

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U50-1



TUESDAY, 23 MAY 2023 – MORNING

MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER

1 hour 45 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 5, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

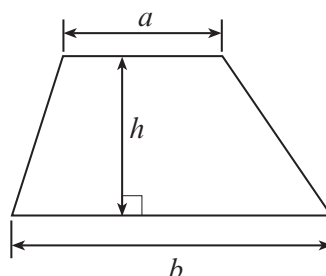
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	2	
2.	4	
3.	6	
4.	4	
5.	7	
6.	3	
7.	5	
8.	3	
9.	4	
10.	4	
11.	5	
12.	4	
13.	7	
14.	5	
15.	6	
16.	6	
17.	5	
Total	80	



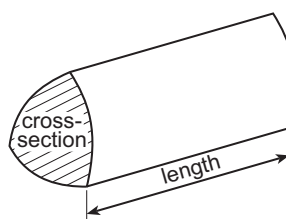
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

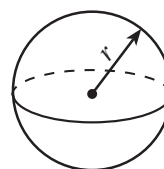


Volume of prism = area of cross-section $\times$ length



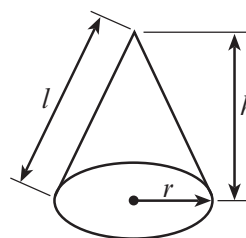
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

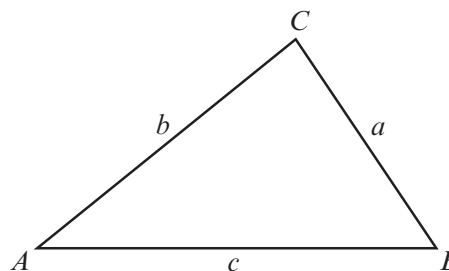


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

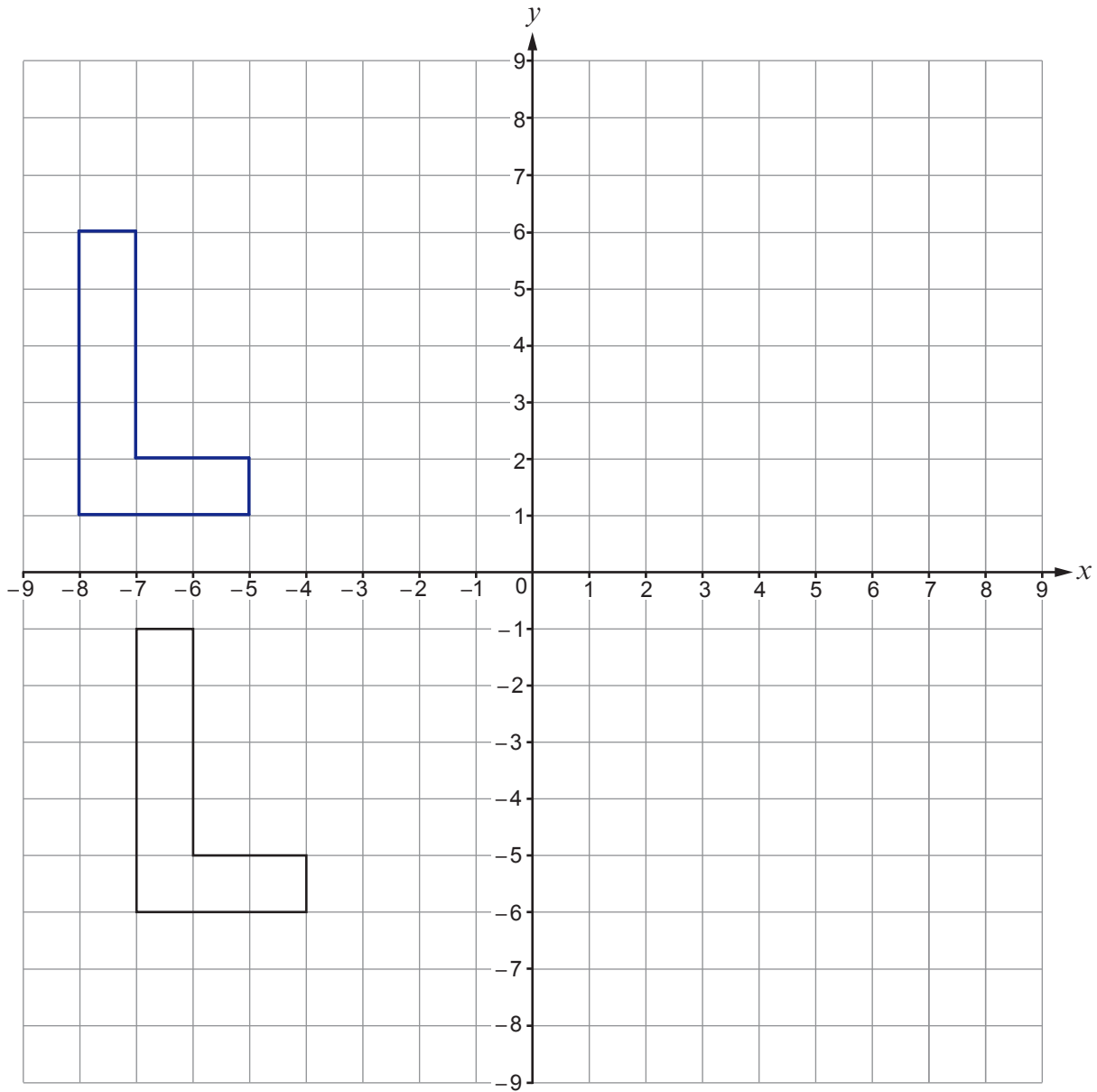
Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) Translate the shape shown below using the column vector $\begin{pmatrix} -1 \\ 7 \end{pmatrix}$.

