

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U60-1



S23-3300U60-1

WEDNESDAY, 14 JUNE 2023 – MORNING

MATHEMATICS

UNIT 2: CALCULATOR-ALLOWED

HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 10(a), the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	6	
2.	4	
3.	4	
4.	6	
5.	4	
6.	7	
7.	4	
8.	5	
9.	2	
10.	8	
11.	3	
12.	6	
13.	5	
14.	4	
15.	6	
16.	6	
Total	80	

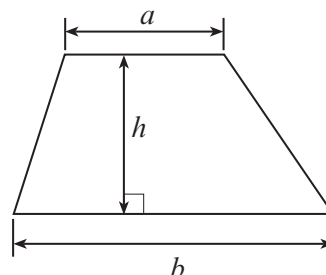
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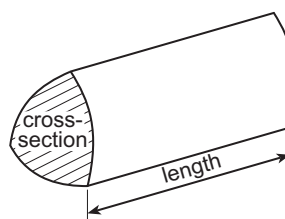
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

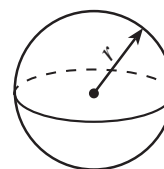


Volume of prism = area of cross-section $\times$ length



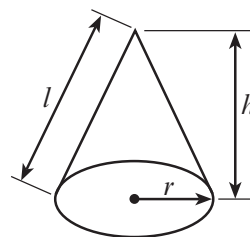
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

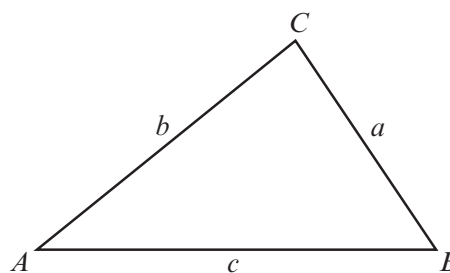


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) Solve the equation $7 + 5(x - 2) = 3x + 8$. [3]

$$7 + 5x - 10 = 3x + 8$$

$$5x - 3 = 3x + 8$$

$$2x = 11$$

$$x = \frac{11}{2}$$

$$x = 5.5$$

- (b) Make f the subject of the formula $h = 13 - 2f$. [2]

$$2f = 13 - h$$

$$f = \frac{13 - h}{2}$$

- (c) Factorise $15x - 35y$. [1]

$$5(3x - 7y)$$



2. A large number of prize tokens are placed in a box.
The tokens are identical in shape and size.

Gold, Silver, Bronze or *No Prize* is written on each token.

One token is chosen at random from the box.

The table below shows the probability of choosing a *Gold* prize token and the probability of choosing a *Silver* prize token.

Token	Gold	Silver	Bronze	No Prize
Probability	0.02	0.18	0.2	0.6

- (a) There are three times as many *No Prize* tokens in the box as there are *Bronze* prize tokens.

Complete the table.

[2]

$$P(\text{Bronze}) + P(\text{No Prize}) = 1 - (0.02 + 0.18) = 0.8$$

$$\text{Bronze} : \text{No Prize} = 1 : 3, \quad 0.8 \div 4 = 0.2$$

$$P(\text{Bronze}) = 0.2$$

$$P(\text{No Prize}) = 3 \times 0.2 = 0.6$$

- (b) There are 15 *Gold* prize tokens in the box.
How many *Silver* prize tokens are there in the box?

[2]

$$\text{Total tokens} = 15 \div 0.02 = 750$$

$$\text{Silver tokens} = 750 \times 0.18$$

$$= 135 \text{ Silver tokens}$$



3. A solution of the equation

$$x^3 - 8x + 3 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 - 8x + 3$	
2	$8 - 16 + 3 = -5$	negative
3	$27 - 24 + 3 = 6$	positive
2.5	$15.625 - 20 + 3 = -1.375$	negative
2.6	$17.576 - 20.8 + 3 = -0.224$	negative
2.7	$19.683 - 21.6 + 3 = 1.083$	positive
2.65	$18.609625 - 21.2 + 3 = 0.409625$	positive

At $x = 2.6$ the value is negative and at $x = 2.65$ it is positive,
so the solution lies between 2.6 and 2.65, which rounds to 2.6.

