

|             |                              |
|-------------|------------------------------|
| Surname     | <i>StemUp Model Solution</i> |
| Other Names |                              |

|               |
|---------------|
| Centre Number |
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|                  |
|------------------|
| Candidate Number |
| 0                |



**GCSE – NEW**

3300U60-1



**MATHEMATICS**  
**UNIT 2: CALCULATOR-ALLOWED**  
**HIGHER TIER**

THURSDAY, 10 NOVEMBER 2016 – MORNING

1 hour 45 minutes

**ADDITIONAL MATERIALS**

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page(s) at the back of the booklet, taking care to number the question(s) correctly.

Take  $\pi$  as 3.14 or use the  $\pi$  button on your calculator.

**INFORMATION FOR CANDIDATES**

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 8, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

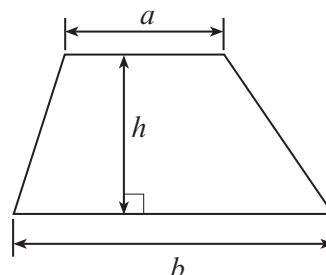
| For Examiner's use only |              |              |
|-------------------------|--------------|--------------|
| Question                | Maximum Mark | Mark Awarded |
| 1.                      | 4            |              |
| 2.                      | 2            |              |
| 3.                      | 3            |              |
| 4.                      | 4            |              |
| 5.                      | 6            |              |
| 6.                      | 5            |              |
| 7.                      | 4            |              |
| 8.                      | 7            |              |
| 9.                      | 7            |              |
| 10.                     | 3            |              |
| 11.                     | 2            |              |
| 12.                     | 5            |              |
| 13.                     | 4            |              |
| 14.                     | 5            |              |
| 15.                     | 2            |              |
| 16.                     | 6            |              |
| 17.                     | 3            |              |
| 18.                     | 8            |              |
| Total                   | 80           |              |



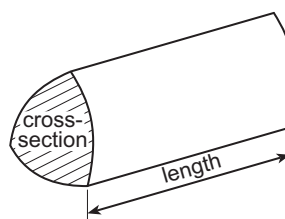
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### Formula List - Higher Tier

**Area of trapezium**  $= \frac{1}{2}(a + b)h$

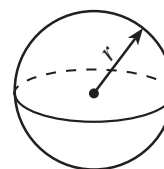


**Volume of prism** = area of cross-section  $\times$  length



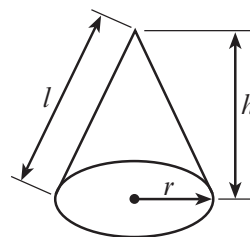
**Volume of sphere**  $= \frac{4}{3}\pi r^3$

**Surface area of sphere**  $= 4\pi r^2$



**Volume of cone**  $= \frac{1}{3}\pi r^2 h$

**Curved surface area of cone**  $= \pi r l$

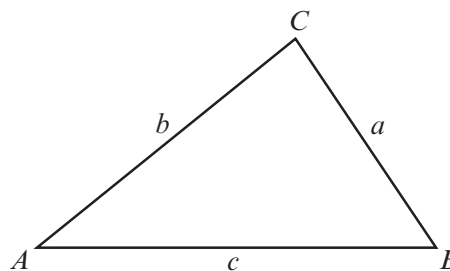


**In any triangle ABC**

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$

**Area of triangle**  $= \frac{1}{2}ab \sin C$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$  where  $a \neq 0$  are given by  $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

