

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U60-1



WEDNESDAY, 12 JUNE 2024 – MORNING

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** the questions in the spaces provided.
If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.
Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
In question 4, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

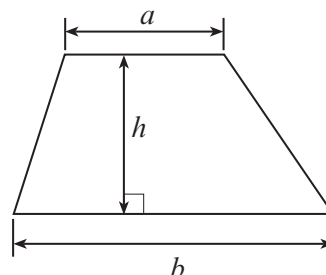
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	5	
3.	4	
4.	8	
5.	2	
6.	6	
7.	4	
8.	3	
9.	3	
10.	4	
11.	1	
12.	6	
13.	3	
14.	2	
15.	3	
16.	4	
17.	3	
18.	4	
19.	5	
20.	6	
Total	80	



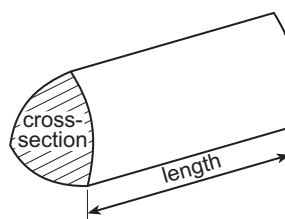
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

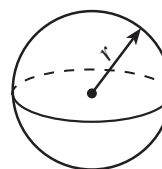


Volume of prism = area of cross-section $\times$ length



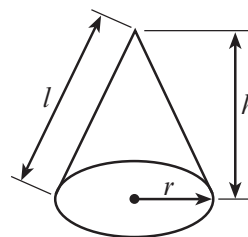
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

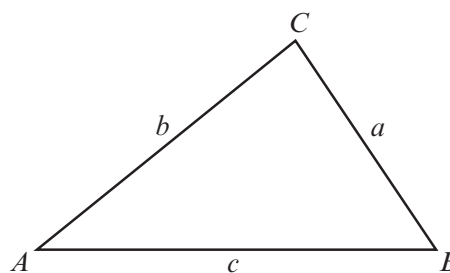


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) Evaluate $\sqrt{0.9^3 - 0.9^4}$.

[2]

$$\sqrt{(0.9^3 - 0.9^4)} = \sqrt{(0.729 - 0.6561)}$$

$$= \sqrt{0.0729}$$

$$= 0.27$$

- (b) What is the greatest integer value of n if $2n < 17$?

[1]

$$2n < 17$$

$$n < 17 \div 2$$

$$n < 8.5$$

Greatest integer value of n = 8

- (c) What is the least integer value of n if $2^n > 125$?

[1]

$$2^6 = 64, \text{ which is less than } 125$$

$$2^7 = 128, \text{ which is greater than } 125$$

Least integer value of n = 7



2. The table below shows some of the values of $y = 2x^2 + 5x - 3$ for values of x from -4 to 2 .

x	-4	-3	-2	-1	0	1	2
$y = 2x^2 + 5x - 3$	9	0	-5	-6	-3	4	15

(a) Complete the table above.

[1]

When $x = 1$: $y = 2 \times 1^2 + 5 \times 1 - 3 = 2 + 5 - 3 = 4$

(b) On the graph paper opposite, draw the graph of $y = 2x^2 + 5x - 3$ for values of x from -4 to 2 .

[2]

