

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
Other Names		0



GCSE – NEW

3300U50-1



**MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER**

TUESDAY, 8 NOVEMBER 2016 – MORNING

1 hour 45 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space use the continuation pages at the back of the booklet, taking care to number the questions correctly.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 6, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

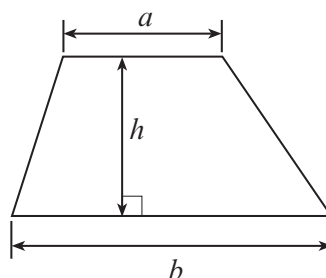
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	3	
2.	7	
3.	6	
4.	4	
5.	4	
6.	7	
7.	6	
8.	4	
9.	2	
10.	3	
11.	5	
12.	3	
13.	4	
14.	5	
15.	5	
16.	4	
17.	5	
18.	3	
Total	80	



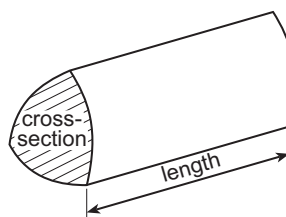
NOV163300U50101

Formula List - Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

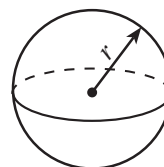


Volume of prism = area of cross-section $\times$ length



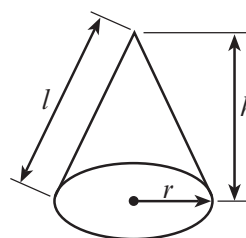
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

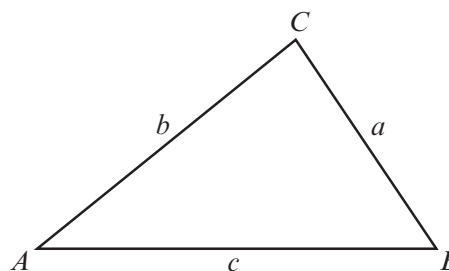


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. A fair six-sided dice and a fair coin are thrown together once.

Circle the correct answer for each of the following statements.

- (a) The number of possible outcomes is [1]

2 6 8 12 24.

- (b) The probability of getting a **4** on the dice and a **tail** on the coin is [1]

$\frac{1}{8}$ $\frac{1}{12}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{24}$.

- (c) The probability of getting a **multiple of 3** on the dice and a **head** on the coin is [1]

$\frac{1}{8}$ $\frac{1}{12}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{24}$.

Space for working:

(a) $6 \times 2 = 12$ outcomes

(b) $P(4 \text{ and tail}) = 1/6 \times 1/2 = 1/12$

(c) Multiples of 3 on the dice: 3, 6, so $P = 2/6 = 1/3$

$P(\text{multiple of 3 and head}) = 1/3 \times 1/2 = 1/6$



2. (a) The table below shows some of the values of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

Complete the table by finding the value of y for $x = -1$ and for $x = 2$.

[2]