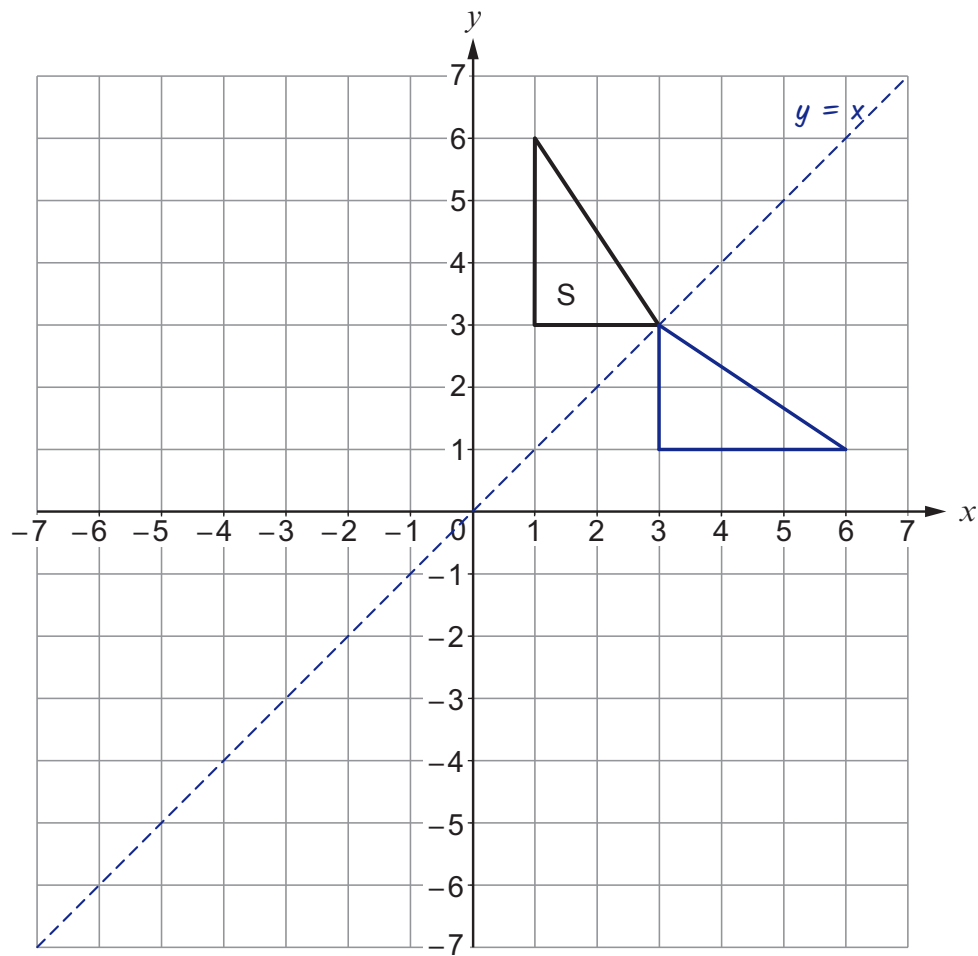
### Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula  $\left(1 + \frac{i}{n}\right)^n - 1$ , where  $i$  is the nominal interest rate per annum as a decimal and  $n$  is the number of compounding periods per annum.



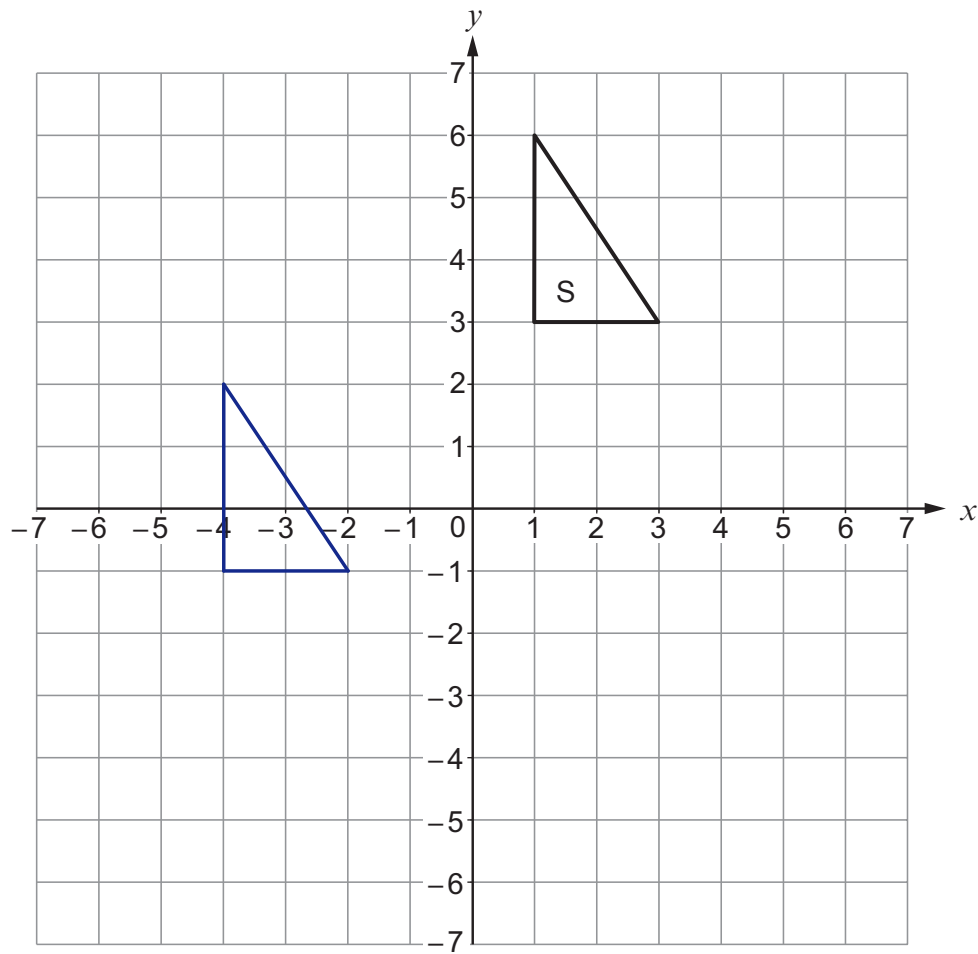
1. (a) Reflect the triangle S in the line  $y = x$ .