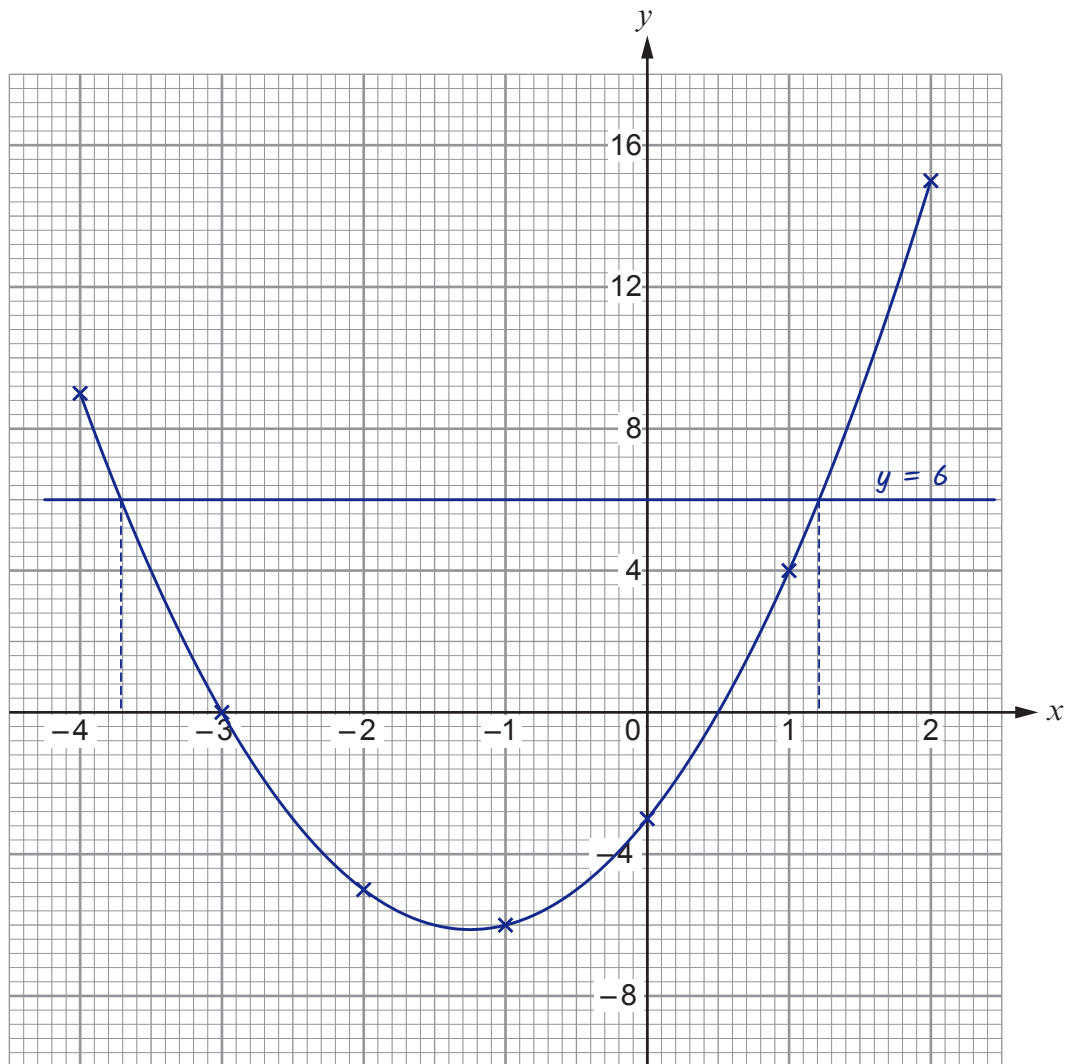
(c) Draw the line $y = 6$ on the graph paper.

Write down the values of x where the line $y = 6$ cuts the curve $y = 2x^2 + 5x - 3$.

[2]

Values of x are -3.7 and 1.2



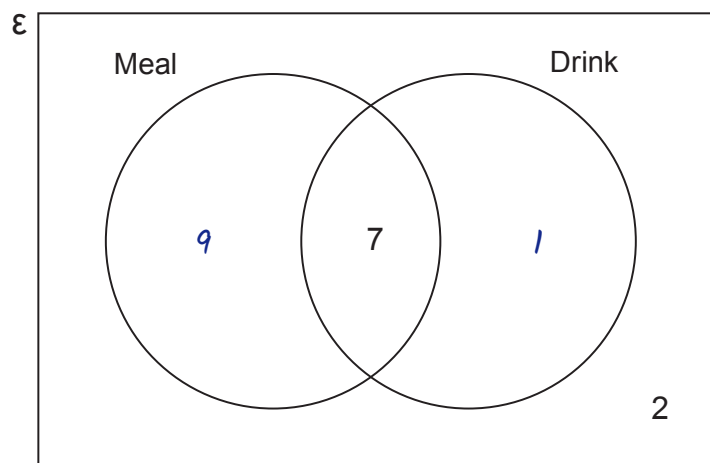


3. At lunchtime on Wednesday, a café had **19 customers**.

- 7 of these customers bought a meal **and** a drink.
- 2 of these customers did not buy a meal **or** a drink.
- The total number of customers who bought a meal was **twice** the total number of customers who bought a drink.

(a) Complete the Venn diagram below to show this information.

[2]



Meal and drink = 7, neither = 2, so $19 - 7 - 2 = 10$ in the other two parts

Let meal only = m and drink only = d : $m + d = 10$

Total meal = 2 × total drink: $7 + m = 2(7 + d)$

$7 + m = 14 + 2d$, so $m = 7 + 2d$

$7 + 2d + d = 10 \rightarrow 3d = 3 \rightarrow d = 1, m = 9$

Check: meal total = $9 + 7 = 16$, drink total = $7 + 1 = 8$, $16 = 2 \times 8$

(b) One of these customers is chosen at random.
What is the probability that this customer bought a meal?

[2]

Number who bought a meal = $9 + 7 = 16$

$$P(\text{bought a meal}) = \frac{16}{19}$$



4. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

The diagram below shows a water container.
It is formed from an open rectangular tank with a solid cylinder attached to its base inside the tank.

