

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U60-1



TUESDAY, 14 JUNE 2022 – MORNING

MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

1 hour 35 minutes

ADDITIONAL MATERIALS

A calculator will be required for this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** the questions in the spaces provided.
If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.
Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
In question 4, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	2	
2.	4	
3.	5	
4.	8	
5.	5	
6.	1	
7.	6	
8.	6	
9.	4	
10.	3	
11.	6	
12.	3	
13.	2	
14.	3	
15.	3	
16.	6	
17.	3	
Total	70	

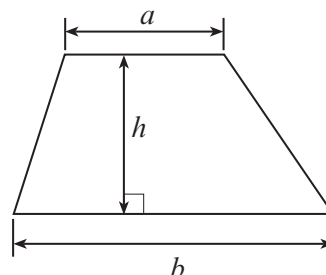
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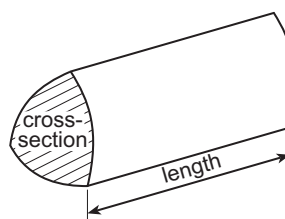
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

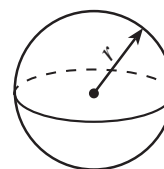


Volume of prism = area of cross-section $\times$ length



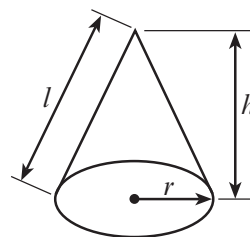
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

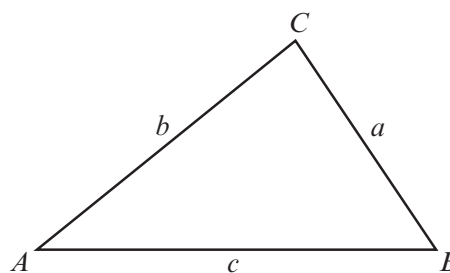


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

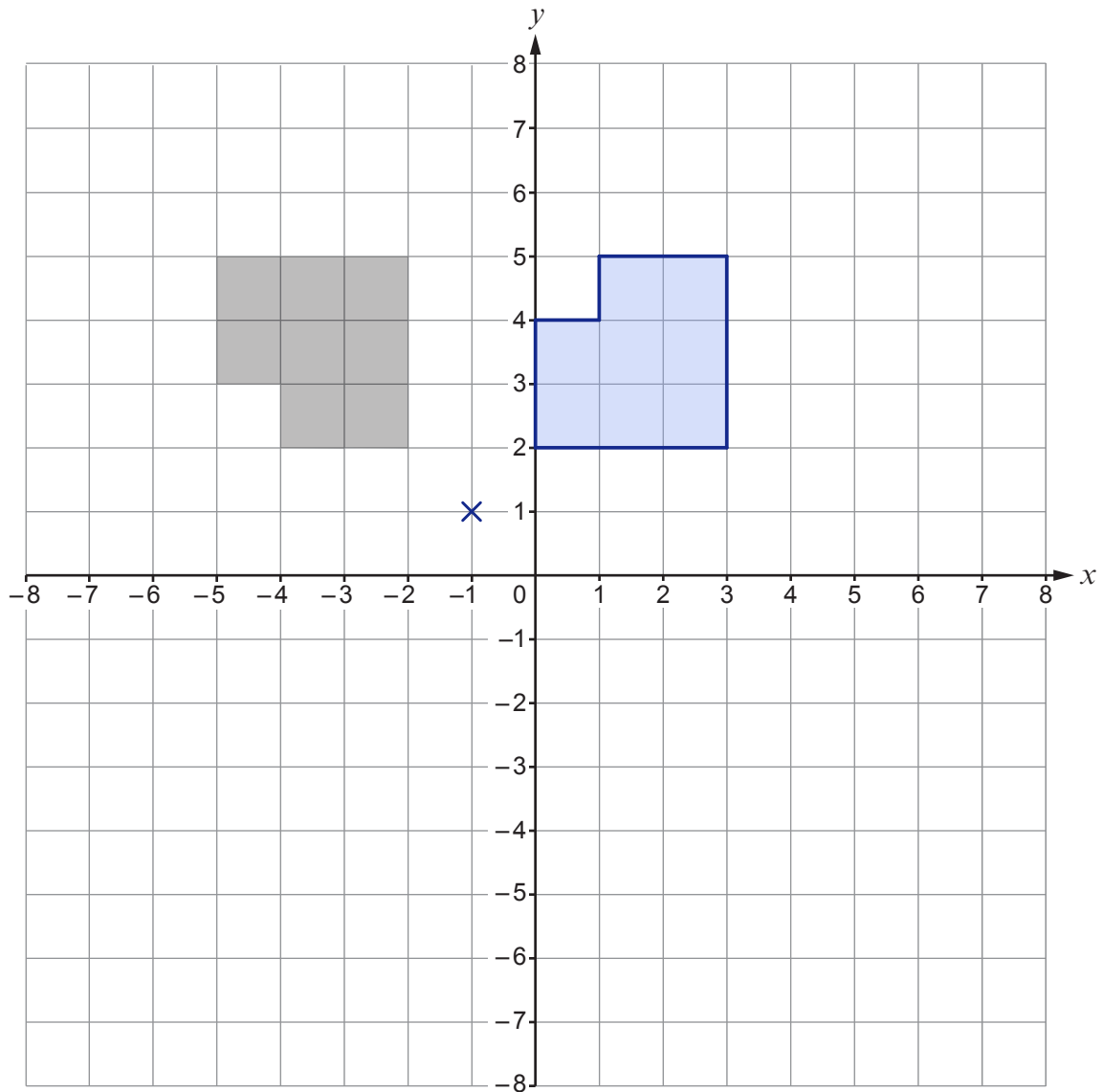
Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. Rotate the shape below through 90° clockwise about the point $(-1, 1)$.