[2]



- (b) (i) Translate the triangle S using the column vector  $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$ .

[1]



- (ii) Write down the column vector that will reverse the translation in part (i).

[1]

$$\begin{pmatrix} 5 \\ 4 \end{pmatrix}$$



2. The  $n$ th term of a sequence is given by  $n^2 + 7$ .

Write down the first three terms of this sequence.

[2]

$$n = 1: 1^2 + 7 = 8$$

$$n = 2: 2^2 + 7 = 11$$

$$n = 3: 3^2 + 7 = 16$$

$$1^{\text{st}} \text{ term} = \dots 8 \dots$$

$$2^{\text{nd}} \text{ term} = \dots 11 \dots$$

$$3^{\text{rd}} \text{ term} = \dots 16 \dots$$

3. Circle the correct answer for each of the following.

(a)  $x^3 \times x^6 =$

[1]

$x^{36}$

$x^{0.5}$

$x^2$

$x^9$

$x^{18}$

$$x^3 + 6 = x^9$$

(b)  $(7x - 5y) - (3x + 2y) =$

[1]

$4x - 3y$

$4x - 7y$

$4x + 3y$

$-4x + 7y$

$-4x - 7y$

$$7x - 5y - 3x - 2y = 4x - 7y$$

- (c) A car travels  $x$  miles in 30 minutes.  
Its average speed in miles per hour is

[1]

$\frac{x}{2}$

$\frac{x}{30}$

$2x$

$\frac{2}{x}$

$30x$

$$30 \text{ minutes} = \frac{1}{2} \text{ hour, so speed} = x \div \frac{1}{2} = 2x \text{ mph}$$



## 4. A solution to the equation

$$2x^3 - 3x - 17 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.  
You must show all your working.

[4]

| $x$  | $2x^3 - 3x - 17$ |          |
|------|------------------|----------|
| 2    | -7               | too low  |
| 3    | 28               | too high |
| 2.2  | -2.304           | too low  |
| 2.3  | 0.434            | too high |
| 2.25 | -0.96875         | too low  |

2.25 gives a negative value and 2.3 gives a positive value,  
so the solution lies between 2.25 and 2.3.

$x = 2.3$  (to 1 d.p.)



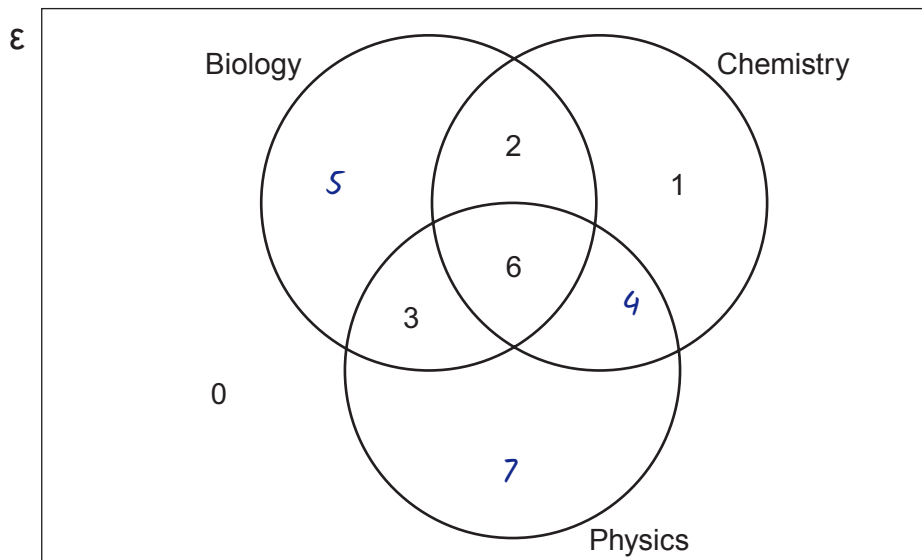
5. At a college, a total of 28 students study one or more of the science subjects: Biology, Chemistry and Physics.  
The 28 students form the universal set,  $\mathcal{E}$ .  
Some parts of the Venn diagram below have already been completed.

It is also known that:

- 5 students study only Biology
- 13 students study Chemistry

- (a) Complete the Venn diagram.

[3]



$$\text{Chemistry only part: } 13 - 2 - 6 - 1 = 4$$

$$\text{Biology only} = 5$$

$$\text{Physics only: } 28 - (5 + 2 + 1 + 6 + 3 + 4 + 0) = 28 - 21 = 7$$

- (b) How many students study Biology and Chemistry but not Physics?