[1]



- (b) Write down the column vector that will reverse the translation in part (a).

[1]

$$\begin{pmatrix} 1 \\ -7 \end{pmatrix}$$



2. (a) Express 675 as a product of its prime factors in index form.

[3]

$$675 \div 5 = 135$$

$$135 \div 5 = 27$$

$$27 \div 3 = 9$$

$$9 \div 3 = 3$$

$$3 \div 3 = 1$$

$$675 = 3 \times 3 \times 3 \times 5 \times 5$$

$$675 = 3^3 \times 5^2$$

- (b) 360 expressed as a product of its prime factors in index form is $2^3 \times 3^2 \times 5$.

What is the smallest whole number that 360 can be multiplied by to give a square number?

[1]

$$360 = 2^3 \times 3^2 \times 5 \quad (\text{indices of 2 and 5 are odd})$$

$$\text{Multiply by } 2 \times 5: 2^4 \times 3^2 \times 5^2 = 3600 = 60^2$$

Smallest whole number is 10.....



3. (a) Simplify each of the following.
Circle your answer in each case.

(i) $m^4 \times m^3 =$

[1]

$$m^7$$

$$m^{12}$$

$$m^{43}$$

$$7m$$

$$12m$$

(ii) $\frac{m^{15}}{m^5} =$

[1]

$$m^{75}$$

$$\frac{1}{m^3}$$

$$m^3$$

$$m^{10}$$

$$\frac{1}{m^{10}}$$

- (b) Write down an expression for the n th term of the following sequence.

[2]

4, 11, 18, 25, 32

Common difference = 7, so $7n$

$7n - 3$ ($n = 1: 7 - 3 = 4$)

n th term = $7n - 3$

- (c) List all of the integers that satisfy the following inequality.

[2]