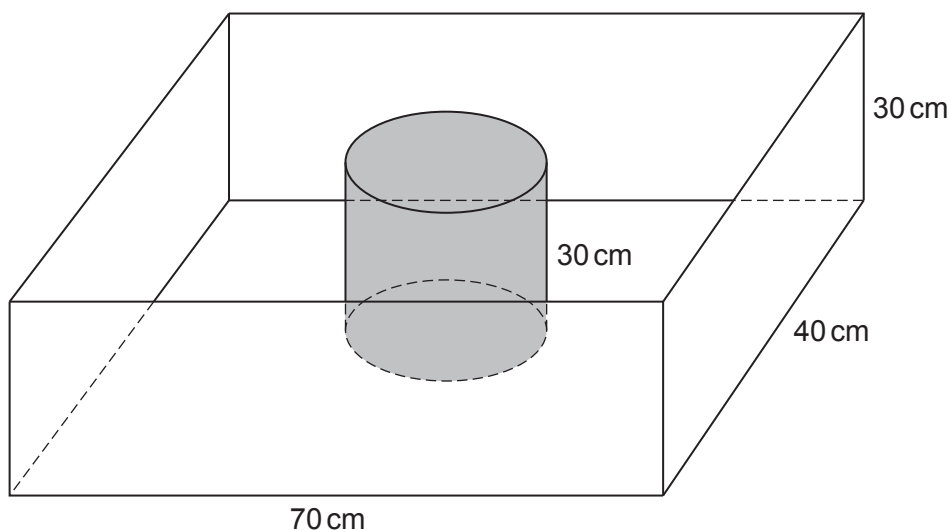


Diagram not drawn to scale

The base radius of the cylinder is 10 cm.
The height of the cylinder is 30 cm.

How many **litres** of water can the container hold?
You must show all your working.

[6 + 2 OCW]

$$\text{Volume of tank} = \text{length} \times \text{width} \times \text{height}$$

$$= 70 \times 40 \times 30 = 84000 \text{ cm}^3$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \pi \times 10^2 \times 30 = 3000\pi = 9424.777... \text{ cm}^3$$

$$\text{Space for water} = \text{volume of tank} - \text{volume of cylinder}$$

$$= 84000 - 9424.777... = 74575.22... \text{ cm}^3$$

$$1 \text{ litre} = 1000 \text{ cm}^3, \text{ so capacity} = 74575.22... \div 1000$$

$$= 74.575... \text{ litres}$$

$$\text{The container can hold } 74.6 \text{ litres of water (to 3 s.f.).}$$



5. The equation of a straight line is $y = -3x + 7$.

- (a) What is the gradient of the line?
Circle the correct answer.

[1]

$\frac{1}{3}$

$-\frac{1}{3}$

3

-3

7

- (b) What are the coordinates of the point where the line intersects the y -axis?
Circle the correct answer.

[1]

$(-3, 7)$

$(0, -3)$

$(0, 3)$

$(0, -7)$

$(0, 7)$



6. In the diagram below, ABD and BCD are both right-angled triangles.

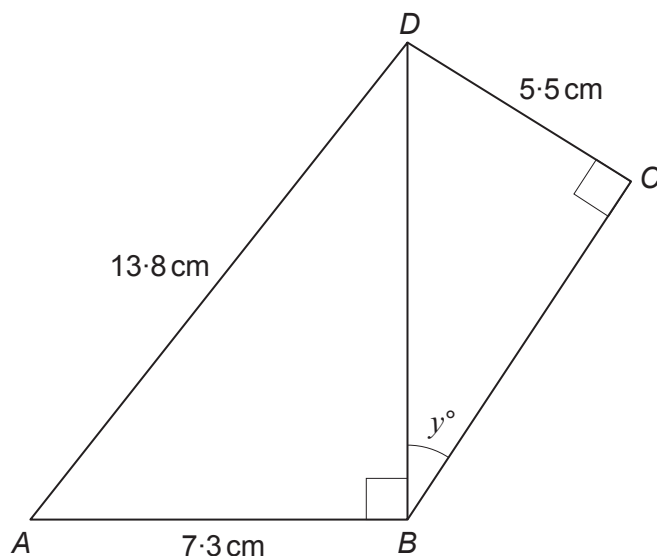


Diagram not drawn to scale

$AB = 7.3$ cm, $AD = 13.8$ cm and $CD = 5.5$ cm.

Calculate the value of y .