[1]

2

- (c) One of the students is chosen at random.  
What is the probability that this student studies Biology?

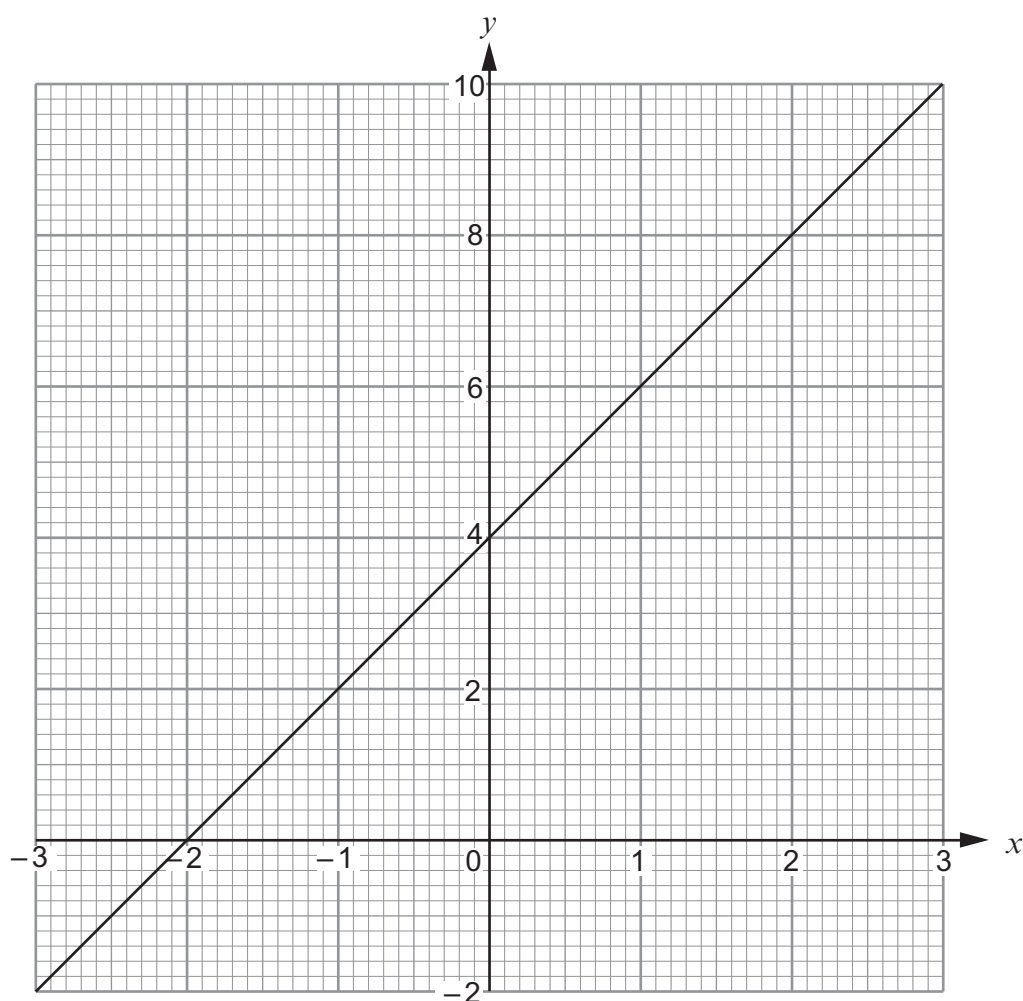
[2]

$$\text{Number studying Biology} = 5 + 2 + 6 + 3 = 16$$

$$P(\text{Biology}) = \frac{16}{28}$$



6. (a) The diagram below shows the graph of a straight line for values of  $x$  from  $-3$  to  $3$ .



- (i) Write down the gradient of the above line.

[1]

$$\text{gradient} = (6 - 4) \div (1 - 0) = 2$$

- (ii) Write down the equation of the line in the form  $y = mx + c$ , where  $m$  and  $c$  are whole numbers.

[2]

$$y = 2x + 4$$

- (b) Without drawing, show that the line  $2y = 5x - 3$  is parallel to the line  $4y = 10x + 7$ . You must show working to support your answer.