$x = 2.6$ (1 d.p.)



4. (a) Evaluate $\frac{\sqrt[3]{154}}{7.9 - 3.26}$.

Give your answer correct to 2 significant figures. [2]

$$\sqrt[3]{154} = 5.360108411...$$

$$7.9 - 3.26 = 4.64$$

$$5.360108411... \div 4.64 = 1.155195778...$$

$$= 1.2 \text{ (2 s.f.)}$$

- (b) Calculate the reciprocal of 23.
Give your answer correct to 3 decimal places. [2]

$$1 \div 23 = 0.0434782608...$$

$$= 0.043 \text{ (3 d.p.)}$$

- (c) Circle the correct answer for each of the following.

- (i) The Lowest Common Multiple (LCM) of 4 and 6 is:

2

4

6

12

24

[1]

- (ii) The Highest Common Factor (HCF) of 10 and 15 is:

5

10

15

30

150

[1]



5. Solve the following simultaneous equations using an algebraic (not graphical) method. You must show all your working. [4]

$$2x + 3y = 16.4$$

$$3x - 2y = 7.7$$

$$(1) \times 2: 4x + 6y = 32.8$$

$$(2) \times 3: 9x - 6y = 23.1$$

$$\text{Add: } 13x = 55.9$$

$$x = 55.9 \div 13 = 4.3$$

Substitute $x = 4.3$ into (1):

$$2 \times 4.3 + 3y = 16.4$$

$$8.6 + 3y = 16.4$$

$$3y = 7.8$$

$$y = 2.6$$

$$\text{Check in (2): } 3 \times 4.3 - 2 \times 2.6 = 12.9 - 5.2 = 7.7 \checkmark$$

$$x = 4.3, y = 2.6$$



6. In the diagram below, AD is a straight line.

$$\hat{BAC} = 90^\circ, \hat{BDE} = 90^\circ \text{ and } \hat{CBE} = 90^\circ.$$

$$AC = 7.7 \text{ cm}, BC = 11.3 \text{ cm and } BD = 13.1 \text{ cm}.$$

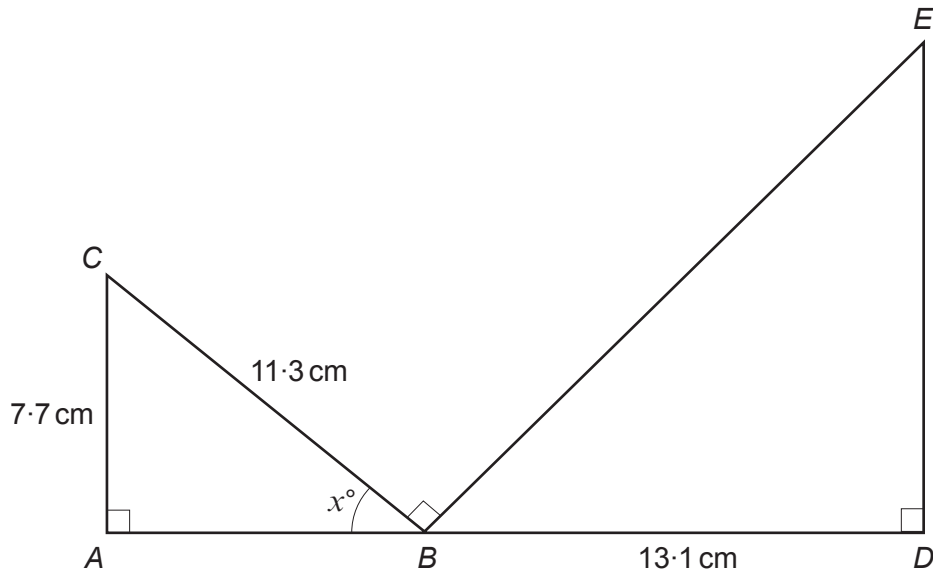


Diagram not drawn to scale

- (a) Calculate the value of x .

[3]

Triangle ABC is right-angled at A, so use SOH CAH TOA:

$$\sin x = \frac{\text{opp}}{\text{hyp}} = \frac{7.7}{11.3} = 0.6814159292...$$

$$x = \sin^{-1}(0.6814159292...)$$

$$x = 42.95438807...$$

$$x = 43.0^\circ \text{ (to 1 d.p.)}$$



- (b) Hence find the length DE .