[6]

In triangle ABD (right angle at B): $AD^2 = AB^2 + BD^2$

$$13.8^2 = 7.3^2 + BD^2$$

$$BD^2 = 13.8^2 - 7.3^2 = 190.44 - 53.29 = 137.15$$

$$BD = \sqrt{137.15} = 11.7111... \text{ cm}$$

In triangle BCD (right angle at C), BD is the hypotenuse:

$$\sin y = \text{opposite} \div \text{hypotenuse} = CD \div BD$$

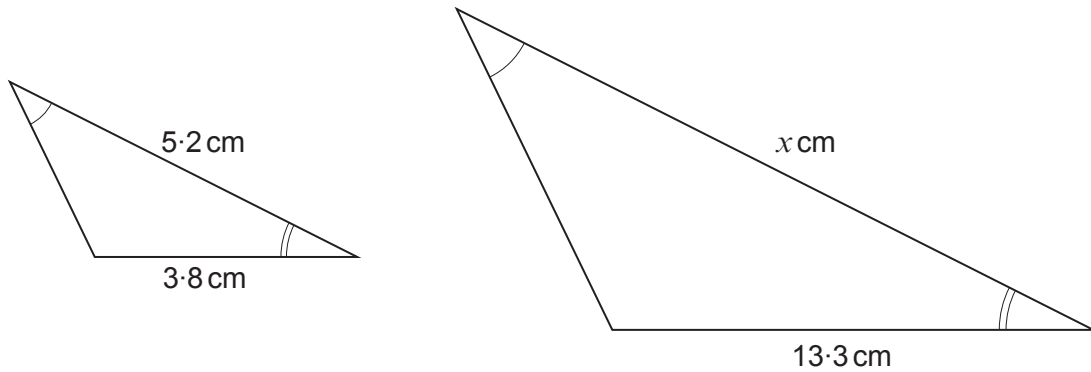
$$\sin y = 5.5 \div 11.7111... = 0.469639...$$

$$y = \sin^{-1}(0.469639...) = 28.0109...$$

$$y = 28.0$$



7. (a) The triangles shown below are similar.



Diagrams not drawn to scale

Calculate the value of x .

[2]

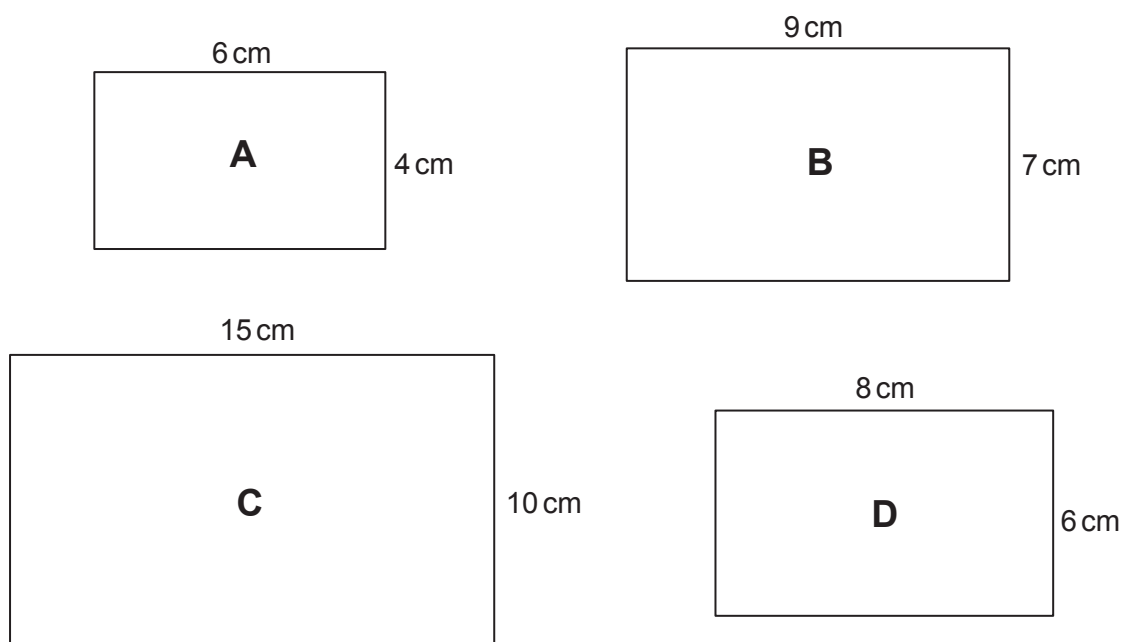
$$\text{Scale factor} = 13.3 \div 3.8 = 3.5$$

$$x = 5.2 \times 3.5$$

$$x = 18.2$$



- (b) Four rectangles labelled **A**, **B**, **C** and **D** are shown below.



Diagrams not drawn to scale

Which two rectangles are similar?
Give a reason for your choice.

[2]

The two rectangles which are similar are rectangles A and C

Reason: For both, the length is 1.5 times the width:

$6 \div 4 = 1.5$ and $15 \div 10 = 1.5$, so the sides are in the same ratio 3 : 2

(scale factor from A to C is $15 \div 6 = 2.5$ and $10 \div 4 = 2.5$).



8. Factorise $x^2 + 3x - 40$, and hence solve $x^2 + 3x - 40 = 0$. [3]

Two numbers that multiply to -40 and add to 3: 8 and -5

$$x^2 + 3x - 40 = (x + 8)(x - 5)$$

$$(x + 8)(x - 5) = 0$$

$$x + 8 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -8 \quad \text{or} \quad x = 5$$

9. a and b are two numbers, where $b > a$.