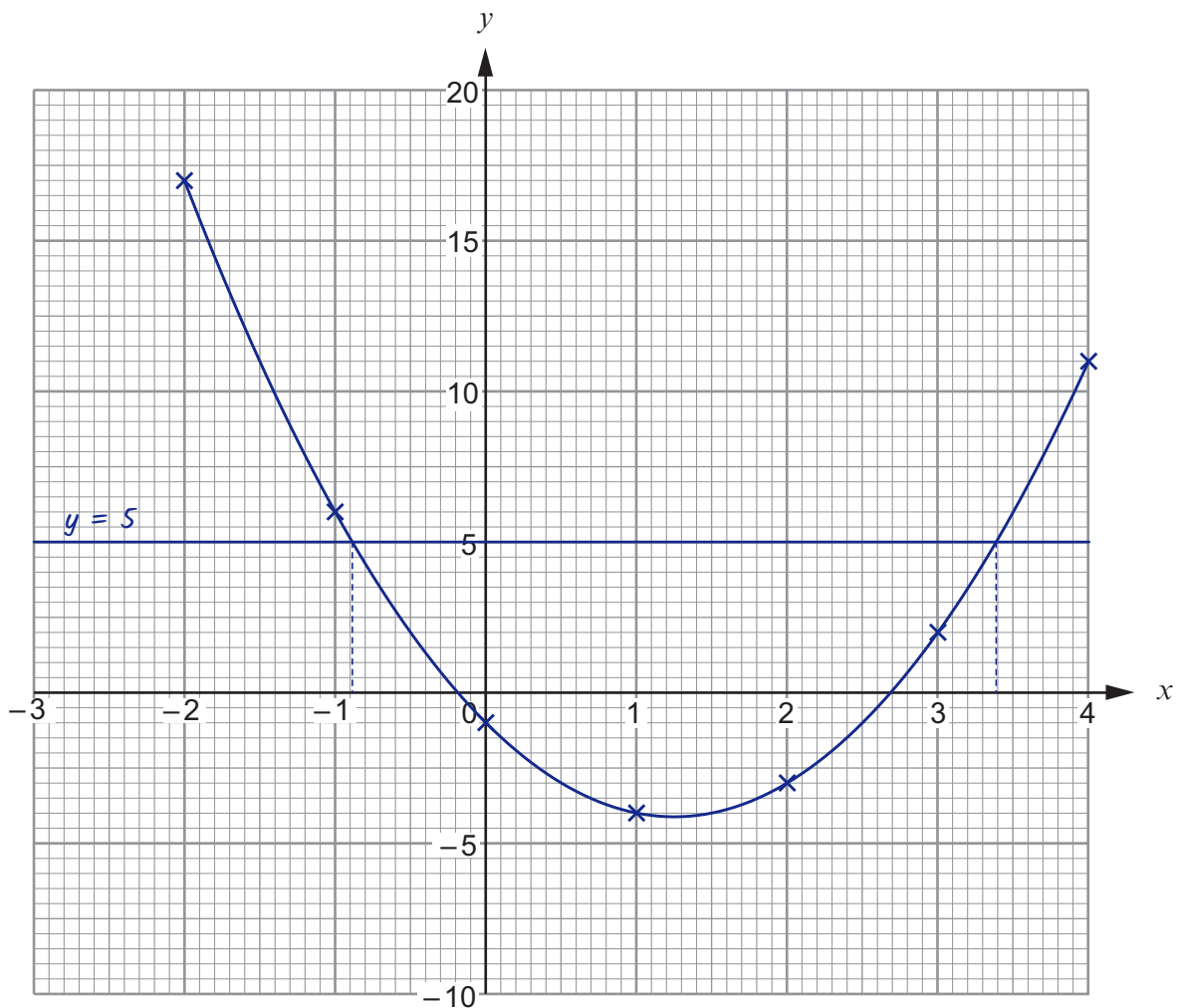
x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x - 1$	17	6	-1	-4	-3	2	11

$$x = -1: y = 2 \times (-1)^2 - 5 \times (-1) - 1 = 2 + 5 - 1 = 6$$

$$x = 2: y = 2 \times 2^2 - 5 \times 2 - 1 = 8 - 10 - 1 = -3$$

- (b) On the graph paper below, draw the graph of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

[2]



- (c) Draw the line $y = 5$ on the graph paper.

Write down the values of x where the line $y = 5$ cuts the curve $y = 2x^2 - 5x - 1$.
Give your answers correct to 1 decimal place.

[2]

Values of x are -0.9 and 3.4

- (d) Circle the equation below whose solutions are the values you have given in (c).