[4]

Angles on a straight line AD : angle $DBE = 180^\circ - 90^\circ - x$

Angle $DBE = 180^\circ - 90^\circ - 42.954\dots^\circ = 47.045\dots^\circ$

Triangle BDE is right-angled at D :

$$\tan 47.045\dots^\circ = \frac{DE}{BD} = \frac{DE}{13.1}$$

$$DE = 13.1 \times \tan 47.045\dots^\circ$$

$$DE = 14.07047054\dots$$

$$DE = 14.07 \text{ cm}$$

7. (a) A number is decreased by 5% of its value.
This is done 4 times in total. Each time, the value decreases by 5%.
Circle the multiplier that you would use to find the value after the 4 decreases.

[1]

$$\times 0.05^4$$

$$\times 0.95^4$$

$$\times 0.20$$

$$\times 1.05^4$$

$$\times 0.04^5$$

- (b) A number has been decreased by 17% to give an answer of 3569.
What was the original number?

[3]

$$\text{Multiplier} = 100\% - 17\% = 83\% = 0.83$$

$$\text{Original} \times 0.83 = 3569$$

$$\text{Original number} = \frac{3569}{0.83}$$

$$= 4300$$

The original number was 4300



8. The diagram below shows a semicircle, with radius r , drawn inside a trapezium.

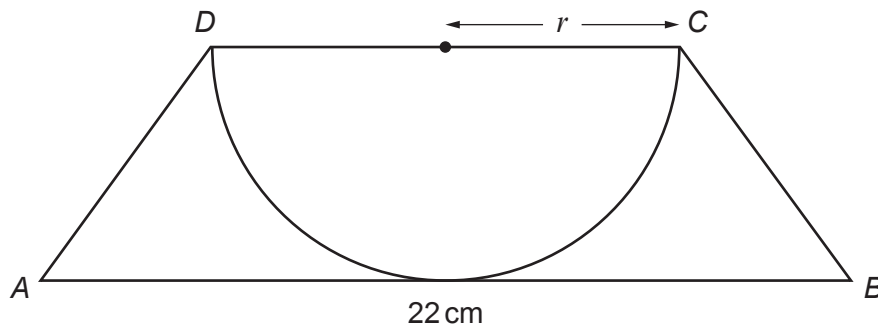


Diagram not drawn to scale

The area of the semicircle is 77 cm^2 .

The semicircle touches the line AB.
 $AB = 22 \text{ cm}$.

Calculate the area of the trapezium ABCD.

[5]