$$13 < 2n < 19$$

$$13 < 2n < 19$$

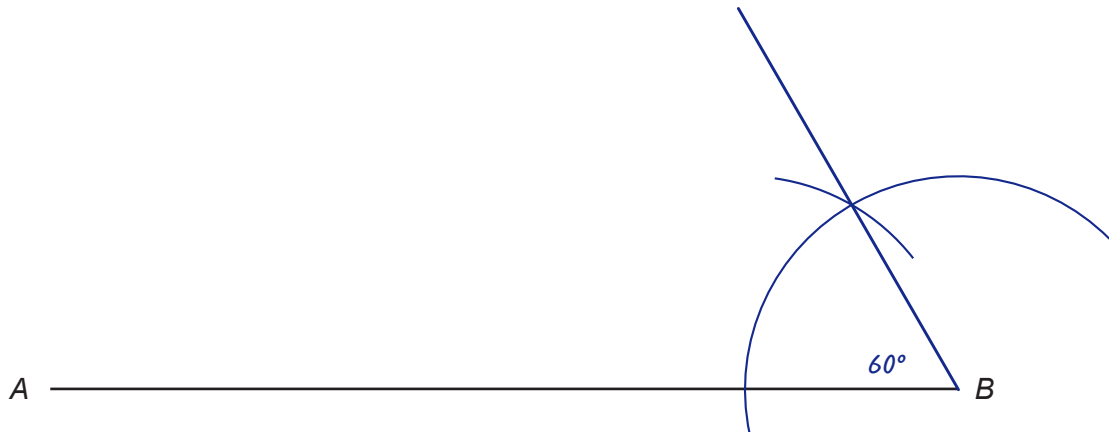
$$6.5 < n < 9.5$$

$$2n = 14, 16, 18$$

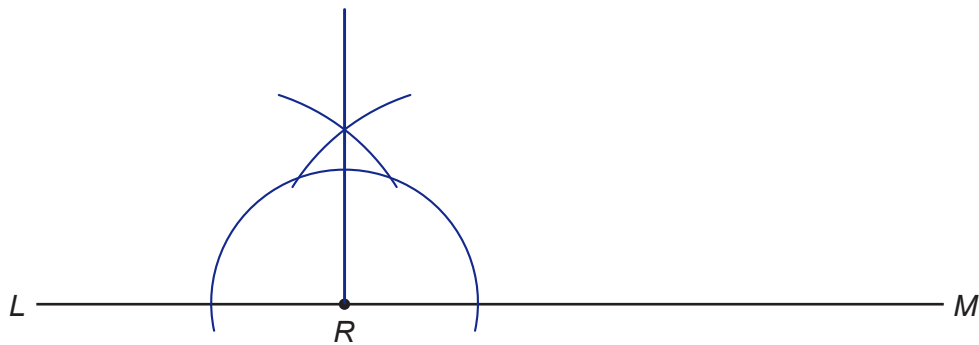
Integers are 7, 8, 9



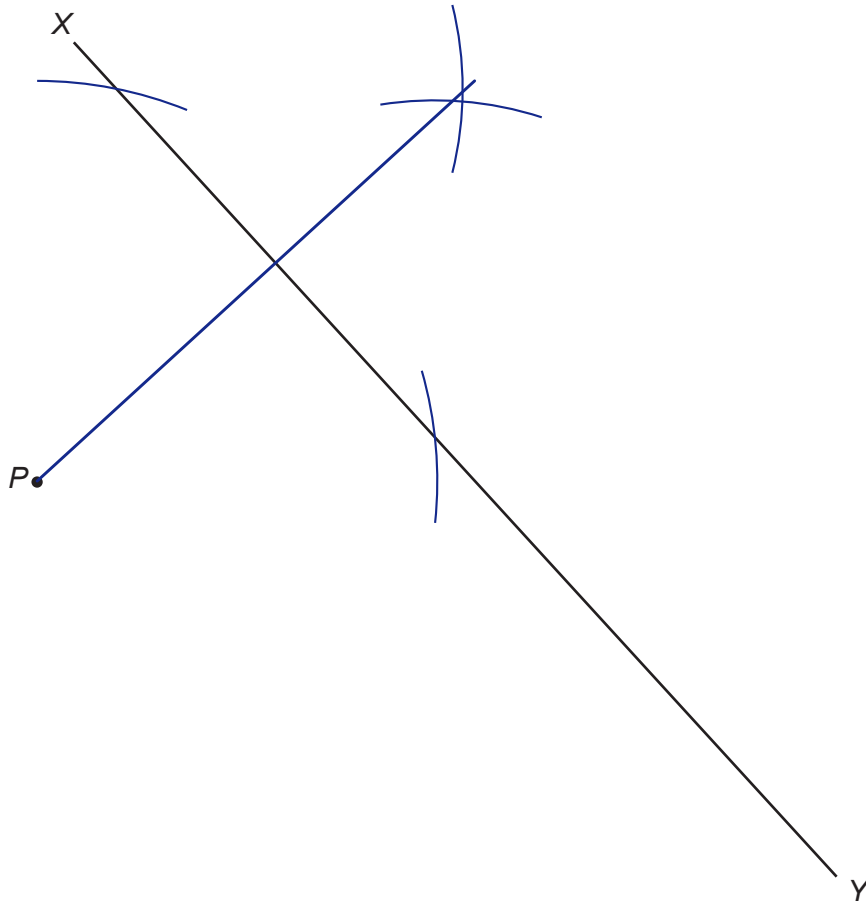
4. (a) Line AB is shown below.
Using only a ruler and a pair of compasses, construct an angle of 60° at point B . [1]



- (b) R is a point on the line LM .
Using only a ruler and a pair of compasses, construct an angle of 90° at point R . [1]



- (c) Using only a ruler and a pair of compasses, construct a perpendicular line from the point P to the line XY . [2]



5. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

The shape below consists of a semicircle attached to one side of a right-angled triangle.

$\angle ABC = 90^\circ$, $AB = 8$ cm, $BC = 6$ cm.

BC is the diameter of the semicircle.

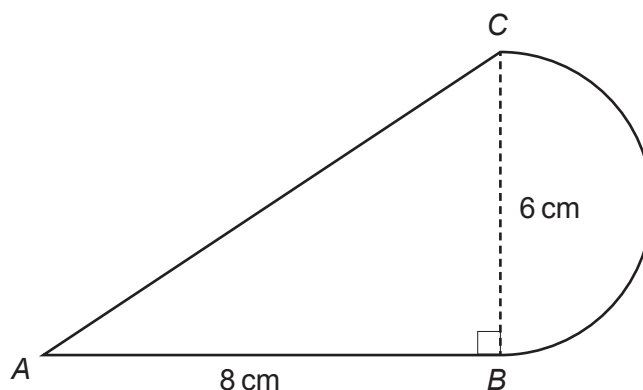


Diagram not drawn to scale

Calculate the perimeter of the shape.

Use $\pi = 3.14$.

You must show all your working.