[1]

$$2x^2 - 5x - 1 = 0$$

$$2x^2 - 5x - 6 = 0$$

$$2x^2 - 5x - 5 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 5x + 4 = 0$$

$$2x^2 - 5x - 1 = 5$$

$$2x^2 - 5x - 6 = 0$$



3. A regular polygon has exterior angles of 45° .

(a) How many sides does this polygon have? [2]

$$\text{Number of sides} = 360 \div 45$$

$$= 8 \text{ sides}$$

(b) Each side of this regular polygon is 7 cm.
A sketch of two sides, AB and BC , of the polygon is shown below.

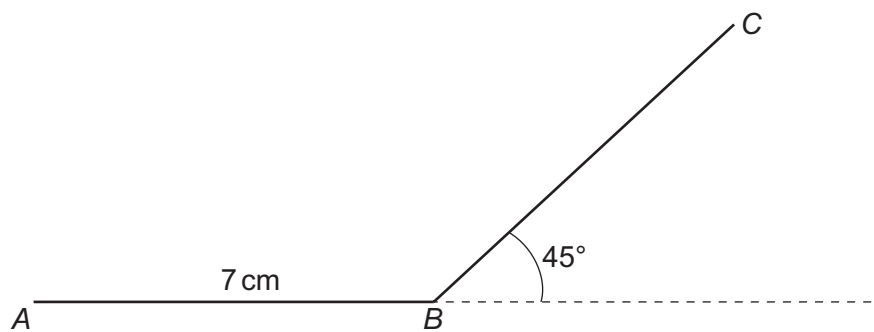
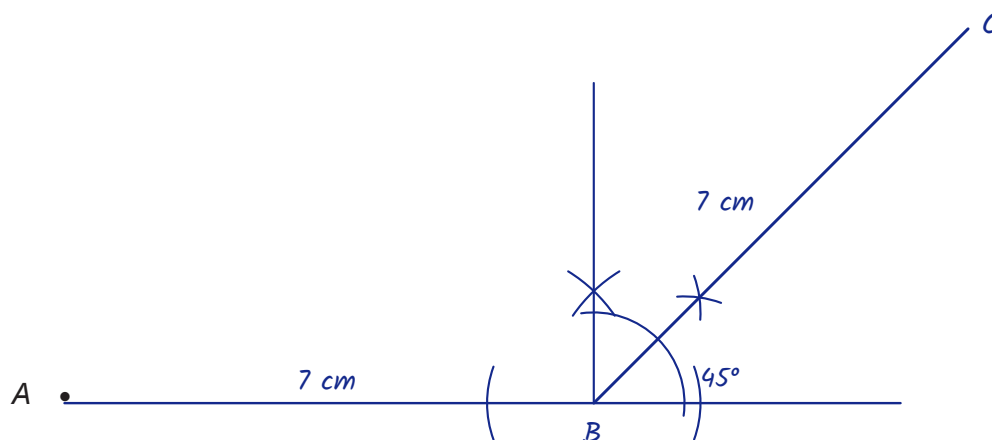


Diagram not drawn to scale

Using only a ruler and a pair of compasses, construct an accurate drawing that shows these **two sides** of the polygon.

The point A has been given.

You must show your construction arcs. [4]



4. (a) Make m the subject of the formula $y = 6m + 7$. [2]

$$y = 6m + 7$$

$$y - 7 = 6m$$

$$m = \frac{y - 7}{6}$$

- (b) Factorise $6x^2 - 12x$. [2]

$$6x(x - 2)$$

5. Find, in standard form, the value of each of the following.

(a) $\frac{7.5 \times 10^6}{5000}$ [2]

$$7.5 \times 10^6 \div 5000 = 7\,500\,000 \div 5000$$

$$= 1500$$

$$= 1.5 \times 10^3$$

(b) $(2.3 \times 10^3) + (6.4 \times 10^4)$ [2]

