

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
Other Names		0



GCSE

3300U60-1



MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

MONDAY, 13 NOVEMBER 2017 – MORNING

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page(s) at the back of the booklet. Question numbers must be given for all work written on the continuation page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 4(a), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	3	
2.	4	
3.	6	
4.	10	
5.	5	
6.	5	
7.	6	
8.	5	
9.	7	
10.	3	
11.	4	
12.	2	
13.	3	
14.	4	
15.	3	
16.	2	
17.	8	
Total	80	

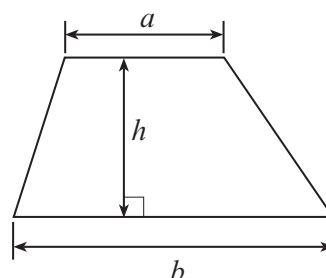
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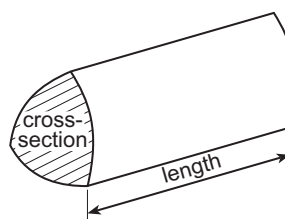
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Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

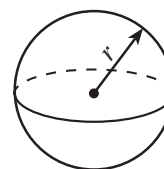


Volume of prism = area of cross-section $\times$ length



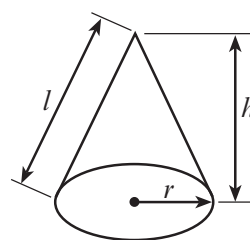
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

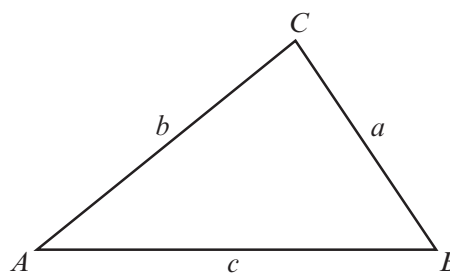


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. Simplify each of the following and circle the correct answer in each case.

(a) $6p^6 \times 3p^3$

[1]

$9p^9$

$9p^{18}$

$18p^{18}$

$18p^2$

$18p^9$

(b) $3.4g^8 \div 13.6g^2$

[1]

$\frac{g^4}{4}$

$\frac{g^6}{4}$

$4g^4$

$4g^6$

$0.4g^6$

(c) $\frac{m^3 \times m^6}{m^9}$

[1]

1

m

m^2

m^4

4



2. A solution of the equation

$$x^3 + 2x = 91$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 + 2x$	
4	$64 + 8 = 72$	too low
5	$125 + 10 = 135$	too high
4.5	$91.125 + 9 = 100.125$	too high
4.3	$79.507 + 8.6 = 88.107$	too low
4.4	$85.184 + 8.8 = 93.984$	too high
4.35	$82.312875 + 8.7 = 91.012875$	too high

4.3 is too low and 4.35 is too high,

so the solution lies between 4.3 and 4.35.

$x = 4.3$ (1 d.p.)



3. ABC is an isosceles triangle with $AB = AC$.

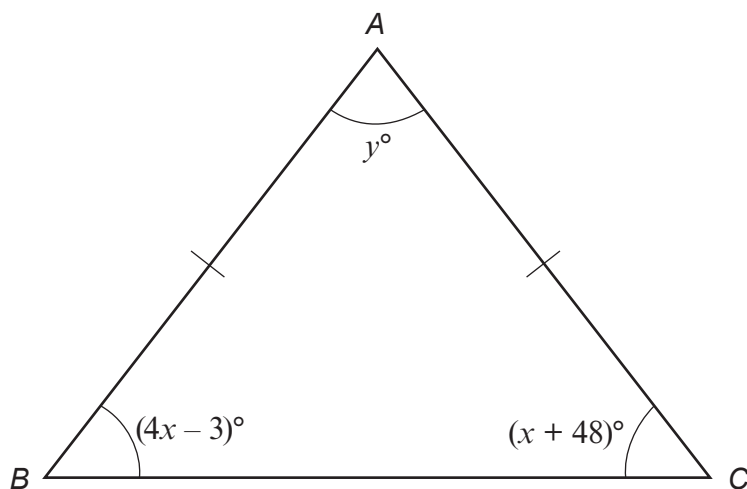


Diagram not drawn to scale

Calculate the value of y .