[5 + 2 OCW]

Triangle ABC is right-angled at B, so by Pythagoras:

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$AC = \sqrt{100} = 10 \text{ cm}$$

The curved edge is half the circumference of a circle, diameter 6 cm.

$$\text{Semicircle arc} = (\pi \times d) \div 2 = (3.14 \times 6) \div 2$$

$$3.14 \times 6 = 18.84, \quad 18.84 \div 2 = 9.42 \text{ cm}$$

BC is inside the shape, so it is not part of the perimeter.

$$\text{Perimeter} = AB + AC + \text{curved edge}$$

$$= 8 + 10 + 9.42$$

$$= 27.42 \text{ cm}$$

The perimeter of the shape is 27.42 cm.



6. Two time periods are measured as 4 hours 40 minutes and 2 hours 50 minutes. Each measurement is correct to the nearest 10 minutes.

What is the least possible sum of these two time periods?
Give your answer in hours and minutes.

[3]

Correct to the nearest 10 minutes, so the lower bound is 5 minutes less.

Lower bound of 4 h 40 min = 4 h 35 min

Lower bound of 2 h 50 min = 2 h 45 min

Least sum = 4 h 35 min + 2 h 45 min

= 6 h 80 min

= 7 h 20 min

(275 min + 165 min = 440 min = 7 h 20 min)

Answer = 7 hours 20 minutes



7. Whitney walks, cycles or travels on the bus to work each day.

On any randomly chosen day:

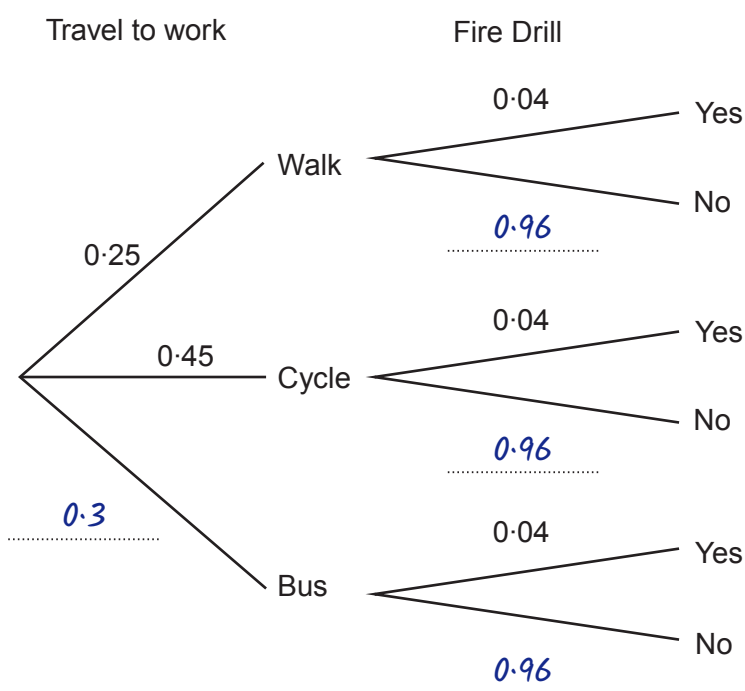
- the probability that she walks to work is 0.25
- the probability that she cycles to work is 0.45.

At work, the probability that there will be a fire drill on any randomly chosen day is 0.04.

How Whitney travels to work is independent of whether or not there is a fire drill.

(a) Complete the tree diagram shown below.

[3]



$$P(\text{Bus}) = 1 - 0.25 - 0.45 = 0.3$$

$$P(\text{No fire drill}) = 1 - 0.04 = 0.96$$

(b) On a randomly chosen day, what is the probability that Whitney walks to work and there is a fire drill? [2]

$$P(\text{Walk and fire drill}) = 0.25 \times 0.04$$

$$= 0.01$$



8. In the following formulae, each measurement of length is represented by a letter.

Consider the dimensions implied by each formula.

For each case, write down whether the formula could be for a length, an area, a volume or none of these.

The first one has been done for you.

[3]

<u>Formula</u>	<u>Formula could be for</u>
$4d + r - 2w$	length
$w(l + b + h)$	 <i>area</i>
$d^3 + 3 \cdot 14r$	 <i>none</i>
$\frac{w^3}{d^2}$	 <i>length</i>
$3 \cdot 14r^3 - lbh$	 <i>volume</i>
$\frac{4w^2}{d}$	 <i>length</i>



9. (a) Express 0.0076 in standard form.

[1]

$$0.0076 = 7.6 \times 10^{-3}$$

- (b) Calculate the value of $(3 \times 10^{17}) \times (2 \times 10^{-12})$.
Give your answer in standard form.

[1]

$$(3 \times 2) \times 10^{17 + (-12)}$$

$$= 6 \times 10^5$$

- (c) Calculate the value of $(2.3 \times 10^4) + (5 \times 10^3)$.
Give your answer in standard form.