[2]



2. (a) Make m the subject of the formula $w = 8m - 3$.

[2]

$$w + 3 = 8m$$

$$m = \frac{w + 3}{8}$$

- (b) Expand and simplify $(y + 5)(y - 4)$.

[2]

$$\begin{aligned}(y + 5)(y - 4) &= y^2 - 4y + 5y - 20 \\ &= y^2 + y - 20\end{aligned}$$

3. The height of a cylinder is 24.8 cm.
The ratio of the **diameter** of the cylinder to the height of the cylinder is 3 : 2.

Find the volume of the cylinder.
Give your answer correct to 2 significant figures.
You must show all your working.

[5]

$$\text{Diameter : height} = 3 : 2$$

$$\text{Diameter} = 24.8 \div 2 \times 3 = 37.2 \text{ cm}$$

$$\text{Radius} = 37.2 \div 2 = 18.6 \text{ cm}$$

$$\text{Volume of cylinder} = \pi \times r^2 \times h$$

$$= \pi \times 18.6^2 \times 24.8$$

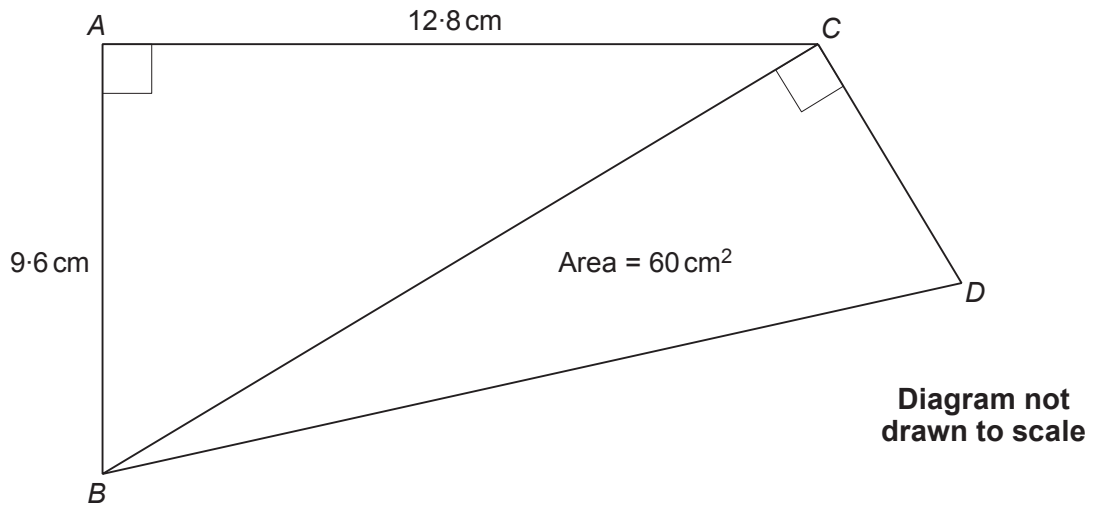
$$= \pi \times 345.96 \times 24.8$$

$$= 26954.26178... \text{ cm}^3$$

$$\text{Volume} = 27000 \text{ cm}^3 \text{ (2 s.f.)}$$



4. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.



In the diagram above, the area of triangle BCD is 60 cm^2 .
Calculate the length of CD .
You must show all your working.

[6 + 2 OCW]

Triangle ABC is right-angled at A, so I use Pythagoras to find BC.

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = 9.6^2 + 12.8^2 = 92.16 + 163.84 = 256$$

$$BC = \sqrt{256} = 16 \text{ cm}$$

*Triangle BCD is right-angled at C, so I take BC as the base and
