has 4 equal sides, so the perimeter is $4 \times \text{side}$.

Length of one side = $76.4 \div 4 = 19.1 \text{ m}$

The diagonal splits the square into two right-angled triangles, with the diagonal as the hypotenuse.

Using Pythagoras' theorem:

$$\text{diagonal}^2 = 19.1^2 + 19.1^2$$

$$\text{diagonal}^2 = 364.81 + 364.81 = 729.62$$

$$\text{diagonal} = \sqrt{729.62}$$

$$\text{diagonal} = 27.01147904... \text{ m}$$

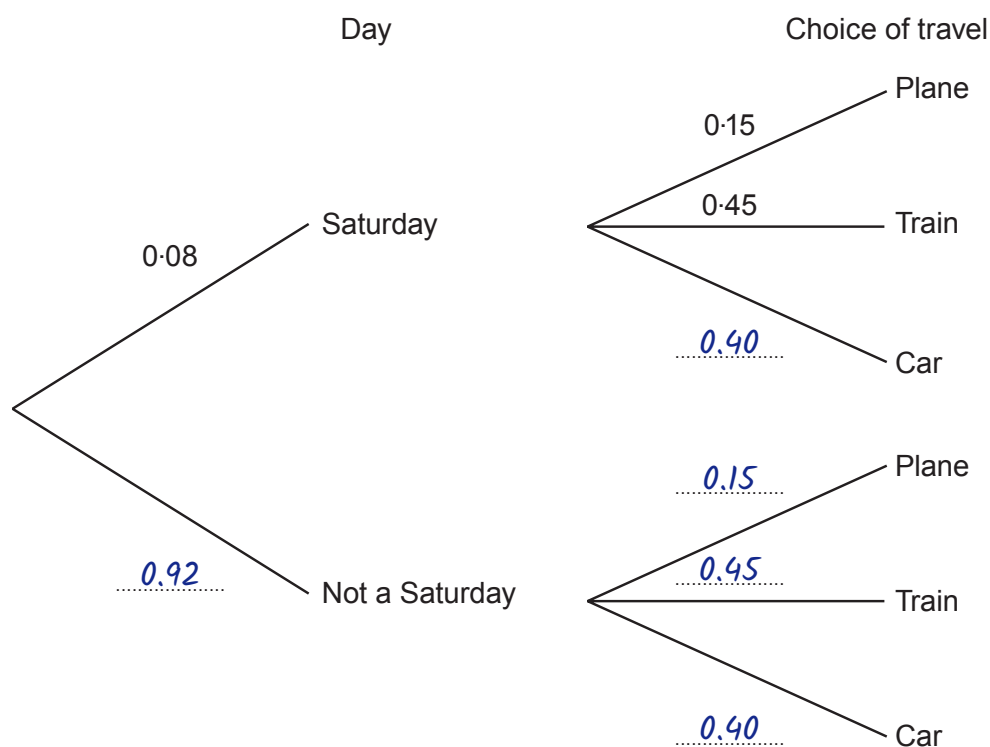
The length of the diagonal is 27.0 m (to 1 decimal place).



8. Alwena regularly travels from Anglesey to Cardiff to attend meetings. For each meeting, she chooses one of three ways to travel: by plane, train or car. The probability of a meeting being held on a Saturday is 0.08. The probability that Alwena travels by plane to a meeting is 0.15. The probability that she travels by train is 0.45. Her decision on how to travel is independent of the day on which the meeting is held.

(a) Complete the following tree diagram.