[2]

$$2.3 \times 10^4 = 23\,000, \quad 5 \times 10^3 = 5\,000$$

$$23\,000 + 5\,000 = 28\,000$$

$$= 2.8 \times 10^4$$



10. XY is a tangent to a circle, centre O , at the point A .

$$\hat{AYO} = 54^\circ.$$

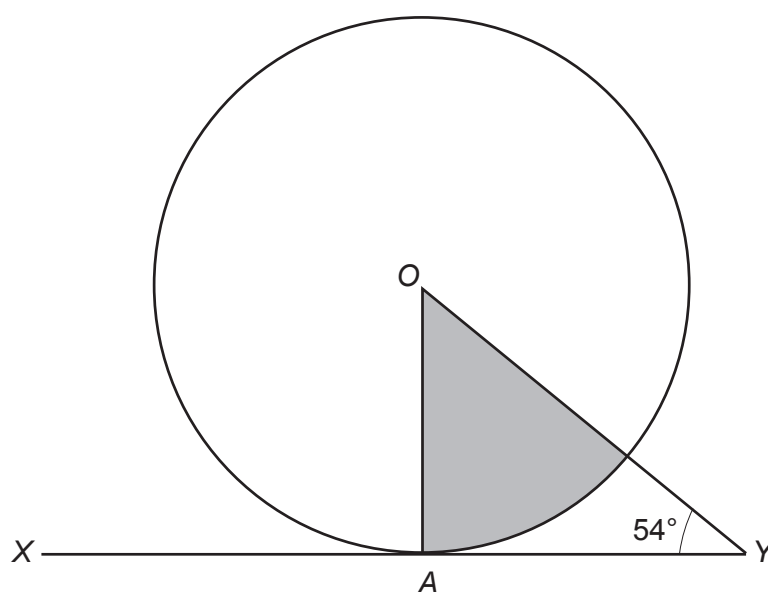


Diagram not drawn to scale

- (a) What percentage of the whole circle is shaded?
You **must** show how you calculated your answer.

[3]

OA is a radius and XY is a tangent at A , so $\hat{OAY} = 90^\circ$

$$\hat{AOY} = 180^\circ - 90^\circ - 54^\circ = 36^\circ \quad (\text{angles in a triangle add to } 180^\circ)$$

The shaded sector has angle 36° .

$$\text{Percentage shaded} = \frac{36}{360} \times 100 = \frac{1}{10} \times 100$$

$$= 10\%$$

- (b) What tangent property of circles did you use in order to answer part (a)?

[1]

A tangent to a circle is perpendicular to the radius

at the point of contact (angle between them is 90°).



11. (a) Given that y is inversely proportional to x and that $y = 0.2$ when $x = 160$, find an expression for y in terms of x . [3]

$$y \propto \frac{1}{x}, \text{ so } y = \frac{k}{x}$$

$$0.2 = \frac{k}{160}$$

$$k = 0.2 \times 160 = 32$$

$$y = \frac{32}{x}$$

- (b) Use the expression you found in part (a) to complete the following table. [2]

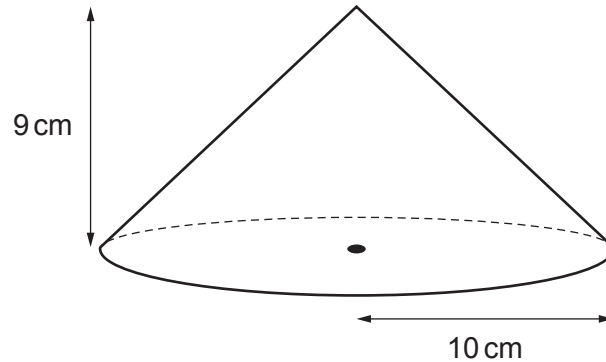
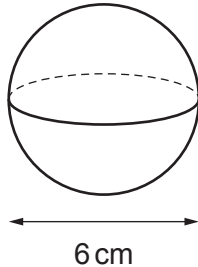
x	160	128	40
y	0.2	0.25	0.8

$$x = 128: y = \frac{32}{128} = \frac{1}{4} = 0.25$$

$$y = 0.8: 0.8 = \frac{32}{x}, x = 32 \div 0.8 = 320 \div 8 = 40$$



12. A sphere has a diameter of 6 cm.
A cone has a base radius of 10 cm and a height of 9 cm.



Diagrams not drawn to scale

Find the ratio of the volume of the sphere to the volume of the cone.
Give your answer in its simplest form.