[2]

$$2y = 5x - 3 \rightarrow y = 2.5x - 1.5, \text{ so gradient} = 2.5$$

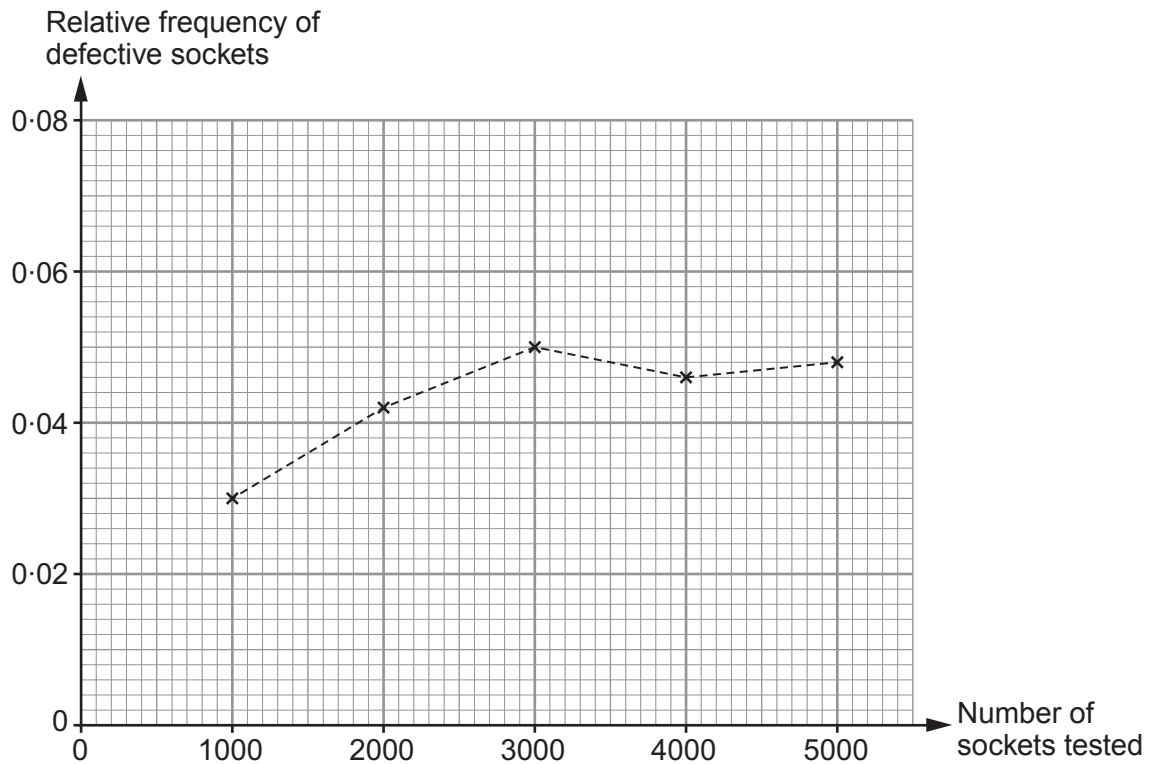
$$4y = 10x + 7 \rightarrow y = 2.5x + 1.75, \text{ so gradient} = 2.5$$

Both lines have equal gradients of 2.5, so they are parallel.



7. A factory uses a machine to produce electrical sockets. The manager carries out a survey to investigate the probability of the machine producing a defective socket.

The relative frequency of defective sockets produced was calculated after testing a total of 1000, 2000, 3000, 4000 and 5000 sockets. The results are plotted on the graph below.



- (a) How many of the first 3000 sockets tested were defective? [2]

*Relative frequency after 3000 sockets = 0.05*

*Defective =  $3000 \times 0.05 = 150$  sockets*

- (b) Write down the best estimate for the probability that one socket, selected at random, will be defective. You must give a reason for your choice. [2]

Probability: *0.048*

Reason: *it uses all the data, the most sockets tested (5000),*

*so the last point plotted gives the most reliable estimate.*



8. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Points A, B, C and D lie on the circumference of a circle, centre O.

BD is a diameter of the circle.

The straight line  $BC = 4.7$  cm and  $\hat{BAC} = 28^\circ$ .

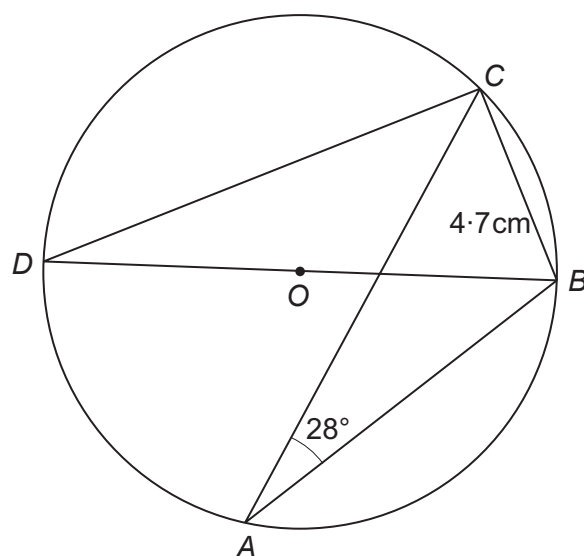


Diagram not drawn to scale

Write down the size of  $\hat{BDC}$ .  
Hence, calculate the length BD.  
You must show all your working.