The mean of the two numbers is equal to the range of the two numbers.

Show that $3a = b$. [3]

$$\text{Mean} = \frac{a+b}{2}, \quad \text{range} = b - a \quad (\text{since } b > a)$$

$$\frac{a+b}{2} = b - a$$

$$a + b = 2(b - a)$$

$$a + b = 2b - 2a$$

$$a + 2a = 2b - b$$

$$3a = b$$



10. Aled and Berwyn share $\pounds x$ in the ratio 2 : 3.

- (a) Aled's share of the money is $\pounds 0.4x$.
What is Berwyn's share of the money in terms of x ? [1]

$$\text{Berwyn} = x - 0.4x \text{ (or } 3/5 \text{ of } x)$$

$$\text{Berwyn's share} = \pounds 0.6x$$

- (b) Carys and Delyth share the same amount, $\pounds x$, in the ratio 3 : 7.

Show that one of these four people receives the same amount as the combined total of two of the other people. [3]

$$\text{Carys and Delyth: } 3 + 7 = 10 \text{ parts, } 1 \text{ part} = \pounds x \div 10 = \pounds 0.1x$$

$$\text{Carys} = 3 \times \pounds 0.1x = \pounds 0.3x$$

$$\text{Delyth} = 7 \times \pounds 0.1x = \pounds 0.7x$$

$$\text{Aled} = \pounds 0.4x, \text{ Berwyn} = \pounds 0.6x$$

$$\text{Carys} + \text{Aled} = \pounds 0.3x + \pounds 0.4x = \pounds 0.7x$$

$$\pounds 0.7x = \text{Delyth's share}$$

So Delyth receives the same amount as Carys and Aled combined.

11. Write $\frac{1}{4} \times 2^{400}$ in the form 2^n . [1]

$$\frac{1}{4} = 2^{-2}, \text{ so } 2^{-2} \times 2^{400} = 2^{-2+400} = 2^{398}$$



12. The pyramid below has a horizontal rectangular base $ABCD$. The point O is the top of the pyramid. It is vertically above the point A . $OC = 113$ cm, $AB = 72$ cm and $CB = 84$ cm.

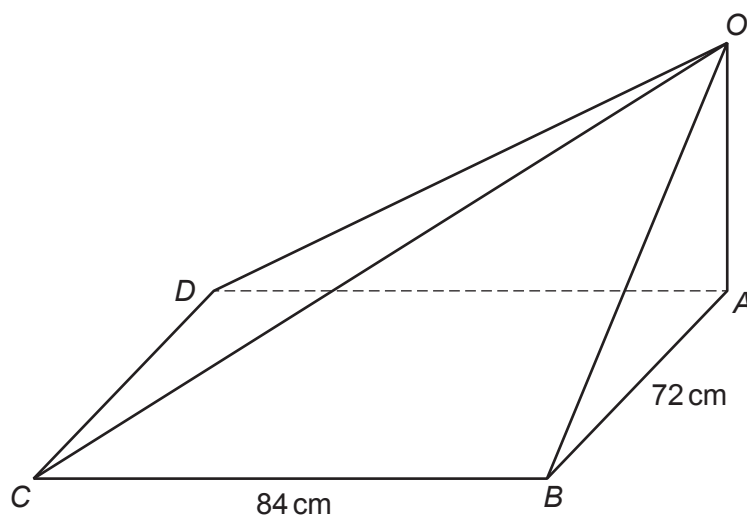


Diagram not drawn to scale

- (a) Calculate the length of OA , the vertical height of the pyramid.

[4]

AC is the diagonal of the base: $AC^2 = AB^2 + BC^2$

$$AC^2 = 72^2 + 84^2 = 5184 + 7056 = 12240$$

Triangle OAC has a right angle at A : $OC^2 = OA^2 + AC^2$

$$OA^2 = 113^2 - 12240 = 12769 - 12240 = 529$$

$$OA = \sqrt{529} = 23 \text{ cm}$$

- (b) Calculate the volume of this pyramid.