[3]



$$P(\text{not a Saturday}) = 1 - 0.08 = 0.92$$

$$P(\text{car}) = 1 - 0.15 - 0.45 = 0.40$$

- (b) A meeting is chosen at random. Calculate the probability that the meeting is held on a Saturday and that Alwena travels by plane or by car. [3]

$$P(\text{Saturday and plane}) + P(\text{Saturday and car})$$

$$= 0.08 \times 0.15 + 0.08 \times 0.40$$

$$= 0.012 + 0.032$$

$$= 0.044$$



9. The diagram below shows two right-angled triangles ABC and APQ .
 $AB = 8.2$ cm, $BC = 6.4$ cm and $PQ = 7.9$ cm.

$$\hat{CAQ} = 90^\circ.$$

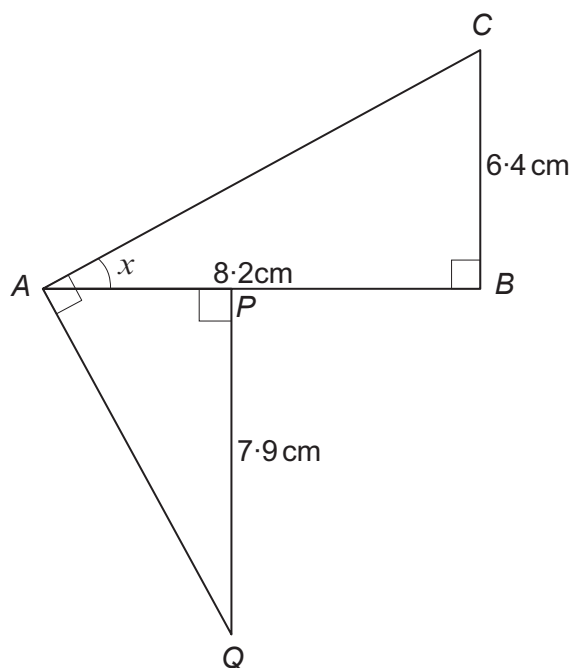


Diagram not drawn to scale

- (a) Calculate the size of angle x .

[3]

$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{6.4}{8.2}$$

$$x = \tan^{-1}(0.7804878\dots)$$

$$x = 37.97160376\dots^\circ$$

$$x = 38.0^\circ$$

- (b) Calculate the length AQ .

[4]

$$\text{Angle } PAQ = 90^\circ - 37.97\dots^\circ = 52.02839624\dots^\circ$$

In right-angled triangle APQ , PQ is opposite angle PAQ :

$$\sin 52.028\dots^\circ = \frac{7.9}{AQ}$$

$$AQ = 7.9 \div \sin 52.028\dots^\circ$$

$$AQ = 10.02136474\dots$$

$$AQ = 10.0 \text{ cm}$$



10. A googol is the number 1×10^{100} .
Circle the value that is 90% of a googol.

[1]