CD as the perpendicular height.*

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$60 = \frac{1}{2} \times 16 \times CD$$

$$60 = 8 \times CD$$

$$CD = 60 \div 8 = 7.5 \text{ cm}$$

Therefore the length of CD is 7.5 cm.



5. (a) Factorise $8x^2 + 6xy$.

[2]

$$8x^2 + 6xy = 2x(4x + 3y)$$

- (b) (i) Factorise $x^2 + 13x + 40$.

[2]

$$8 \times 5 = 40 \text{ and } 8 + 5 = 13$$

$$x^2 + 13x + 40 = (x + 8)(x + 5)$$

- (ii) Explain how you can check that your answer to part (i) is correct.

[1]

Expand the brackets and check you get the original expression:

$$(x + 8)(x + 5) = x^2 + 5x + 8x + 40 = x^2 + 13x + 40$$

This matches $x^2 + 13x + 40$, so the factorisation is correct.

6. Calculate $5.7 \times 10^5 \times 6.4 \times 10^{-2}$.
Circle the correct answer.

[1]

$$3.648 \times 10^8$$

$$3.648 \times 10^4$$

$$-3.648 \times 10^6$$

$$3.648 \times 10^3$$

$$3.648 \times 10^6$$

$$5.7 \times 6.4 = 36.48, \quad 10^5 \times 10^{-2} = 10^3, \quad 36.48 \times 10^3 = 3.648 \times 10^4$$



7. (a) The diagram below shows a right-angled triangle.

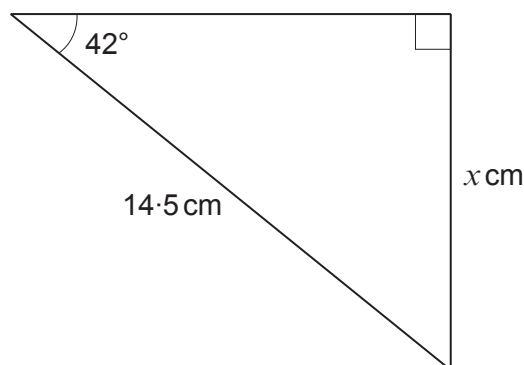


Diagram not
drawn to scale

Calculate the value of x .

[3]

$$\sin 42^\circ = \frac{x}{14.5}$$

$$x = 14.5 \times \sin 42^\circ$$

$$x = 9.702393792...$$

$$x = 9.70 \text{ cm}$$

- (b) The diagram below shows a different right-angled triangle.