[6]

$AB = AC$, so angle $ABC = \text{angle } ACB$ (base angles of an isosceles triangle)

$$4x - 3 = x + 48$$

$$3x = 51$$

$$x = 17$$

$$\text{Angle } ABC = 4 \times 17 - 3 = 65^\circ$$

$$\text{Angle } ACB = 17 + 48 = 65^\circ$$

Angles in a triangle add up to 180° :

$$y = 180 - 2 \times 65$$

$$y = 50$$



4. A triangular prism of length 2 metres is shown below.

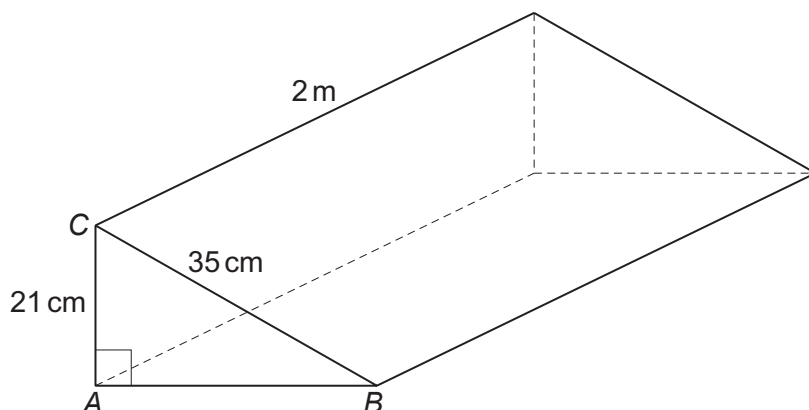


Diagram not drawn to scale

$AC = 21 \text{ cm}$, $BC = 35 \text{ cm}$ and $\hat{BAC} = 90^\circ$.

- (a) In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Calculate the area of triangle ABC.

Give your answer in cm^2 .

You must show all your working.

[5 + 2 OCW]

BC is the hypotenuse, so using Pythagoras' theorem:

$$35^2 = 21^2 + AB^2$$

$$AB^2 = 35^2 - 21^2 = 1225 - 441 = 784$$

$$AB = \sqrt{784} = 28 \text{ cm}$$

Area of triangle ABC = $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$$\text{Area} = \frac{1}{2} \times 21 \times 28$$

$$\text{Area} = 588 \div 2$$

$$\text{Area of triangle ABC} = 294 \text{ cm}^2$$



- (b) Calculate the volume of the prism.
You must give the units of your answer.

[3]

$$\text{Length of prism} = 2 \text{ m} = 200 \text{ cm}$$

$$\text{Volume} = \text{area of cross-section} \times \text{length}$$

$$\text{Volume} = 294 \times 200$$

$$\text{Volume} = 58\,800 \text{ cm}^3$$



5. Find the answer to the following number problem.

[5]

'(the LCM of 12, 18 and 24) $\div$ (the HCF of 36 and 54)'.

$$12 = 2^2 \times 3$$

$$18 = 2 \times 3^2$$

$$24 = 2^3 \times 3$$

$$\text{LCM of 12, 18 and 24} = 2^3 \times 3^2 = 72$$

$$36 = 2^2 \times 3^2$$

$$54 = 2 \times 3^3$$

$$\text{HCF of 36 and 54} = 2 \times 3^2 = 18$$

$$72 \div 18 = 4$$

$$\text{Answer} = 4$$



6. (a) Rearrange the following formula to make x the subject.
Give your answer in its simplest form. [3]

$$2(x + y) = 7y - 3$$

$$2x + 2y = 7y - 3$$

$$2x = 7y - 3 - 2y$$

$$2x = 5y - 3$$

$$x = \frac{5y - 3}{2}$$

- (b) Write down the n th term of the following sequence. [2]

3, 6, 11, 18, 27, ...