1×9^{100}

1×10^{90}

1×9^{90}

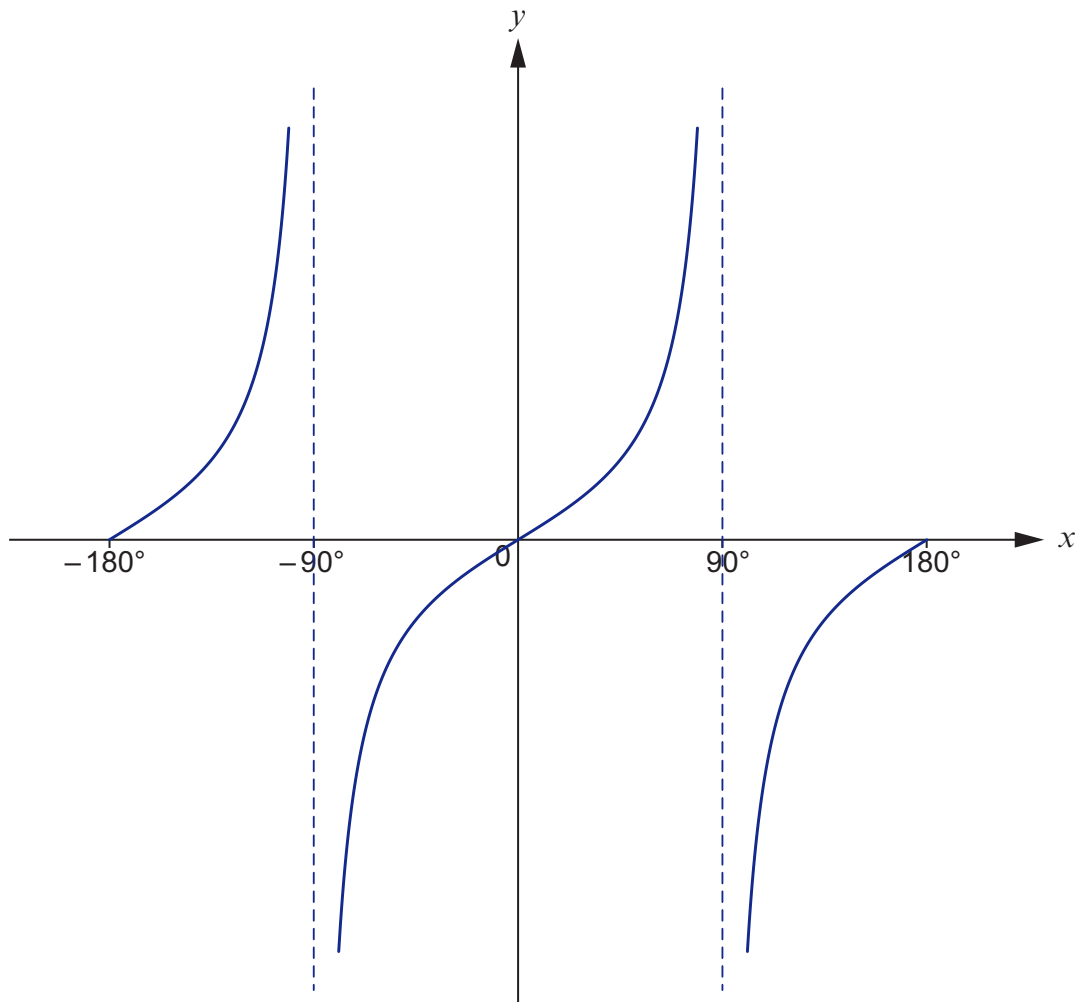
9×10^{90}

9×10^{99}

$$90\% \text{ of } 1 \times 10^{100} = 0.9 \times 10^{100} = 9 \times 10^{99}$$

11. Sketch the curve $y = \tan x$, for values of x in the range $x = -180^\circ$ to $x = 180^\circ$.

[2]



12. Calculate the **total** surface area of a solid hemisphere with a base radius of 29 cm.

[4]

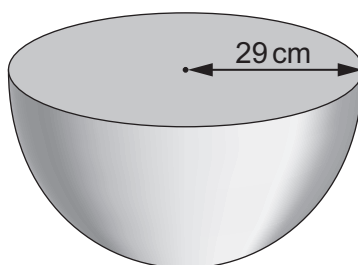


Diagram not drawn to scale

$$\text{Curved surface} = \frac{1}{2} \times \text{surface area of sphere} = \frac{1}{2} \times 4\pi r^2$$

$$= 2 \times \pi \times 29^2 = 1682\pi = 5284.158843...$$

$$\text{Flat base is a circle: area} = \pi r^2$$

$$= \pi \times 29^2 = 841\pi = 2642.079422...$$

$$\text{Total surface area} = 1682\pi + 841\pi$$

$$= 2523\pi$$

$$= 7926.238265...$$

$$\text{Total surface area} = 7926.24 \text{ cm}^2$$



13. (a) Fully factorise the expression $c^3 - cd^2$. [3]

$$c^3 - cd^2 = c(c^2 - d^2)$$

$$= c(c + d)(c - d)$$

- (b) Factorise and simplify $5(e - 1)^2 + 3(e - 1)$. [2]