[2]

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= \frac{1}{3} \times 84 \times 72 \times 23 = 139104 \div 3$$

$$= 46368 \text{ cm}^3$$



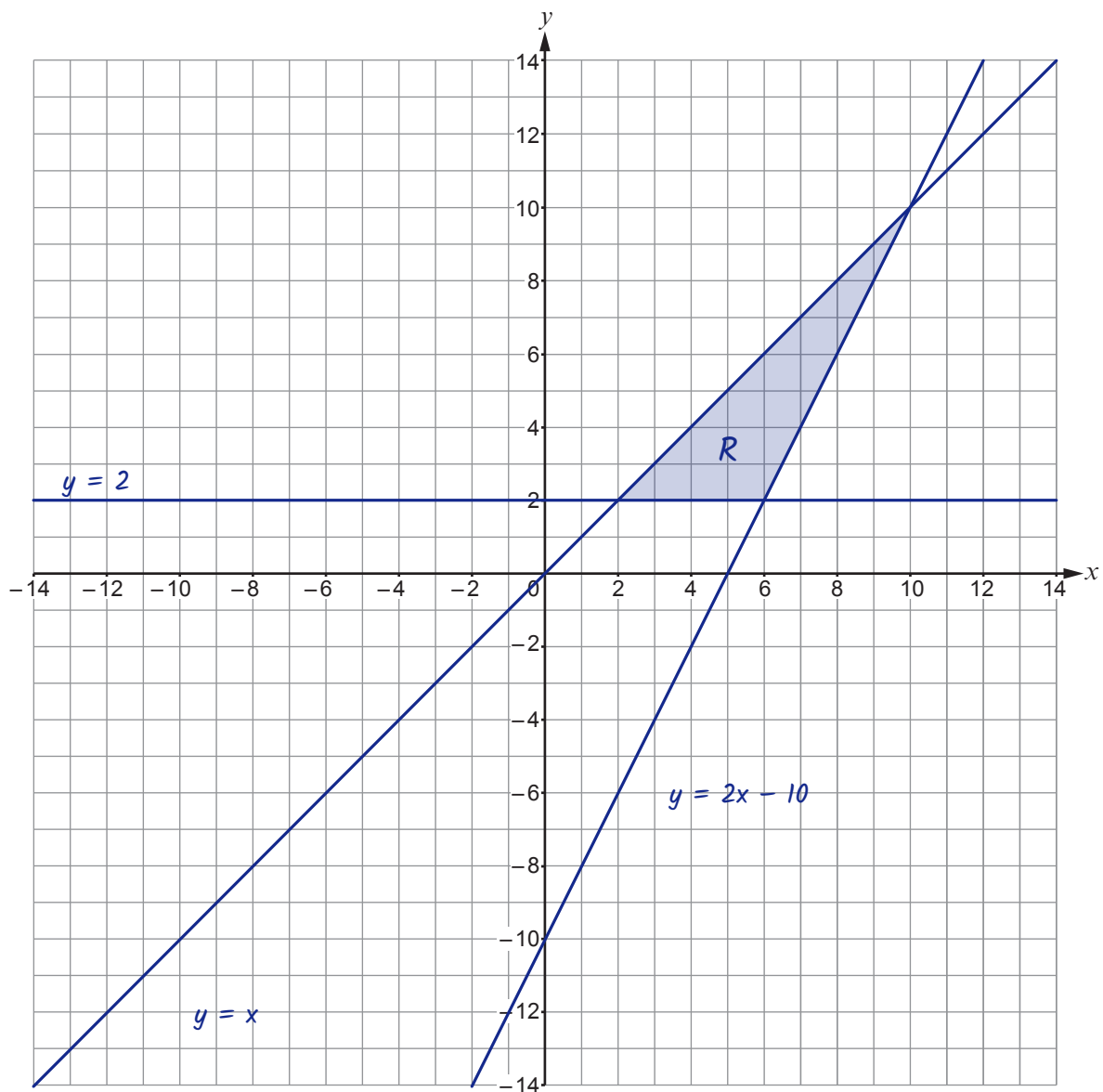
13. Using the axes below, find the region which satisfies the following inequalities:

$$\begin{aligned} y &\geq 2 \\ y &\geq 2x - 10 \\ y &\leq x \end{aligned}$$

Make sure that you clearly indicate the region that represents your answer.

[3]

The required region is shaded and labelled R.



14. Triangle ABC has sides $AB = 24.1$ cm and $AC = 17.9$ cm, as shown below.
 $\hat{BAC} = 37^\circ$.

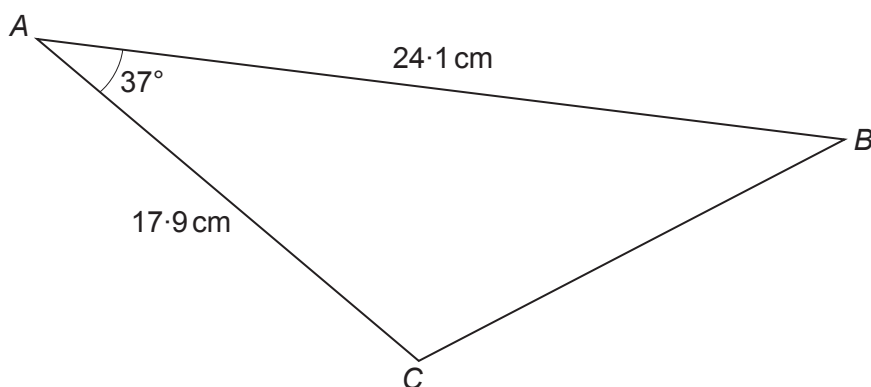


Diagram not drawn to scale

Calculate the area of the triangle ABC.