$$2.3 \times 10^3 = 2300, \quad 6.4 \times 10^4 = 64\,000$$

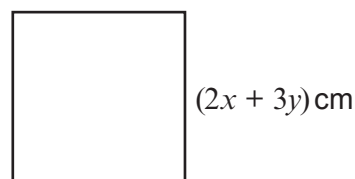
$$2300 + 64\,000 = 66\,300$$

$$= 6.63 \times 10^4$$

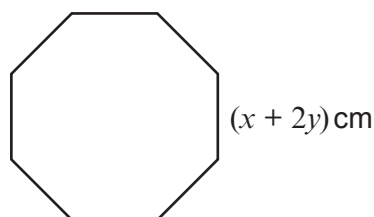


6. In this question you will be assessed on the quality of your organisation, communication and accuracy in writing.

Each side of a square is of length $(2x + 3y)$ cm.
The perimeter of the square is 62 cm.



Each side of a regular octagon is of length $(x + 2y)$ cm.
The perimeter of the octagon is 72 cm.



Use an algebraic method to find the value of x and the value of y .

[5 + 2 OCW]

Square: perimeter = $4 \times (2x + 3y) = 62$

so $8x + 12y = 62$... (1)

Octagon: perimeter = $8 \times (x + 2y) = 72$

so $8x + 16y = 72$... (2)

Subtract (1) from (2) to eliminate x :

$4y = 10$

$y = 2.5$

Substitute $y = 2.5$ into (2):

$8x + 16 \times 2.5 = 72$

$8x + 40 = 72$

$8x = 32$, so $x = 4$

Check in (1): $8 \times 4 + 12 \times 2.5 = 32 + 30 = 62$, which is correct.

So $x = 4$ cm and $y = 2.5$ cm.

$x = 4$ $y = 2.5$



7. Alwyn often drives from Bangor to Cardiff.
He always chooses one of two routes for these journeys.
He either travels through Rhayader or through Hereford.
The probability that he travels through Rhayader is 0.7.

Sometimes he decides to stop for a break during his journey.
His decision is independent of the route he takes.

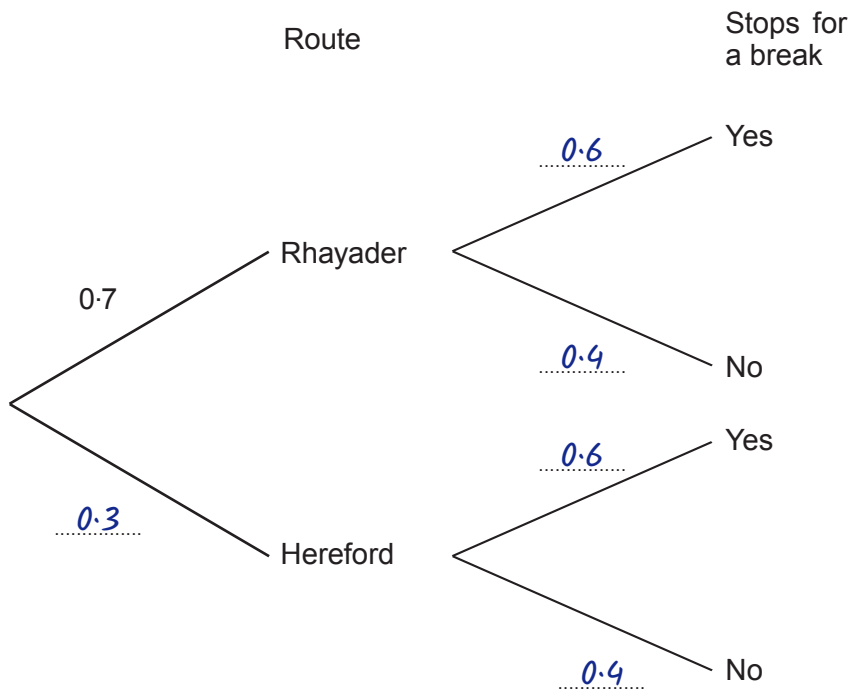
The probability that he travels through Rhayader **and** stops for a break is 0.42.

- (a) Complete the following tree diagram.