[5 + 2 OCW]

$$\text{Angle } BDC = \text{angle } BAC = 28^\circ$$

(angles in the same segment are equal).

Angle  $BCD = 90^\circ$  (the angle in a semicircle is a right angle,  
because BD is a diameter).

In right-angled triangle BCD, BD is the hypotenuse and

$BC = 4.7$  cm is opposite the  $28^\circ$  angle at D.

$$\sin 28^\circ = \text{opposite} / \text{hypotenuse} = 4.7 / BD$$

$$BD = 4.7 \div \sin 28^\circ$$

$$BD = 4.7 \div 0.4694715...$$

$$BD = 10.011256...$$

So  $BD = 10.0$  cm (to 1 d.p.)



9. (a) Factorise  $x^2 - 2x - 24$ , and hence solve  $x^2 - 2x - 24 = 0$ . [3]

$$x^2 - 2x - 24 = (x - 6)(x + 4)$$

$$(x - 6)(x + 4) = 0$$

$$x - 6 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 6 \quad \text{or} \quad x = -4$$

- (b) Solve the equation  $\frac{4x-3}{2} + \frac{7x+1}{6} = \frac{29}{2}$ . [4]

Multiply every term by 6:

$$3(4x - 3) + (7x + 1) = 87$$

$$12x - 9 + 7x + 1 = 87$$

$$19x - 8 = 87$$

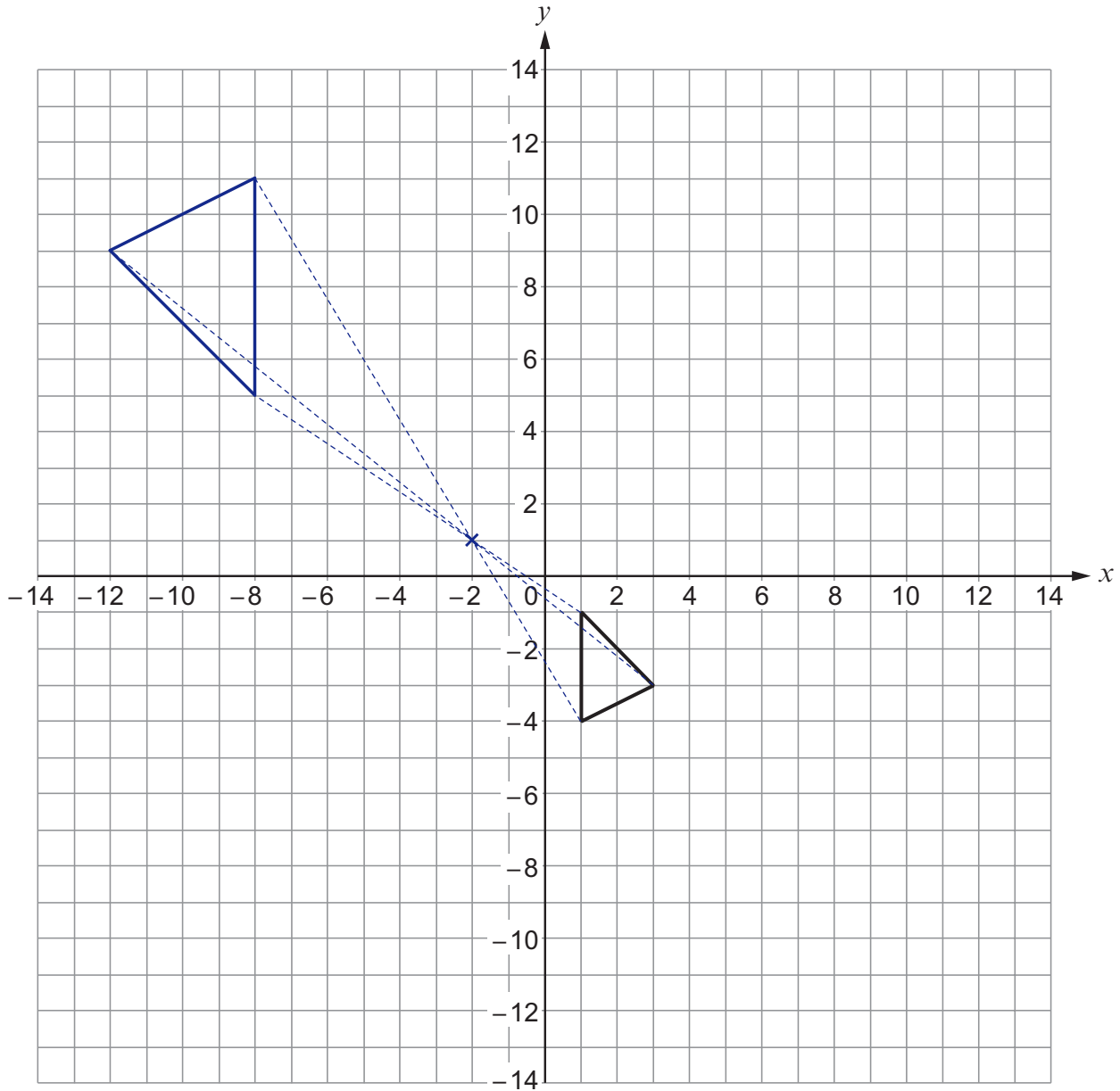
$$19x = 95$$

$$x = 5$$



10. Draw the enlargement of the given triangle, using
- a scale factor of  $-2$ ,
  - $(-2, 1)$  as the centre of enlargement.

[3]



11. A rectangle measures 38 cm by 26 cm.  
Each measurement is correct to the nearest cm.  
Calculate the least possible area of the rectangle.

[2]

Lower bounds: 37.5 cm and 25.5 cm