$$5(e - 1)^2 + 3(e - 1) = (e - 1)[5(e - 1) + 3]$$

$$= (e - 1)(5e - 5 + 3)$$

$$= (e - 1)(5e - 2)$$



14. The diagram below shows a cuboid with $HG = 3\text{ cm}$, $AE = 5\text{ cm}$ and $EH = 7\text{ cm}$.

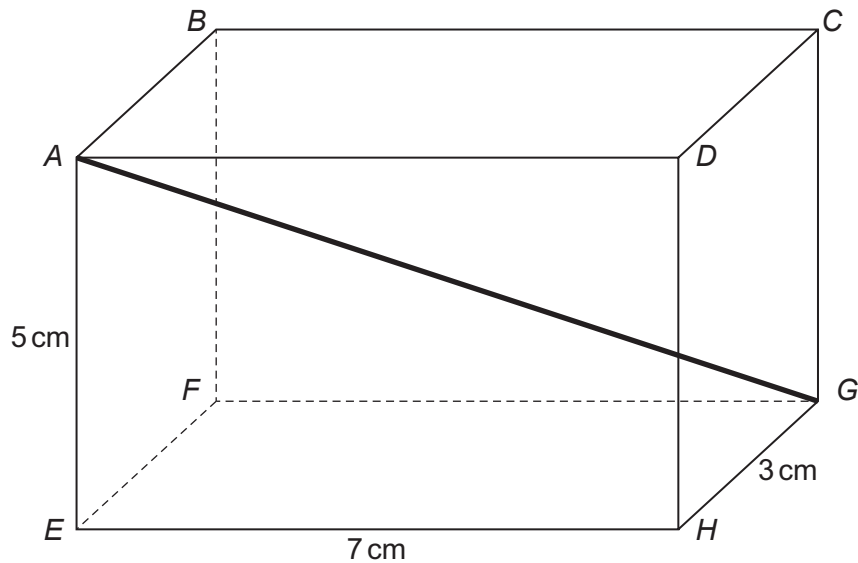


Diagram not drawn to scale

Calculate the length of the line AG.
Give your answer correct to 2 decimal places.

[4]

Diagonal of base EG: $EG^2 = EH^2 + HG^2$

$$EG^2 = 7^2 + 3^2 = 49 + 9 = 58$$

Triangle AEG is right-angled at E:

$$AG^2 = AE^2 + EG^2 = 5^2 + 58$$

$$AG^2 = 25 + 58 = 83$$

$$AG = \sqrt{83} = 9.110433579\dots$$

$$AG = 9.11\text{ cm (2 d.p.)}$$



15. (a) Three unbiased 6-sided dice are rolled at the same time.
Calculate the probability they will all land on a 5.

[2]

$$P(5) = \frac{1}{6} \text{ on each die}$$

$$P(\text{all three land on } 5) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{216}$$

- (b) Four unbiased 6-sided dice are now rolled.
Calculate the probability that at least three of the dice will land on a 5.

[4]

$$P(5) = \frac{1}{6}, \quad P(\text{not } 5) = \frac{5}{6}$$

$$P(\text{four } 5\text{s}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{1296}$$