$$\text{Area of semicircle} = \frac{\pi r^2}{2} = 77$$

$$r^2 = 154 \div \pi = 49.01972247...$$

$$r = \sqrt{49.01972247...} = 7.001408606... \text{ cm}$$

$$DC = 2r = 14.00281721... \text{ cm, height} = r = 7.001408606... \text{ cm}$$

$$\text{Area of trapezium} = \frac{(a + b)}{2} \times h$$

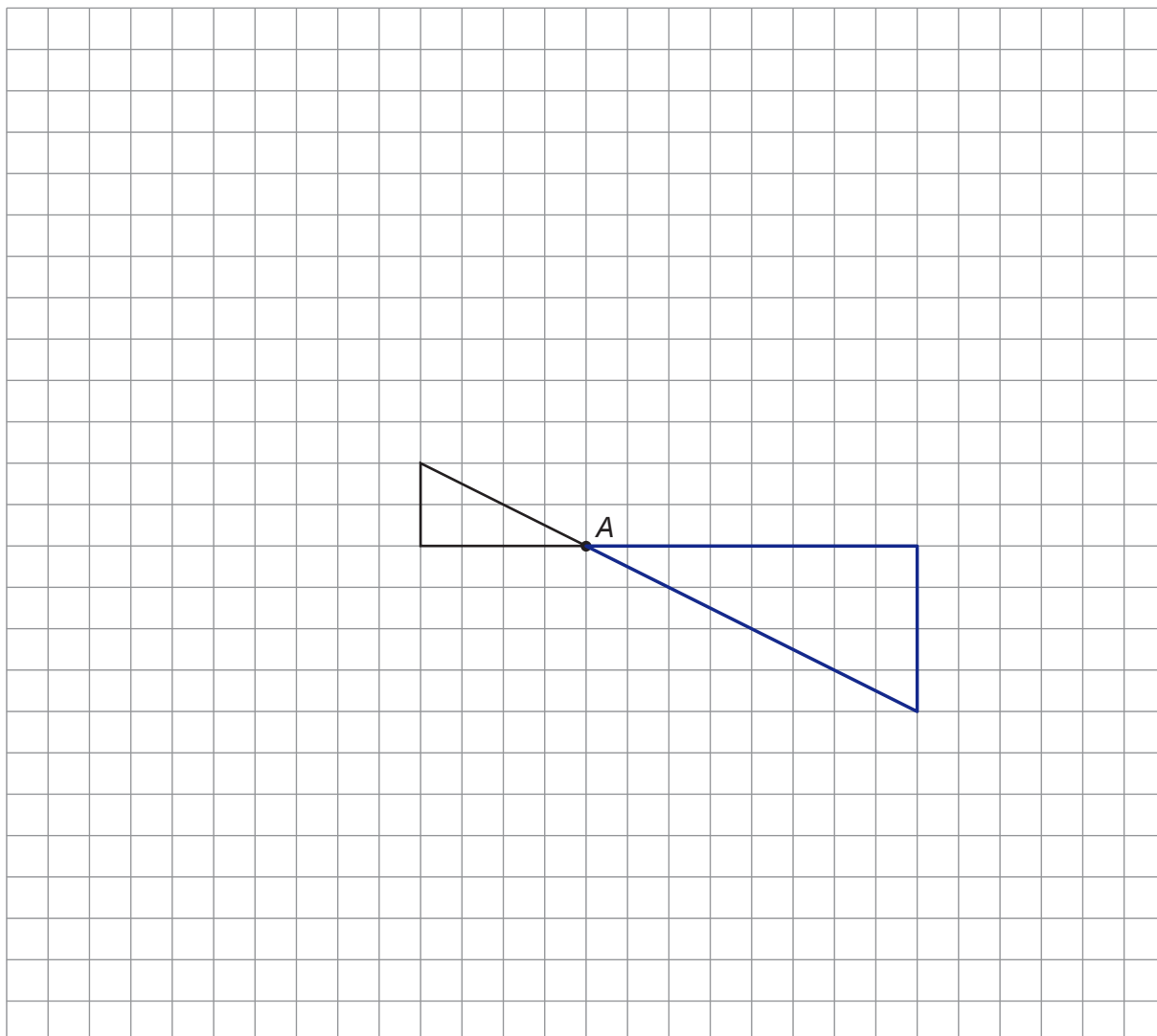
$$= \frac{14.00281721... + 22}{2} \times 7.001408606...$$

$$= 126.0352171...$$

$$\text{Area of trapezium} = 126.0 \text{ cm}^2 \text{ (1 d.p.)}$$



9. Enlarge the given triangle by a scale factor of -2 , using point A as the centre of enlargement. [2]



10. $ABCDE$ is a regular pentagon with sides of length 11 cm.
 CDE is a sector of a circle with centre D and radius 11 cm.
 The two shapes are joined together, as shown below.