[4]

$$\text{Radius of sphere} = 6 \div 2 = 3 \text{ cm}$$

$$\text{Volume of sphere} = \frac{4}{3} \times \pi \times 3^3 = \frac{4}{3} \times 27\pi = 36\pi \text{ cm}^3$$

$$\text{Volume of cone} = \frac{1}{3} \times \pi \times 10^2 \times 9 = \frac{1}{3} \times 900\pi = 300\pi \text{ cm}^3$$

$$\text{Ratio} = 36\pi : 300\pi$$

$$= 36 : 300 \quad (\div 12)$$

$$= 3 : 25$$

Volume of the sphere : Volume of the cone

$$= 3 : 25$$



13. Triangle ABC is right-angled.
The area of triangle ABC is 0.75 m^2 .

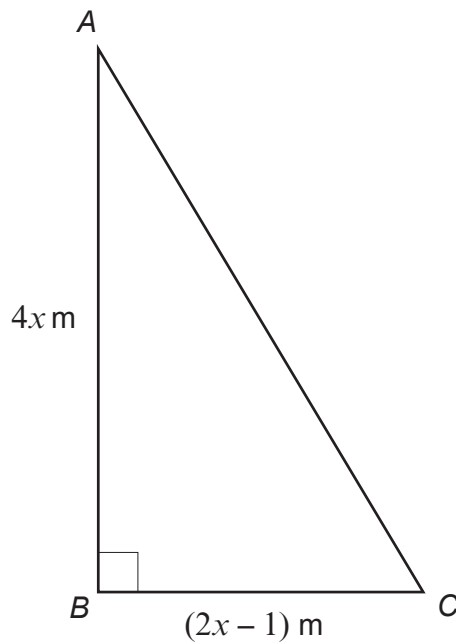


Diagram not drawn to scale

- (a) Show that $16x^2 - 8x - 3 = 0$.

[3]

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4x \times (2x - 1)$$

$$\frac{1}{2} \times 4x \times (2x - 1) = 0.75$$

$$2x(2x - 1) = 0.75$$

$$4x^2 - 2x = 0.75$$

$$16x^2 - 8x = 3 \quad (\times 4)$$

$$16x^2 - 8x - 3 = 0$$



- (b) (i) Solve the equation $16x^2 - 8x - 3 = 0$.
You must use an algebraic method.

[3]

$$16x^2 - 8x - 3 = 0$$

$$(4x - 3)(4x + 1) = 0$$

$$4x - 3 = 0 \quad \text{or} \quad 4x + 1 = 0$$

$$x = \frac{3}{4} \quad \text{or} \quad x = -\frac{1}{4}$$

- (ii) Find the length of BC .
You must **justify** any decision that you make.

[1]

$$x = -\frac{1}{4} \text{ gives } 4x < 0, \text{ but a length cannot be negative, so } x = \frac{3}{4}$$

$$BC = 2x - 1 = 2 \times \frac{3}{4} - 1 = 1.5 - 1 = 0.5 \text{ m}$$

$$BC = 0.5 \text{ m}$$



14. The table below shows some of the values of $y = x + \frac{1}{x}$ for values of x from 0.2 to 4.

x	0.2	0.5	0.8	1	2	3	4
$y = x + \frac{1}{x}$	5.2	2.5	2.05	2	2.5	3.3	4.25

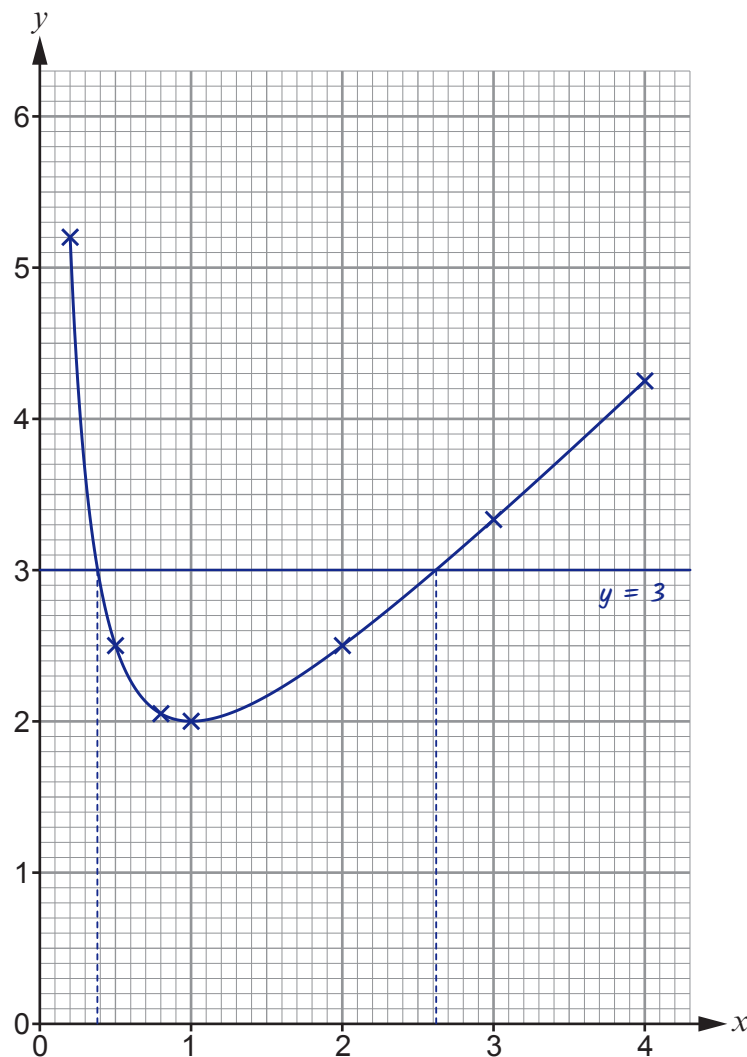
- (a) (i) Complete the table by finding the value of y for $x = 0.5$.