First differences: 3, 5, 7, 9 second difference 2, so n^2

n^2 : 1, 4, 9, 16, 25

sequence - n^2 : 2, 2, 2, 2, 2

n th term = $n^2 + 2$



7. The diagram shows two right-angled triangles, joined together along a common side.

$\hat{SPQ} = 90^\circ$, $\hat{SQR} = 90^\circ$, $\hat{SQP} = 38^\circ$, $PS = 8 \text{ cm}$ and $QR = 15 \text{ cm}$.

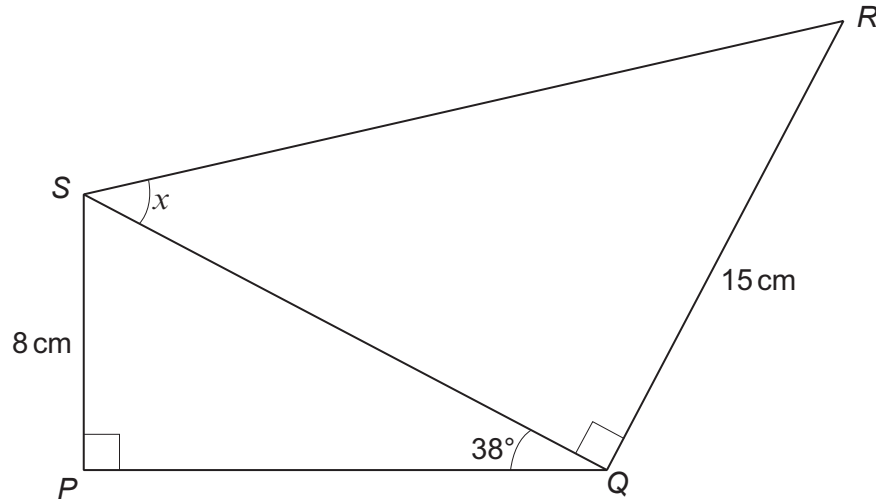


Diagram not drawn to scale

Calculate the size of angle x .

[6]

Triangle SPQ: $\sin 38^\circ = \frac{8}{QS}$

$QS = \frac{8}{\sin 38^\circ}$

$QS = 12.99415396... \text{ cm}$

Triangle SQR: $\tan x = \frac{15}{QS} = \frac{15}{12.99415396...}$

$x = \tan^{-1}(15 \div 12.99415396...)$

$x = 49.09837127...^\circ$

$x = 49.1^\circ$



8. All the members of a farming club visited the Royal Welsh Agricultural Show. They all travelled to the show either by bus or by car. None of them visited the show on more than one day. The decision to travel by car or by bus was independent of the day of the visit.