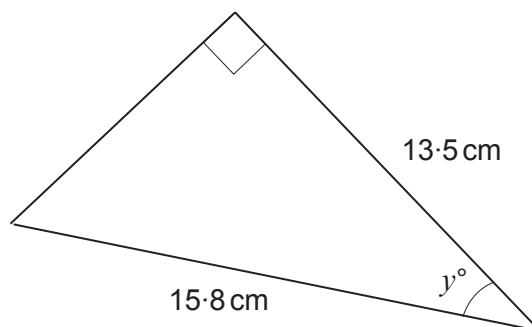


Diagram not
drawn to scale

Calculate the value of y .

[3]

$$\cos y = \frac{13.5}{15.8} = 0.8544303797...$$

$$y = \cos^{-1}(0.8544303797...)$$

$$y = 31.30313754...$$

$$y = 31.3$$



8. This cuboid has:
- length = 5 cm
 - width = x cm
 - height = $(x^2 + 3)$ cm
 - volume = 132 cm^3 .

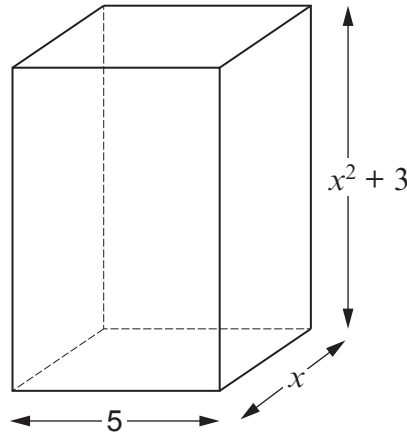


Diagram not
drawn to scale

- (a) Show that $5x^3 + 15x = 132$.

[1]

Volume = length $\times$ width $\times$ height, so $5 \times x \times (x^2 + 3) = 132$

$$5x(x^2 + 3) = 132 \Rightarrow 5x^3 + 15x = 132$$

- (b) (i) A solution of the equation

$$5x^3 + 15x = 132$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.

You must show all your working.

[4]

x	$5x^3 + 15x$	
2.5	115.625	too low
2.6	126.88	too low
2.7	138.915	too high
2.65	132.798125	too high

Solution is between 2.6 and 2.65, so it rounds to 2.6

$$x = 2.6 \text{ (1 d.p.)}$$



(ii) Hence, find the height of the cuboid.

[1]

$$\text{Height} = x^2 + 3 = 2.6^2 + 3 = 6.76 + 3 = 9.76$$

Height of the cuboid = ... 9.76 cm



9. Calculate the **total** surface area of a **solid** hemisphere with a radius of 34 cm.

[4]

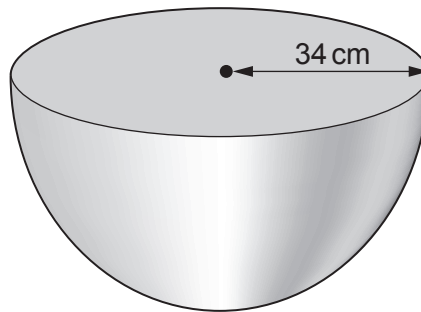


Diagram not
drawn to scale

$$\text{Curved surface} = \frac{1}{2} \times 4\pi r^2 = 2 \times \pi \times 34^2$$

$$= 2312\pi = 7263.362215... \text{ cm}^2$$

$$\text{Circular flat face} = \pi \times 34^2$$

$$= 1156\pi = 3631.681108... \text{ cm}^2$$

$$\text{Total surface area} = 2312\pi + 1156\pi$$

$$= 3468\pi$$

$$= 10895.04332... \text{ cm}^2$$

$$\text{Total surface area} = \dots\dots\dots 10895.0 \dots\dots\dots \text{ cm}^2$$



10. The value of y is found using the formula $y = \frac{t}{w}$.

$t = 98$, correct to 2 significant figures.
 $w = 0.5$, correct to 1 significant figure.

Calculate the **least** value of y .
 Give your answer correct to 1 decimal place.
 You must show all your working.