Exactly three 5s: the non-5 can be any one of the 4 dice

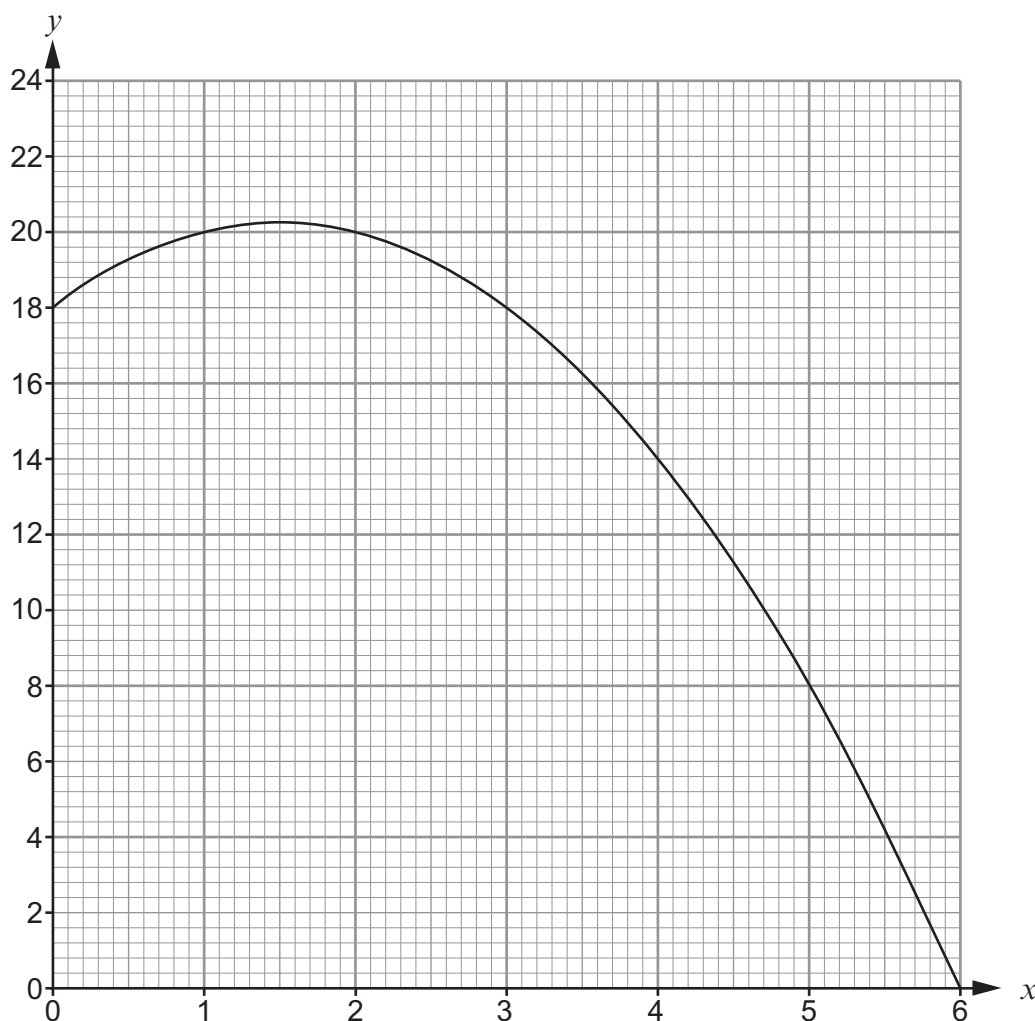
$$P(\text{three } 5\text{s}) = \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times 4 = \frac{20}{1296}$$

$$P(\text{at least three } 5\text{s}) = \frac{1}{1296} + \frac{20}{1296}$$

$$= \frac{21}{1296} = \frac{7}{432}$$



16. The graph of $y = 18 + 3x - x^2$, for values of x from $x = 0$ to $x = 6$, is drawn below.



Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$, $x = 5$ and $x = 6$, to estimate the area of the region bounded by the curve, the positive x -axis and the positive y -axis. [3]

$h = 1$. Reading y from the graph:

$x:$ 0 1 2 3 4 5 6

$y:$ 18 20 20 18 14 8 0

Area $\approx \frac{1}{2}h[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$

$= \frac{1}{2} \times 1 \times [18 + 0 + 2(20 + 20 + 18 + 14 + 8)]$

$= \frac{1}{2} \times [18 + 160] = \frac{1}{2} \times 178$

$= 89 \text{ square units}$



17. The diagram below shows two triangles ABE and BCD , which are constructed from the straight lines AD and EC crossing at point B .