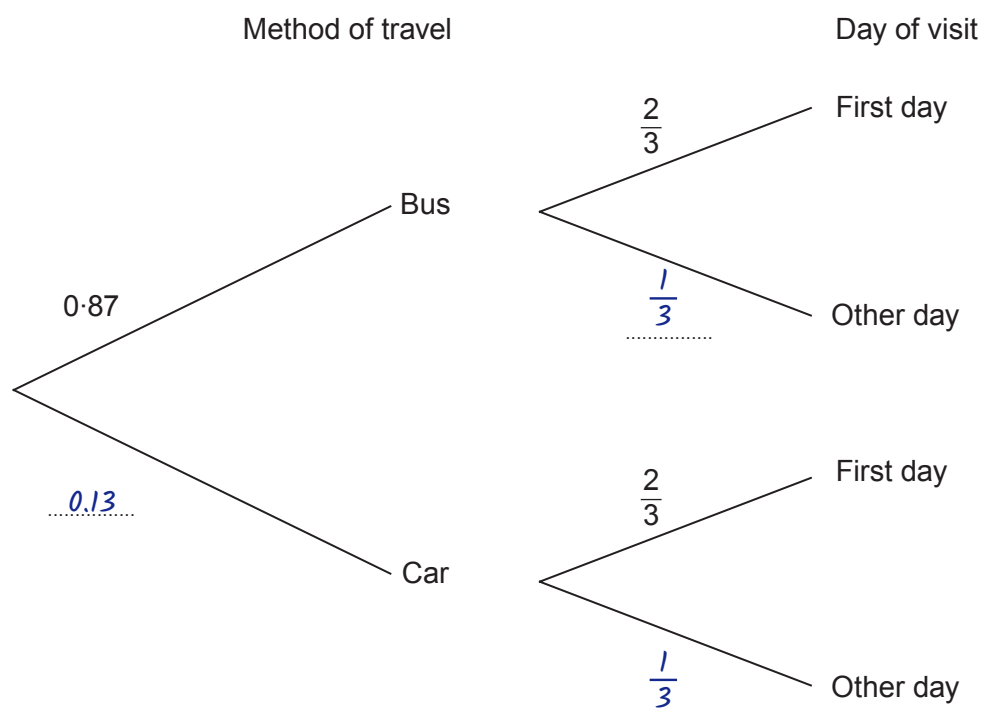
A member of the club was selected at random.

The probability that this member travelled by bus was 0.87.

The probability that this member visited the show on the first day was $\frac{2}{3}$.

- (a) Complete the tree diagram shown below.

[2]



- (b) What is the probability that a member, chosen at random, was **not** one of those who travelled by bus on the first day of the show? [3]

$$P(\text{bus and first day}) = 0.87 \times \frac{2}{3} = 0.58$$

$$P(\text{not bus on the first day}) = 1 - 0.58$$

$$= 0.42$$



9. (a) Show that $(10w + 3)(w - 1) - (2 - 3w)^2 \equiv w^2 + 5w - 7$. [4]

$$(10w + 3)(w - 1) = 10w^2 - 10w + 3w - 3$$

$$= 10w^2 - 7w - 3$$

$$(2 - 3w)^2 = (2 - 3w)(2 - 3w) = 4 - 6w - 6w + 9w^2$$

$$= 4 - 12w + 9w^2$$

$$(10w^2 - 7w - 3) - (4 - 12w + 9w^2)$$

$$= 10w^2 - 7w - 3 - 4 + 12w - 9w^2$$

$$= w^2 + 5w - 7 \text{ as required}$$

- (b) Use the quadratic formula to solve the equation $w^2 + 5w - 7 = 0$.
Give your answers correct to 2 decimal places. [3]

$$a = 1, b = 5, c = -7$$

$$w = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$w = \frac{-5 \pm \sqrt{53}}{2}$$

$$w = \frac{-5 + 7.280109889...}{2} \quad \text{or} \quad w = \frac{-5 - 7.280109889...}{2}$$

$$w = 1.140054945... \quad \text{or} \quad w = -6.140054945...$$

$$w = 1.14 \quad \text{or} \quad w = -6.14$$



10. The line GH is a tangent to the circle at point Y .
 The line EF is parallel to the line GH .
 The vertices of triangle EFY lie on the circle.
 $\hat{EYG} = 60^\circ$.