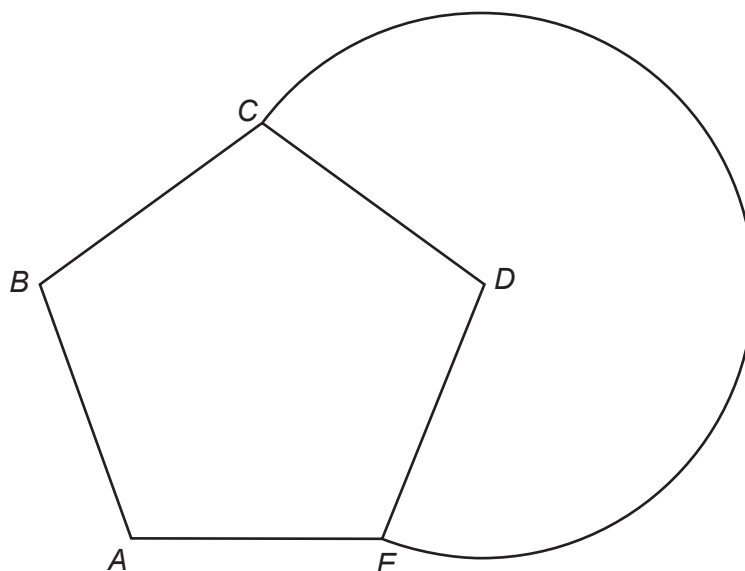


Diagram not drawn to scale

- (a) In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

Calculate the length of the arc CE shown in the diagram.
 You must show all your working.

[4 + 2 OCW]

Interior angle of a regular pentagon:

$$\text{exterior angle} = 360^\circ \div 5 = 72^\circ, \text{ so interior angle} = 180^\circ - 72^\circ = 108^\circ$$

So angle CDE inside the pentagon $= 108^\circ$.

The sector uses the reflex angle at D : $360^\circ - 108^\circ = 252^\circ$

$$\text{Arc length } CE = \frac{252}{360} \times 2\pi r = \frac{252}{360} \times 2 \times \pi \times 11$$

$$\text{Arc length } CE = 0.7 \times 2 \times \pi \times 11 = 15.4\pi$$

$$\text{Arc length } CE = 48.38052687\dots$$

The length of the arc CE is 48.4 cm (to 1 d.p.).



- (b) In a shape **similar** to the one shown on the previous page, the regular pentagon has sides of length 671 cm.

Complete the following statement.

Total area of new shape = ...3721..... × total area of original shape

You must show all your working.

[2]

$$\text{Length scale factor} = 671 \div 11 = 61$$

$$\text{Area scale factor} = (\text{length scale factor})^2$$

$$= 61^2$$

$$= 3721$$

11. Make x the subject of the formula $ax^2 + x^2 = b$.

[3]

$$ax^2 + x^2 = b$$

$$x^2(a + 1) = b$$

$$x^2 = \frac{b}{a + 1}$$

$$x = \pm \sqrt{\frac{b}{a + 1}}$$



12. (a) Factorise $8x^2 - 18$.

[3]

$$8x^2 - 18 = 2(4x^2 - 9)$$

$$4x^2 - 9 = (2x)^2 - 3^2, \text{ a difference of two squares}$$

$$8x^2 - 18 = 2(2x + 3)(2x - 3)$$

- (b) Hence solve $8x^2 - 18 = 0$.

[1]