[4]

$$0.7 \times P(\text{break}) = 0.42, \text{ so } P(\text{break}) = 0.42 \div 0.7 = 0.6$$

$$P(\text{Hereford}) = 1 - 0.7 = 0.3, \quad P(\text{no break}) = 1 - 0.6 = 0.4$$



- (b) Calculate the probability that Alwyn travels through Hereford but **does not** stop for a break. [2]

$$P(\text{Hereford and no break}) = 0.3 \times 0.4$$

$$= 0.12$$



8. William has n marbles.
Lois had 4 times as many marbles as William, but she has now lost 23 of them.

Lois still has more marbles than William.

Write down an inequality in terms of n to show the above information.
Use your inequality to find the least number of marbles that William may have. [4]

Lois now has $4n - 23$ marbles.

Lois has more marbles than William, so:

$$4n - 23 > n$$

$$3n > 23$$

$$n > \frac{23}{3} = 7.66\dots$$

n must be a whole number, so the least value of n is 8.

Least number of marbles William may have = 8



9. Circle the correct answer for each of the following statements.

(a) $9^{-\frac{1}{2}}$ is equal to

-3

$-\frac{1}{3}$

$\frac{1}{4\frac{1}{2}}$

$-4\frac{1}{2}$

$\frac{1}{3}$

[1]

$$9^{-\frac{1}{2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

(b) $8^{\frac{2}{3}}$ is equal to

$5\frac{1}{3}$

4

6

$8\frac{2}{3}$

$\frac{16}{24}$

[1]

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$$

10. The radius of a hemisphere and the radius of a cylinder are equal.
The hemisphere and cylinder have equal volumes.

Calculate the ratio of the height of the cylinder to the radius of the cylinder.

[3]

$$\text{Volume of hemisphere} = \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\pi r^2 h = \frac{2}{3} \pi r^3$$

$$h = \frac{2}{3} r, \text{ so } 3h = 2r$$

$$h : r = 2 : 3$$

height of cylinder : radius of cylinder

$$= 2 : 3$$



11. Given that y is inversely proportional to x , and that $y = 4$ when $x = 3$,

(a) find an expression for y in terms of x , [3]

$$y \propto \frac{1}{x}, \text{ so } y = \frac{k}{x}$$

$$4 = \frac{k}{3}, \text{ so } k = 4 \times 3 = 12$$

$$y = \frac{12}{x}$$

(b) use the expression you found in (a) to complete the following table. [2]

x	3	0.25	60
y	4	48	$\frac{1}{5}$

$$x = 0.25: y = 12 \div 0.25 = 48$$

$$y = \frac{1}{5}: \frac{1}{5} = \frac{12}{x}, \text{ so } x = 12 \times 5 = 60$$



12. Express $\frac{3x}{3x+2} - \frac{2x}{2x+7}$ as a single fraction in its simplest form. [3]

$$\frac{3x(2x+7) - 2x(3x+2)}{(3x+2)(2x+7)}$$

$$= \frac{6x^2 + 21x - 6x^2 - 4x}{(3x+2)(2x+7)}$$

$$= \frac{17x}{(3x+2)(2x+7)}$$



13. The points P , Q and R lie on the circumference of a circle, centre O .
 PQ is a diameter of the circle.
 The straight line ARB is a tangent to the circle.

$\hat{QRB} = x$, where x is measured in degrees.