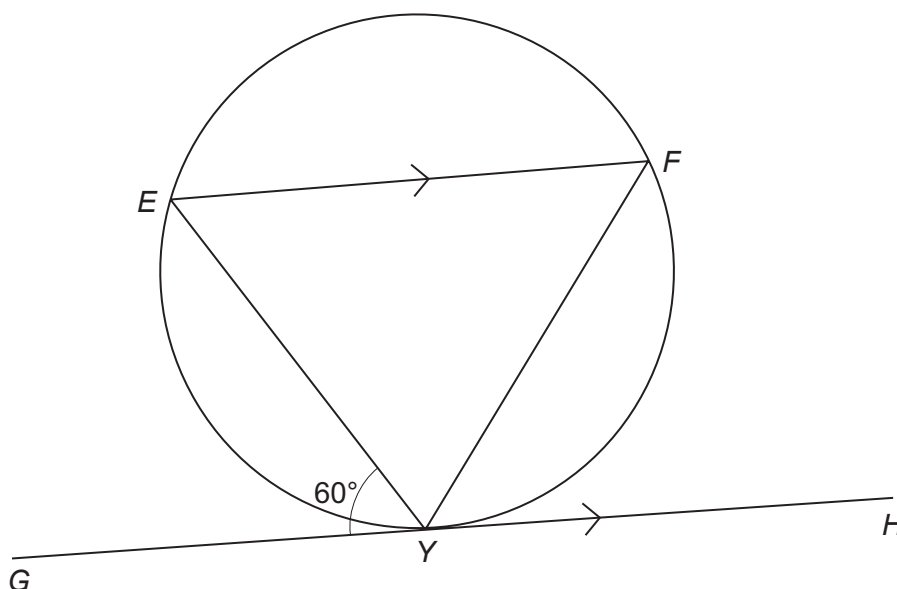


Diagram not drawn to scale

Prove that EFY is an equilateral triangle.
 Give a reason for each step to justify your proof.

[3]

Angle $EFY = 60^\circ$

(alternate segment theorem)

Angle $FEY = \text{angle } EYG = 60^\circ$

(alternate angles, EF is parallel to GH)

Angle $EYF = 180^\circ - 60^\circ - 60^\circ = 60^\circ$

(angles in a triangle add up to 180°)

All three angles are 60° , so EFY is an equilateral triangle.



11. A cone is joined to a cylinder, as shown below.
The cone has a base radius of 11 cm and a slant height of 13 cm.
The cylinder has the same radius, 11 cm, and a height of 17 cm.
Calculate the **total** surface area of the composite solid.

[4]

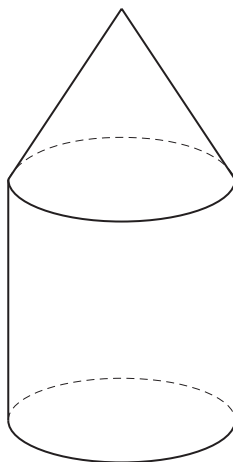


Diagram not drawn to scale

$$\text{Curved surface area of cone} = \pi r l = \pi \times 11 \times 13 = 143\pi$$

$$= 449.2477495... \text{ cm}^2$$

$$\text{Curved surface area of cylinder} = 2\pi r h = 2 \times \pi \times 11 \times 17 = 374\pi$$

$$= 1174.955652... \text{ cm}^2$$

$$\text{Base of cylinder} = \pi r^2 = \pi \times 11^2 = 121\pi$$

$$= 380.1327111... \text{ cm}^2$$

$$\text{Total} = 143\pi + 374\pi + 121\pi = 638\pi$$

$$= 2004.336113... \text{ cm}^2$$

$$\text{Total surface area} = 2004.3 \text{ cm}^2$$



12. The area of a rectangle is 137 cm^2 , correct to the nearest cm^2 .
Its width is 11 cm , correct to the nearest cm .

Calculate the greatest possible length of the rectangle.
Give your answer correct to 3 significant figures.

[2]

$$\text{Upper bound of area} = 137.5 \text{ cm}^2$$

$$\text{Lower bound of width} = 10.5 \text{ cm}$$

$$\text{Greatest length} = 137.5 \div 10.5$$

$$= 13.0952381\dots$$

$$= 13.1 \text{ cm (3 s.f.)}$$

13. A bag contains 5 red counters and 5 blue counters.
Three counters are drawn at random from the bag at the same time.
Calculate the probability that the three counters will be the same colour.