Least area =  $37.5 \times 25.5$

=  $956.25 \text{ cm}^2$



12. (a) Factorise  $(x - 7)^2 + 2(x - 7)$ .

[2]

$$(x - 7)[(x - 7) + 2]$$

$$= (x - 7)(x - 5)$$

(b) Factorise  $12x^2 - 27y^2$ .

[3]

$$12x^2 - 27y^2 = 3(4x^2 - 9y^2)$$

$$= 3[(2x)^2 - (3y)^2]$$

$$= 3(2x + 3y)(2x - 3y)$$



13. Make  $x$  the subject of the following formula.

[4]

$$a(x - b) = x(c - d)$$

$$ax - ab = cx - dx$$

$$ax - cx + dx = ab$$

$$x(a - c + d) = ab$$

$$x = \frac{ab}{a - c + d}$$



14. Points  $E$  and  $F$  lie on a circle, centre  $O$ .  
The radius of the circle is 10 cm.  
The area of the shaded sector is  $65 \text{ cm}^2$ .

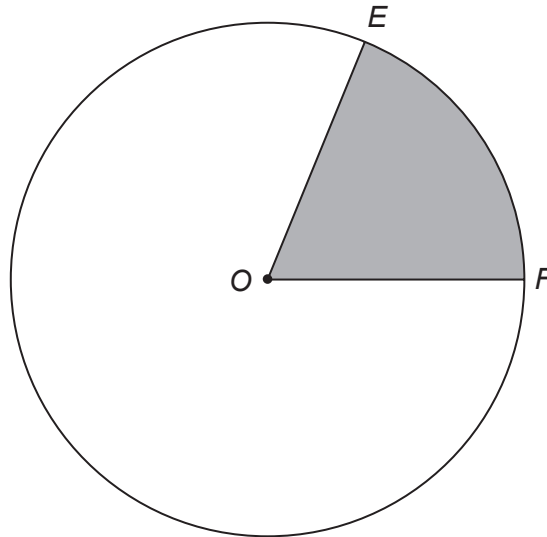


Diagram not drawn to scale

- (a) Calculate the size of  $\widehat{EOF}$ .

[3]

$$\theta/360 \times \pi \times r^2 = \text{sector area}$$

$$\theta/360 \times \pi \times 10^2 = 65$$

$$\theta = 65 \times 360 \div (\pi \times 10^2)$$

$$\theta = 74.48451\dots$$

$$\text{Angle EOF} = 74.5^\circ$$

- (b) Hence, calculate the length of the arc  $EF$ .

[2]

$$\text{Arc EF} = \theta/360 \times 2 \times \pi \times r$$

$$= 74.48451\dots/360 \times 2 \times \pi \times 10$$

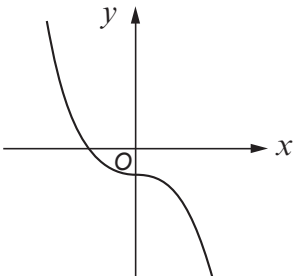
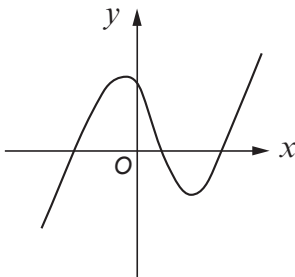
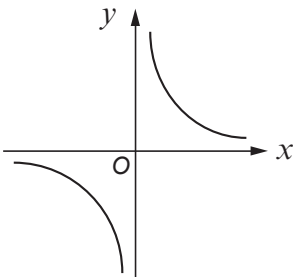
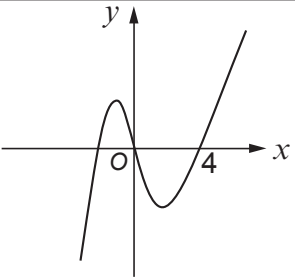
$$= 12.99999\dots$$