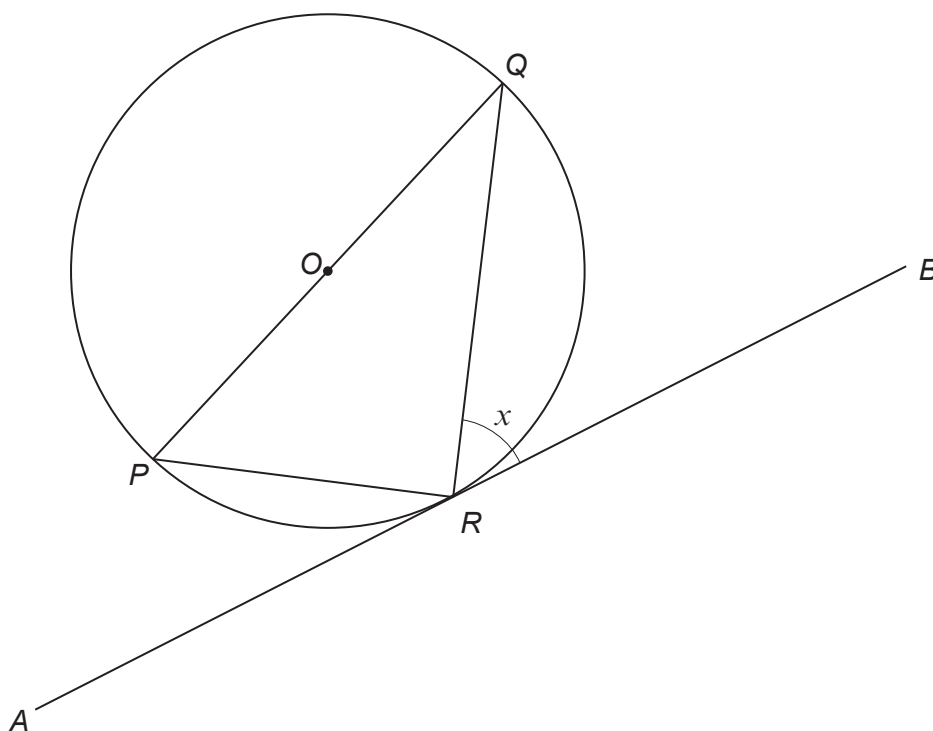


Diagram not drawn to scale

Calculate the size of $\hat{PQR}$ in terms of x .
 You must give a reason for each step of your solution.

[4]

Angle $RPQ = x$ (alternate segment theorem: the angle between
 the tangent and the chord RQ equals the angle in the alternate segment)

Angle $PRQ = 90^\circ$ (angle in a semicircle is 90° , as PQ is a diameter)

Angle $PQR = 180^\circ - 90^\circ - x$

(Angles in a triangle add up to 180°)

Angle $PQR = 90^\circ - x$



14. Aled has three concrete slabs.

Two of the slabs are square, with each side of length x metres.

The third slab is rectangular and measures 1 metre by $(x + 1)$ metres.

The three concrete slabs cover an area of 7 m^2 .

- (a) Show that $2x^2 + x - 6 = 0$.

[1]

$$\text{Total area} = x^2 + x^2 + 1 \times (x + 1) = 7$$

$$2x^2 + x + 1 = 7$$

$$2x^2 + x - 6 = 0$$

- (b) Solve the equation to find the length of each side of the square slabs.
You must justify any decisions that you make.

[4]

$$(2x - 3)(x + 2) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x + 2 = 0$$

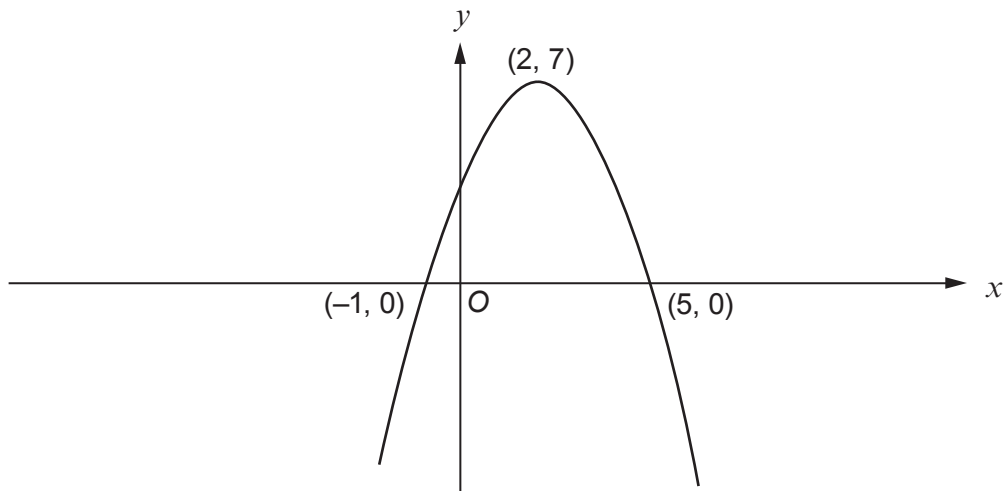
$$x = 1.5 \quad \text{or} \quad x = -2$$