[3]

$$P(\text{all red}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720} = \frac{1}{12}$$

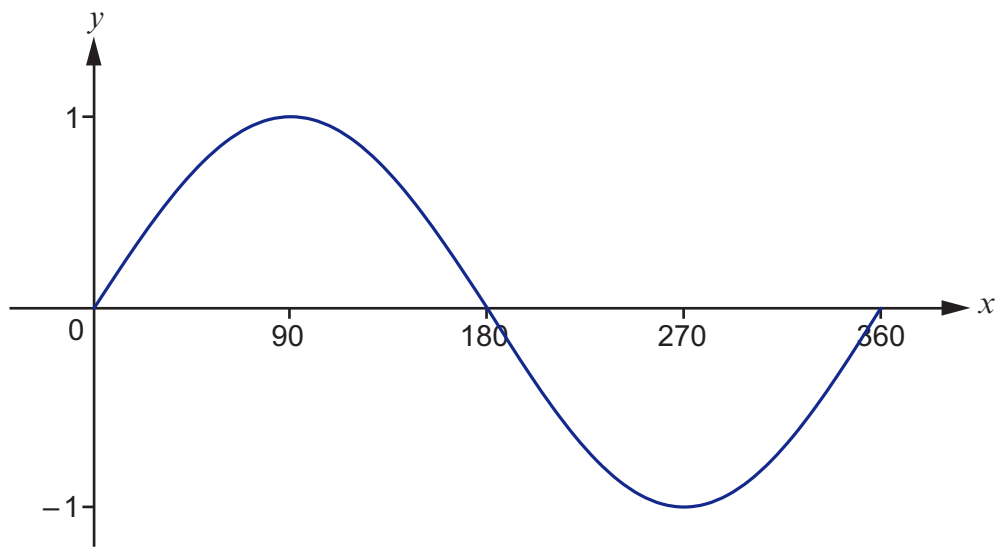
$$P(\text{all blue}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$$

$$P(\text{same colour}) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$



14. (a) Sketch the curve $y = \sin x$, for values of x in the range $x = 0^\circ$ to $x = 360^\circ$.

[1]



- (b) Solve each of the following equations.
Give all answers in the range $x = 0^\circ$ to $x = 360^\circ$.

(i) $\sin x = 0.3$

[2]

$$x = \sin^{-1}(0.3) = 17.45760312...^\circ$$

$$x = 17.5^\circ$$

$$\text{Second solution: } x = 180^\circ - 17.5^\circ = 162.5^\circ$$

$$x = 17.5^\circ \text{ and } x = 162.5^\circ$$

(ii) $\sin x + 1 = 0$

[1]

$$\sin x = -1$$

$$x = 270^\circ$$



15. Two **similar** pyramids have volumes of 3970 cm^3 and 3100 cm^3 respectively. The height of the larger pyramid is 25 cm. Calculate the height of the smaller pyramid. [3]