$$\text{Arc EF} = 13 \text{ cm}$$



15. Circle either TRUE or FALSE for each statement given below.

[2]

| GRAPH                                                                               | STATEMENT                                            |      |       |
|-------------------------------------------------------------------------------------|------------------------------------------------------|------|-------|
|    | The equation of this graph could be $y = -x^3 - 2$ . | TRUE | FALSE |
|    | The equation of this graph could be $y = x^3 - 9x$ . | TRUE | FALSE |
|   | The equation of this graph could be $y = x^{-1}$ .   | TRUE | FALSE |
|  | The equation of this graph could be $y = x^3 + 4$ .  | TRUE | FALSE |

.....

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16. Use the quadratic formula to solve  $(3x - 1)^2 = x(2x + 3) + 7$ .  
Give your answers correct to 2 decimal places.

[6]

$$(3x - 1)^2 = 9x^2 - 3x - 3x + 1 = 9x^2 - 6x + 1$$

$$x(2x + 3) + 7 = 2x^2 + 3x + 7$$

$$9x^2 - 6x + 1 = 2x^2 + 3x + 7$$

$$7x^2 - 9x - 6 = 0$$

$$a = 7, b = -9, c = -6$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 7 \times (-6)}}{2 \times 7}$$

$$x = \frac{9 \pm \sqrt{249}}{14}$$

$$x = (9 + 15.779733...) \div 14 = 1.769980...$$

$$x = (9 - 15.779733...) \div 14 = -0.484266...$$

$$x = 1.77 \text{ or } x = -0.48 \text{ (to 2 d.p.)}$$



17. Two similar shapes have areas of  $700 \text{ cm}^2$  and  $140 \text{ cm}^2$ .  
The perimeter of the smaller shape is  $83 \text{ cm}$ .  
Calculate the perimeter of the larger shape.

[3]

$$\text{Area scale factor} = 700 \div 140 = 5$$

$$\text{Linear scale factor} = \sqrt{5} = 2.236067\dots$$

$$\text{Perimeter of larger shape} = 83 \times \sqrt{5}$$

$$= 185.5936\dots$$

$$= 185.6 \text{ cm (to 1 d.p.)}$$



18. A 9-pointed star, with centre  $O$ , is shown below.  
Each side of the star is of length  $x$  cm.

The distance from the centre to every **inner** vertex of the star is 7 cm.  
The distance from the centre to every **outer** vertex of the star is 10 cm.

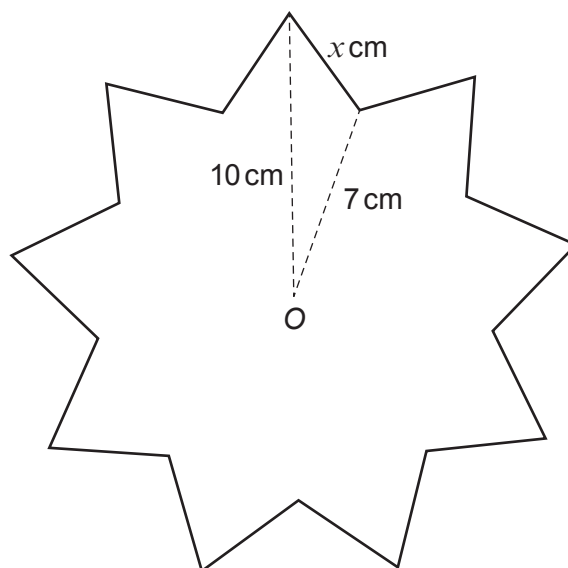


Diagram not drawn to scale

- (a) Calculate the perimeter of the star.

[5]

Angle at  $O$  between an outer and the next inner vertex  $= 360^\circ \div 18 = 20^\circ$

Cosine rule:  $x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 20^\circ$

$$x^2 = 149 - 131.556966...$$

$$x^2 = 17.443033...$$

$$x = 4.176485...$$

$$\text{Perimeter} = 18 \times x = 18 \times 4.176485...$$

$$= 75.17674...$$

$$\text{Perimeter} = 75.2 \text{ cm (to 1 d.p.)}$$



(b) Calculate the area of the star.

[3]

$$\text{Area of one triangle} = \frac{1}{2} \times 10 \times 7 \times \sin 20^\circ$$

$$= 11.970705...$$

$$\text{Area of star} = 18 \times 11.970705...$$

$$= 215.47269...$$

$$\text{Area of star} = 215.5 \text{ cm}^2 \text{ (to 1 d.p.)}$$

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