[1]

$$y = 0.5 + \frac{1}{0.5} = 0.5 + 2 = 2.5$$

- (ii) On the graph paper below, draw the graph of $y = x + \frac{1}{x}$ for values of x from 0.2 to 4.

[2]



- (b) Use your graph to solve the equation $x + \frac{1}{x} = 3$. [2]

Draw $y = 3$ and read off where it meets the curve:

$$x = 0.4 \quad \text{or} \quad x = 2.6$$

15. A box contains 5 blue discs and 3 yellow discs.
Three discs are to be chosen at random, without replacement.

- (a) Calculate the probability that the three discs chosen will all be the same colour. [3]

$$P(BBB) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{60}{336}$$

$$P(YYY) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{6}{336}$$

$$P(\text{same colour}) = \frac{60}{336} + \frac{6}{336} = \frac{66}{336} = \frac{11}{56}$$

- (b) Calculate the probability that exactly one blue disc is selected. [3]

Exactly one blue: BYY, YBY or YYB

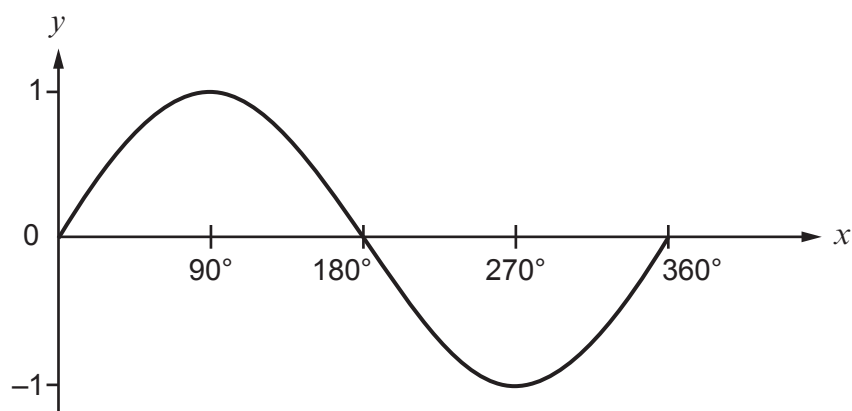
$$P(BYY) = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} = \frac{30}{336}, \quad P(YBY) = \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{30}{336}$$

$$P(YYB) = \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} = \frac{30}{336}$$

$$P(\text{exactly one blue}) = 3 \times \frac{30}{336} = \frac{90}{336} = \frac{15}{56}$$



16. The following diagram shows a sketch of $y = \sin x$ for values of x from 0° to 360° .



- (a) Given that $\sin 62^\circ = 0.8829$, correct to 4 decimal places, write down all the solutions of the equation

$$\sin x = -0.8829$$

for values of x from 0° to 360° .

[2]

$\sin x$ is negative for $180^\circ < x < 360^\circ$

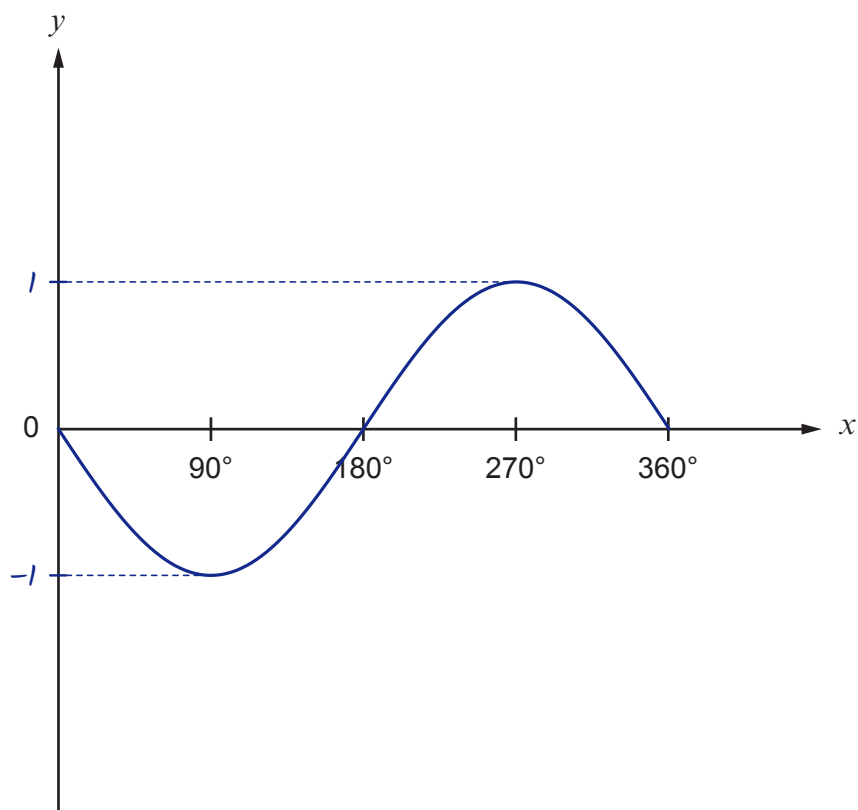
$$x = 180^\circ + 62^\circ = 242^\circ$$