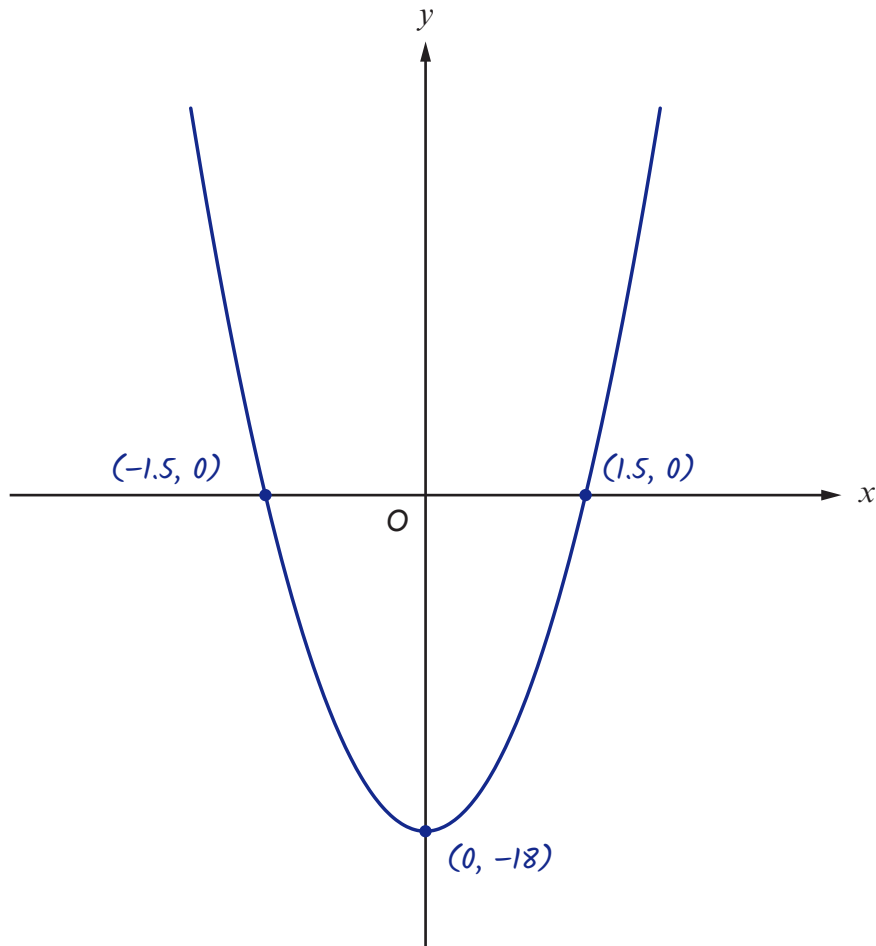
$$2(2x + 3)(2x - 3) = 0$$

$$2x + 3 = 0 \text{ or } 2x - 3 = 0$$

$$x = -1.5 \text{ or } x = 1.5$$



- (c) Hence, sketch the graph of $y = 8x^2 - 18$ on the axes below.
Mark clearly the coordinates of any point where this graph crosses an axis. [2]



Space for working:

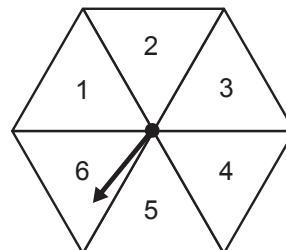
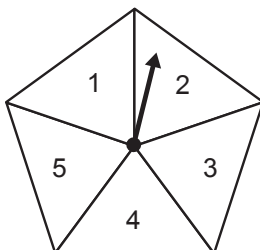
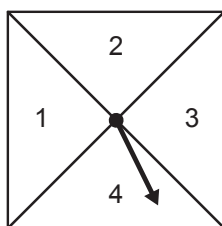
Crosses the x-axis where $8x^2 - 18 = 0$: $x = -1.5$ and $x = 1.5$

Crosses the y-axis at $x = 0$: $y = 8 \times 0^2 - 18 = -18$

Positive x^2 term, so a U-shaped curve with minimum at $(0, -18)$



13. Catherine has three spinners, as shown below.



She spins each spinner once.

- (a) Calculate the probability that all three spinners land on prime numbers.

[2]

$P(\text{prime})$: spinner 1 = $2/4$ (2, 3), spinner 2 = $3/5$ (2, 3, 5), spinner 3 = $3/6$ (2, 3, 5)

$$P(\text{all prime}) = \frac{2}{4} \times \frac{3}{5} \times \frac{3}{6}$$

$$= \frac{18}{120} = \frac{3}{20}$$

- (b) The numbers that the three spinners land on are added together.
Calculate the probability that the total is **greater than 4**.

[3]

$$\text{Total outcomes} = 4 \times 5 \times 6 = 120$$

$$\text{Totals of 4 or less: } 1+1+1 = 3, 1+1+2, 1+2+1, 2+1+1 = 4 \text{ (4 ways)}$$

$$P(\text{total} \leq 4) = 4 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6} = \frac{4}{120}$$