[2]

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 24.1 \times 17.9 \times \sin 37^\circ = 129.808...$$

$$\text{Area} = 129.8 \text{ cm}^2$$

15. The values $e = 7.1$, $f = 73.9$ and $g = 65.7$ are each given correct to 1 decimal place.

The value of h is found using the formula $h = \frac{e}{f - g}$.

Calculate the **greatest** value of h .

Give your answer correct to four decimal places.

You must show all your working.

[3]

For the greatest h use the greatest e and the smallest $f - g$.

Greatest $e = 7.15$

Smallest $f - g = \text{lower bound of } f - \text{upper bound of } g$

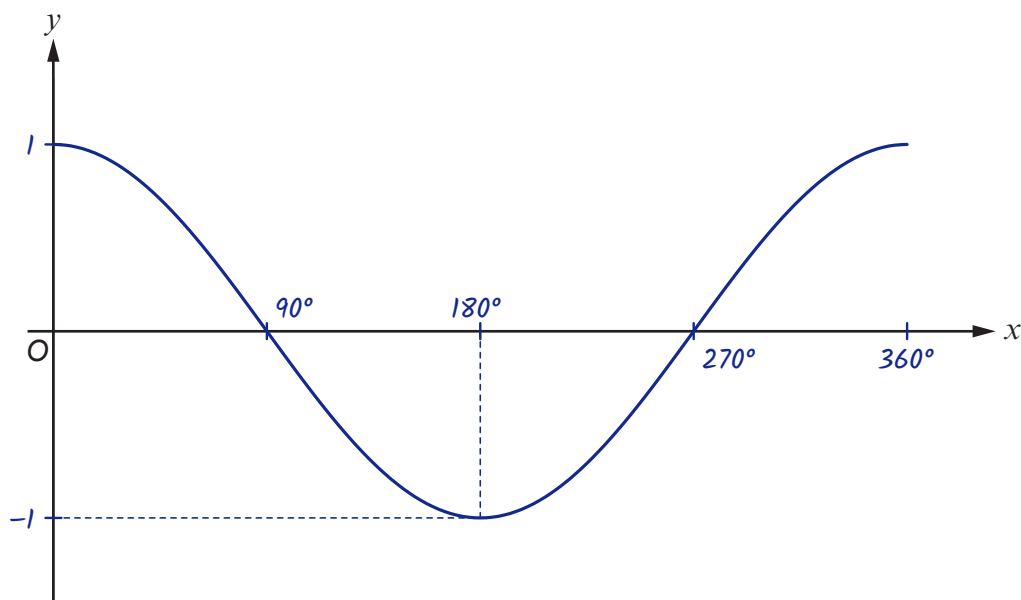
$$= 73.85 - 65.75 = 8.1$$

$$h = \frac{7.15}{73.85 - 65.75} = \frac{7.15}{8.1} = 0.882716...$$

Greatest $h = 0.8827$ (to 4 d.p.)



16. (a) Sketch the curve $y = \cos x$ on the axes below, for values of x from 0° to 360° . You must indicate any important values on both axes. [2]



- (b) Solve the equation $2\cos x = 1$.
Give all answers in the range $x = 0^\circ$ to $x = 360^\circ$. [2]

$$2\cos x = 1$$

$$\cos x = 0.5$$

$$x = \cos^{-1}(0.5) = 60^\circ$$

$$\text{Other solution: } x = 360^\circ - 60^\circ = 300^\circ$$

$$x = 60^\circ \text{ and } x = 300^\circ$$



17. Zoltan rolls three unbiased 6-sided dice.
Calculate the probability that the sum of the values shown on the dice is 17. [3]

A total of 17 needs 6, 6 and 5 in some order:

(6, 6, 5), (6, 5, 6), (5, 6, 6) so there are 3 ways

$$P(\text{each way}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

$$P(\text{sum is 17}) = 3 \times \frac{1}{216} = \frac{3}{216} = \frac{1}{72}$$



18. Two **similar** prisms have volumes of 569 cm^3 and 3821 cm^3 , as shown below. The uniform cross-sectional area of the smaller prism is 29 cm^2 .