$$x = 360^\circ - 62^\circ = 298^\circ$$

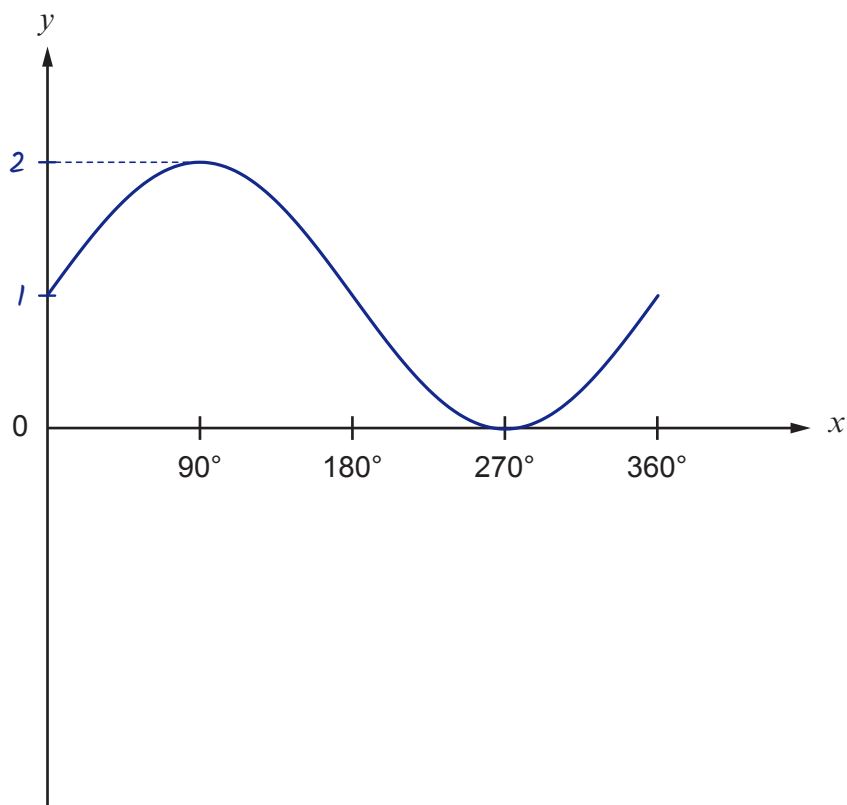
$$x = 242^\circ \text{ and } x = 298^\circ$$



- (b) (i) Use the following axes to sketch the graph of $y = -\sin x$ for values of x from 0° to 360° . You must indicate any important values on the y -axis. [2]



- (ii) Use the following axes to sketch the graph of $y = \sin x + 1$ for values of x from 0° to 360° . You must indicate any important values on the y -axis. [2]



17. (a) Expand and simplify $(4 - \sqrt{6})(1 + \sqrt{6})$. [2]

$$(4 - \sqrt{6})(1 + \sqrt{6})$$

$$= 4 + 4\sqrt{6} - \sqrt{6} - \sqrt{6} \times \sqrt{6}$$

$$= 4 + 4\sqrt{6} - \sqrt{6} - 6$$

$$= 3\sqrt{6} - 2$$

- (b) (i) Write down an **integer** value of x that is greater than 5, for which $x^{\frac{3}{2}}$ is rational. [1]

$$x^{\frac{3}{2}} = (\sqrt{x})^3, \text{ so } x \text{ must be a square number}$$

$$\sqrt{9} = 3, \quad 3^3 = 27 \text{ (rational)}$$

$$x = 9$$

- (ii) Write down an **integer** value of x that is greater than 5, for which $x^{\frac{2}{3}}$ is rational. [1]

$$x^{\frac{2}{3}} = (\sqrt[3]{x})^2, \text{ so } x \text{ must be a cube number}$$

$$\sqrt[3]{8} = 2, \quad 2^2 = 4 \text{ (rational)}$$

$$x = 8$$

- (iii) Write down an **integer** value of x that is greater than 5, for which $x^{\frac{3}{2}}$ and $x^{\frac{2}{3}}$ are both rational. [1]

$$x \text{ must be both a square and a cube: } x = 2^6 = 64$$

$$64^{\frac{3}{2}} = 8^3 = 512, \quad 64^{\frac{2}{3}} = 4^2 = 16$$

$$x = 64$$

END OF PAPER



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