$$P(\text{total} > 4) = 1 - \frac{4}{120} = \frac{116}{120}$$

$$= \frac{29}{30}$$



14. The cube shown below has a volume of 10648 cm^3 .

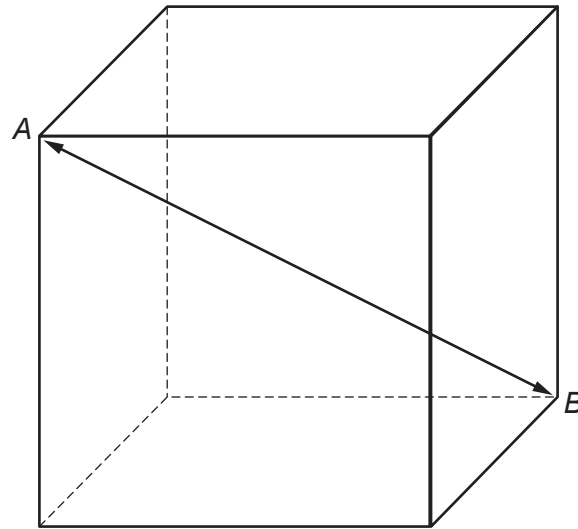


Diagram not drawn to scale

Calculate the length of the internal diagonal AB.

[4]

$$\text{Side length} = \sqrt[3]{10648} = 22 \text{ cm}$$

$$\text{Diagonal of base: } 22^2 + 22^2 = 968$$

$$AB^2 = 22^2 + 22^2 + 22^2 = 1452$$

$$AB = \sqrt{1452}$$

$$AB = 38.10511777\dots$$

$$AB = 38.1 \text{ cm (1 d.p.)}$$



15. Use the quadratic formula to solve the equation $\frac{1}{x-2} + \frac{1}{3x-7} = 1$.

Give your answers correct to 2 decimal places.

You **must** show all your working.

[6]

Multiply every term by $(x-2)(3x-7)$:

$$(3x-7) + (x-2) = (x-2)(3x-7)$$

$$4x-9 = 3x^2-7x-6x+14$$

$$4x-9 = 3x^2-13x+14$$

$$0 = 3x^2-17x+23$$

$$a = 3, b = -17, c = 23$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \times 3 \times 23}}{2 \times 3}$$

$$x = \frac{17 \pm \sqrt{289 - 276}}{6} = \frac{17 \pm \sqrt{13}}{6}$$

$$x = \frac{17 + 3.605551275...}{6} \quad \text{or} \quad x = \frac{17 - 3.605551275...}{6}$$

$$x = 3.434258546... \quad \text{or} \quad x = 2.232408121...$$

$$x = 3.43 \quad \text{or} \quad x = 2.23 \quad (2 \text{ d.p.})$$



16. In triangle ABC shown below, $AB = 13$ cm and $BC = 11$ cm.
 D is a point on AC where $BD = 7$ cm and $DC = 5$ cm.

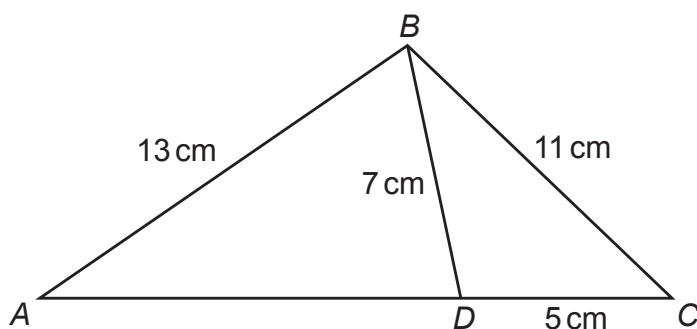


Diagram not drawn to scale

Calculate the size of $\hat{BAD}$.
 You must show all your working.

[6]

Cosine rule in triangle BDC to find angle BDC:

$$\cos BDC = \frac{5^2 + 7^2 - 11^2}{2 \times 5 \times 7} = \frac{25 + 49 - 121}{70} = \frac{-47}{70}$$

$$\text{Angle BDC} = \cos^{-1}(-47/70) = 132.1774186...^\circ$$

$$\text{Angle ADB} = 180^\circ - 132.1774186...^\circ = 47.82258140...^\circ$$

(angles on a straight line AC)

Sine rule in triangle ABD:

$$\frac{\sin BAD}{7} = \frac{\sin 47.82258140...^\circ}{13}$$

$$\sin BAD = 7 \times \sin 47.82258140...^\circ \div 13 = 0.3990373031...$$

$$\text{Angle BAD} = \sin^{-1}(0.3990373031...) = 23.51800943...^\circ$$

$$\text{Angle BAD} = 23.5^\circ \text{ (1 d.p.)}$$

END OF PAPER



[illegible]