$x = -2$ is rejected because a length cannot be negative.

Each side of the square slabs is 1.5 metres.



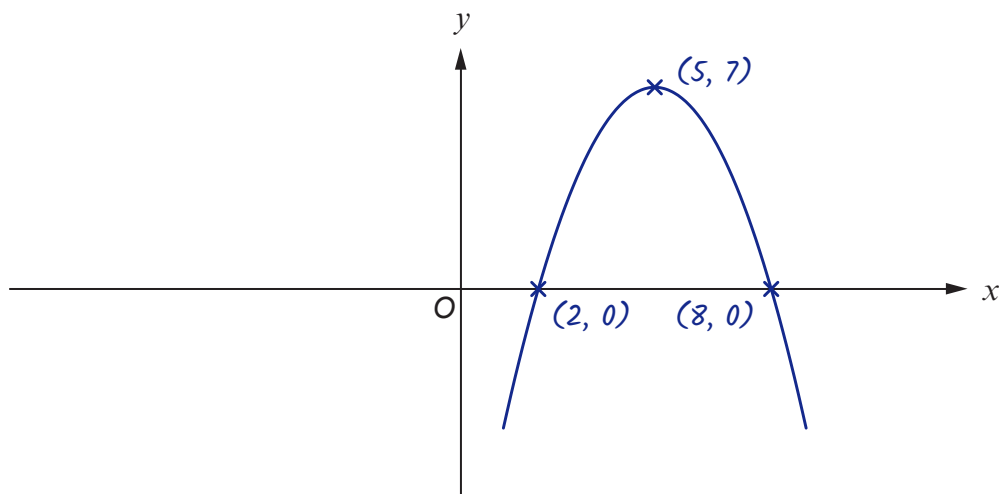
15. (a) The diagram shows a sketch of the graph $y = f(x)$.
The graph passes through the points $(-1, 0)$ and $(5, 0)$ and its highest point is at $(2, 7)$.



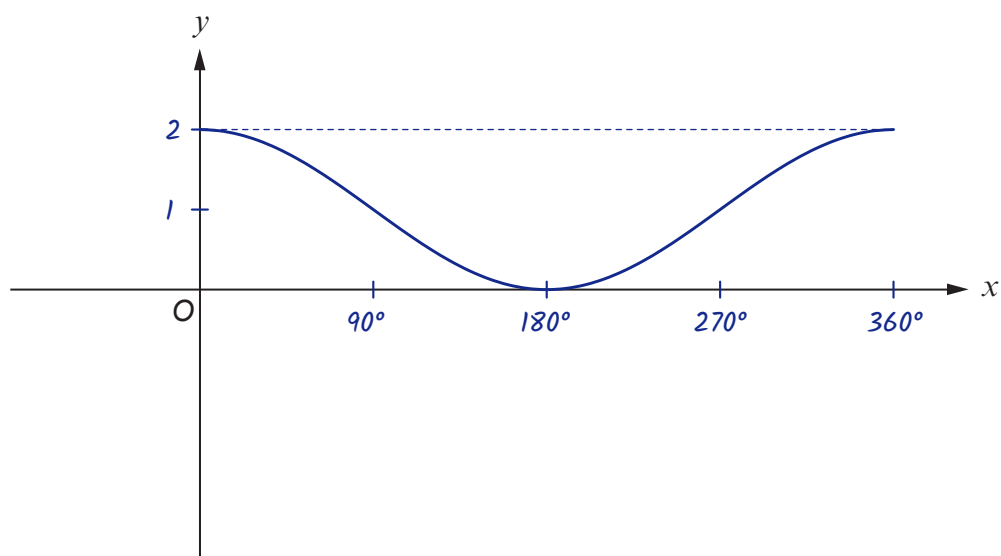
Sketch the graph of $y = f(x - 3)$ on the axes below.
You must indicate