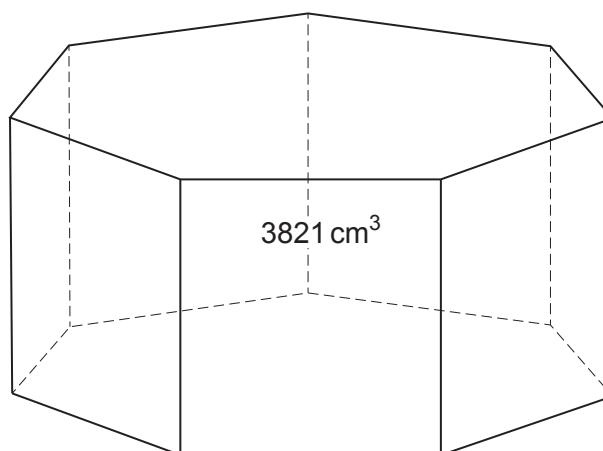
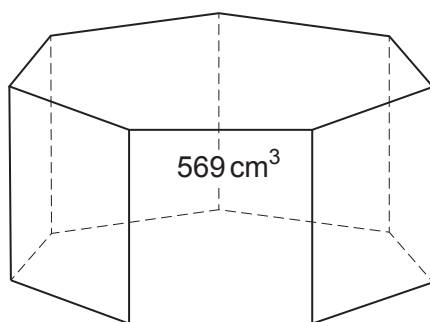


Diagram not drawn to scale

Calculate the uniform cross-sectional area of the larger prism.

[4]

$$\text{Volume scale factor} = 3821 \div 569 = 6.71528...$$

$$\text{Length scale factor} = \sqrt[3]{(3821 \div 569)} = 1.88663...$$

$$\text{Area scale factor} = (1.88663...) ^2 = 3.55939...$$

$$\text{Cross-sectional area of larger prism} = 29 \times 3.55939...$$

$$= 103.222... \text{ cm}^2$$

$$= 103.2 \text{ cm}^2$$



19. Solve the equation $7x(x + 3) + 1 = x(x + 2)$.
You must use the quadratic formula and show all your working.
Give your answers correct to 2 decimal places.

[5]

$$7x(x + 3) + 1 = x(x + 2)$$

$$7x^2 + 21x + 1 = x^2 + 2x$$

$$6x^2 + 19x + 1 = 0$$

$$a = 6, b = 19, c = 1$$

$$x = \frac{-19 \pm \sqrt{19^2 - 4 \times 6 \times 1}}{2 \times 6}$$

$$x = \frac{-19 \pm \sqrt{337}}{12}$$

$$x = (-19 + 18.35755...) \div 12 = -0.053536...$$

$$x = (-19 - 18.35755...) \div 12 = -3.113129...$$

$$x = -0.05 \text{ or } x = -3.11 \text{ (to 2 d.p.)}$$



20. In the diagram below, $AB = 13\text{ cm}$, $AD = 17\text{ cm}$, $BC = 19\text{ cm}$ and $DC = 11\text{ cm}$.

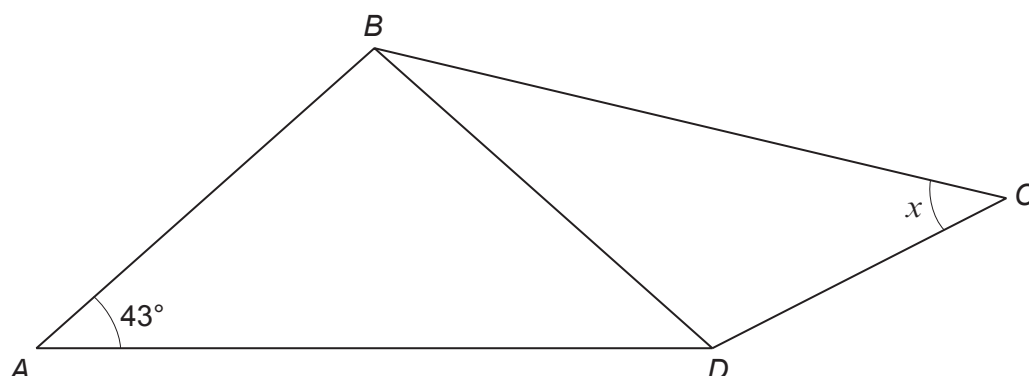


Diagram not drawn to scale

Calculate the size of angle x .
You must show all your working.

[6]

Cosine rule in triangle ABD to find BD:

$$BD^2 = 13^2 + 17^2 - 2 \times 13 \times 17 \times \cos 43^\circ$$

$$BD^2 = 458 - 323.258... = 134.741...$$

$$BD = \sqrt{134.741...} = 11.6078... \text{ cm}$$

Cosine rule in triangle BCD to find x :

$$\cos x = \frac{11^2 + 19^2 - 134.741...}{2 \times 11 \times 19}$$

$$\cos x = 347.258... \div 418 = 0.830761...$$

$$x = \cos^{-1}(0.830761...) = 33.8229...^\circ$$

$$x = 33.8^\circ$$

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