$AE = 11 \text{ cm}$, $EB = 13 \text{ cm}$, $BC = 10 \text{ cm}$ and $BD = 7 \text{ cm}$.

$\angle EAB = 37^\circ$.

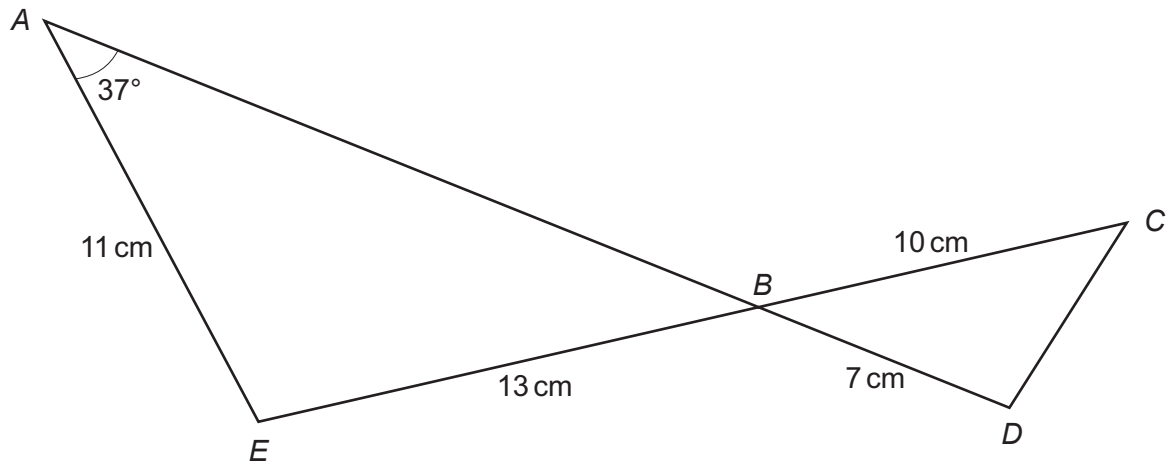


Diagram not drawn to scale.

Calculate the length CD .

[7]

Triangle ABE , sine rule: $\frac{\sin ABE}{11} = \frac{\sin 37^\circ}{13}$

$\sin ABE = 11 \times \sin 37^\circ \div 13 = 0.5092280965\dots$

$\text{Angle } ABE = \sin^{-1}(0.5092\dots) = 30.61242731\dots^\circ$

$\text{Angle } CBD = \text{angle } ABE = 30.612\dots^\circ$

(vertically opposite angles are equal)

Triangle BCD , cosine rule:

$CD^2 = BC^2 + BD^2 - 2 \times BC \times BD \times \cos CBD$

$CD^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 30.612\dots^\circ$

$CD^2 = 149 - 120.4884236\dots = 28.51157642\dots$

$CD = \sqrt{28.51157642\dots} = 5.339623247\dots$

$CD = 5.3 \text{ cm (1 d.p.)}$



18. Solve the equation $\frac{7x+1}{5x+2} = \frac{1}{x+3}$.

Give your answers correct to 2 decimal places.
You must show all your working.

[6]

Cross-multiply: $(7x + 1)(x + 3) = 5x + 2$

$$7x^2 + 21x + x + 3 = 5x + 2$$

$$7x^2 + 22x + 3 = 5x + 2$$

$$7x^2 + 17x + 1 = 0$$

$$a = 7, b = 17, c = 1$$

$$x = \frac{-17 \pm \sqrt{17^2 - 4 \times 7 \times 1}}{2 \times 7}$$

$$x = \frac{-17 \pm \sqrt{261}}{14}$$

$$x = \frac{-17 + 16.15549442...}{14} = -0.0603218...$$

$$x = \frac{-17 - 16.15549442...}{14} = -2.3682496...$$

$$x = -0.06 \text{ or } x = -2.37 \text{ (2 d.p.)}$$

END OF PAPER



[illegible]