- the coordinates of the points of intersection of the graph with the x -axis
- the coordinates of the highest or lowest point.

[3]



- (b) Using the axes below, **sketch** the graph of $y = \cos x + 1$ for values of x from 0° to 360° . [2]



16. Triangle ABC is an isosceles triangle with $\angle ABC = \angle ACB$.

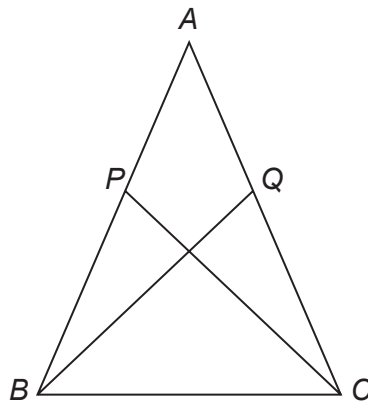


Diagram not drawn to scale

P and Q are points on AB and AC respectively such that $AP = AQ$.

Prove that triangle ABQ is congruent to triangle ACP .
You must give reasons for each step of your proof.

[4]

In triangles ABQ and ACP :

*$AB = AC$ (triangle ABC is isosceles: $\angle ABC = \angle ACB$,
so the sides opposite these equal angles are equal)*

$AQ = AP$ (given)

Angle $BAQ = \text{angle } CAP$ (angle A is common to both triangles)

So triangle ABQ is congruent to triangle ACP

(SAS: two sides and the included angle are equal).



17. Simplify

$$\frac{(5\sqrt{3})^2 - \frac{2\sqrt{18}}{\sqrt{2}}}{\sqrt{32} \times \sqrt{2}}$$

and state whether your answer is rational or irrational.

[5]

$$(5\sqrt{3})^2 = 5^2 \times (\sqrt{3})^2 = 25 \times 3 = 75$$

$$\frac{2\sqrt{18}}{\sqrt{2}} = 2\sqrt{9} = 2 \times 3 = 6$$

$$\sqrt{32} \times \sqrt{2} = \sqrt{64} = 8$$

$$\text{So the expression} = \frac{75 - 6}{8} = \frac{69}{8}$$

$\frac{69}{8}$ is a ratio of two integers, so the answer is rational.

Answer: $\frac{69}{8}$, rational



18. A game played at a children's party involves throwing a ball into a bucket. Each child tries to get the ball into the bucket in the least number of throws. On each attempt, the probability that Sofia gets the ball into the bucket is 0.8. Each attempt is independent of any previous attempt.

Show that she is 5 times more likely to get the ball into the bucket on her first attempt than to have her first successful throw on her second attempt.

You must show all your working.

[3]

$$P(\text{success on 1st attempt}) = 0.8$$

$$P(\text{miss}) = 1 - 0.8 = 0.2$$

$$P(\text{first success on 2nd attempt}) = P(\text{miss}) \times P(\text{success})$$

$$= 0.2 \times 0.8 = 0.16$$

$$0.8 \div 0.16 = 5, \text{ so } 0.8 = 5 \times 0.16$$

So she is 5 times more likely to get the ball in on her first attempt than to have her first successful throw on her second attempt.

END OF PAPER



BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**



[illegible]

BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**



BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**