$$\text{Volume scale factor} = 3100 \div 3970 = 0.7808564232...$$

$$\text{Linear scale factor} = \sqrt[3]{(3100 \div 3970)} = 0.9208531874...$$

$$\text{Height of smaller pyramid} = 25 \times 0.9208531874...$$

$$= 23.02132969...$$

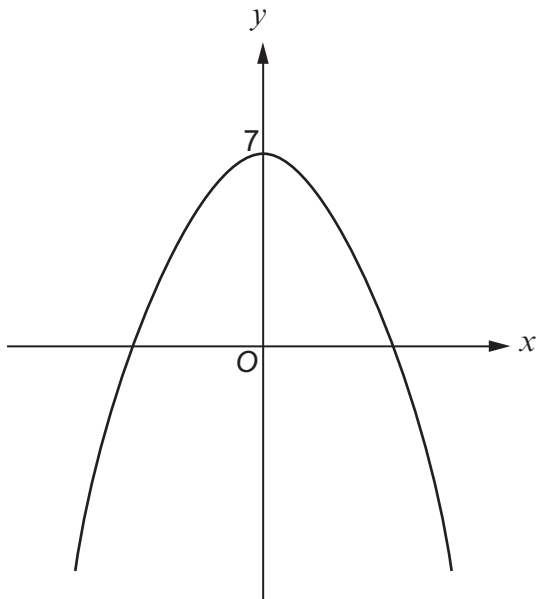
$$= 23.0 \text{ cm}$$

$$\text{Height} = 23.0 \text{ cm}$$



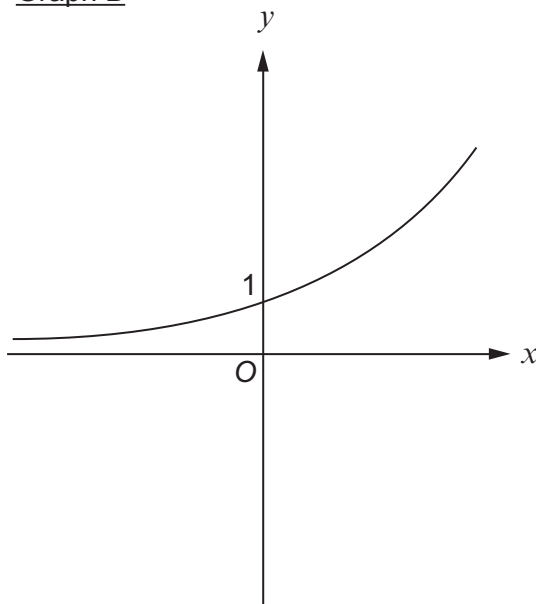
16. Each of the two graphs below is described by **one** of the equations on the right. Put a **tick** in the box next to the equation which correctly describes each graph. [2]

Graph A



	Equation describing graph A
$y = 7x^2$	
$y = -(x + 7)^2$	
$y = (x - 7)^2$	
$y = 7 - x^2$	✓
$y = x^2 + 7$	

Graph B



	Equation describing graph B
$y = x^2 + 1$	
$y = 2^x$	✓
$y + 1 = x^2$	
$y = \frac{1}{x}$	
$y = x^0$	

.....

.....

.....

.....

.....

.....



17. ABC represents the **sector** of a circle with radius 7 cm and centre A , as shown below.
 $\widehat{BAC} = x^\circ$, $AD = 3$ cm and $BD = 6$ cm.

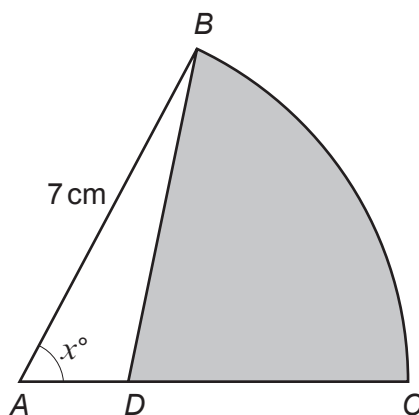


Diagram not drawn to scale

Find the area of the shaded region BCD .

[8]

Cosine rule in triangle ABD:

$$6^2 = 3^2 + 7^2 - 2 \times 3 \times 7 \times \cos x$$

$$\cos x = \frac{3^2 + 7^2 - 6^2}{2 \times 3 \times 7} = \frac{22}{42}$$

$$x = \cos^{-1}(22 \div 42) = 58.41186449...^\circ$$

$$\text{Area of sector } ABC = \frac{58.41186449...}{360} \times \pi \times 7^2$$

$$= 24.97724426... \text{ cm}^2$$

$$\text{Area of triangle } ABD = \frac{1}{2} \times 3 \times 7 \times \sin 58.41186449...^\circ$$

$$= 8.94427191... \text{ cm}^2$$

$$\text{Area of shaded region } BCD = 24.97724426... - 8.94427191...$$

$$= 16.03297235...$$

$$= 16.0 \text{ cm}^2$$

END OF PAPER



[illegible]