[3]

$$97.5 \leq t < 98.5$$

$$0.45 \leq w < 0.55$$

$$\text{Least } y = \text{least } t \div \text{greatest } w$$

$$= 97.5 \div 0.55$$

$$= 177.2727272\dots$$

$$= 177.3 \text{ (1 d.p.)}$$

$$\text{Least value of } y = 177.3$$



11. The diagram shows a triangle ABC and a circle with centre A . The points B and D lie on the circumference of the circle.

The length of the line AC is 27 cm.
The area of triangle ABC is 112 cm^2 .

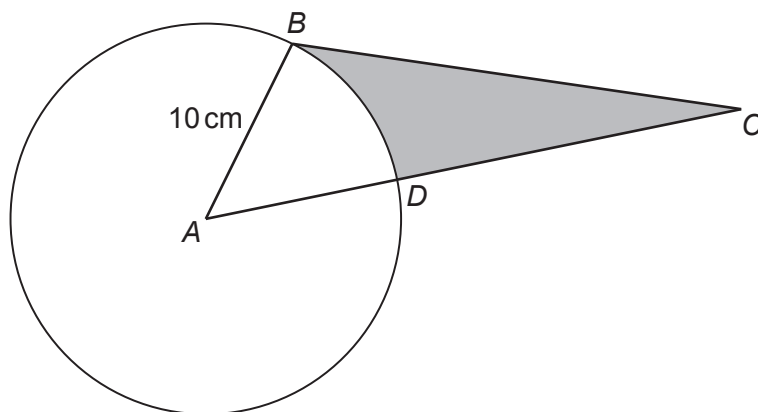


Diagram not
drawn to scale

Calculate the area of the shaded region.

[6]

$$\text{Area of triangle } ABC = \frac{1}{2} \times AB \times AC \times \sin BAD$$

$$112 = \frac{1}{2} \times 10 \times 27 \times \sin BAD$$

$$\sin BAD = \frac{2 \times 112}{10 \times 27} = \frac{224}{270} = 0.8296296...$$

$$BAD = \sin^{-1}(0.8296296...) = 56.06071079...^\circ$$

$$\text{Sector area} = \frac{56.06071079...}{360} \times \pi \times 10^2 = 48.92219922... \text{ cm}^2$$

$$\text{Shaded area} = 112 - 48.92219922...$$

$$= 63.07780078... \text{ cm}^2$$

$$\text{Area of shaded region} = \dots\dots\dots 63.1 \dots\dots\dots \text{ cm}^2$$



12. Express the following as a single fraction in its simplest form.

[3]

$$\frac{4}{3x-7} + \frac{5}{2x+9}$$

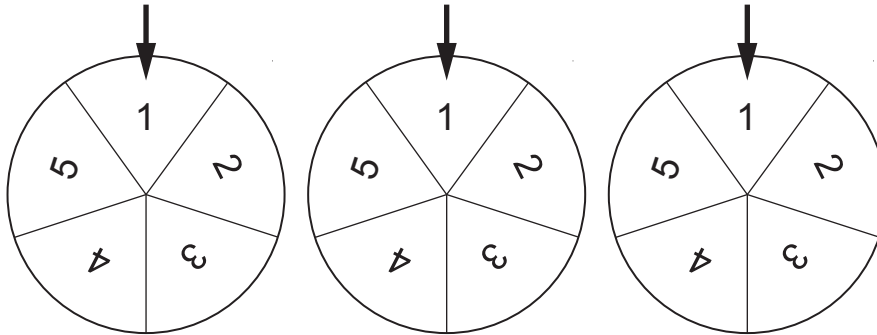
$$\frac{4}{3x-7} + \frac{5}{2x+9} = \frac{4(2x+9) + 5(3x-7)}{(3x-7)(2x+9)}$$

$$= \frac{8x + 36 + 15x - 35}{(3x-7)(2x+9)}$$

$$= \frac{23x + 1}{(3x-7)(2x+9)}$$



13. Three fair spinners are shown in the diagram below.



The three spinners are spun.

Calculate the probability that all the spinners will land on an even number.

[2]

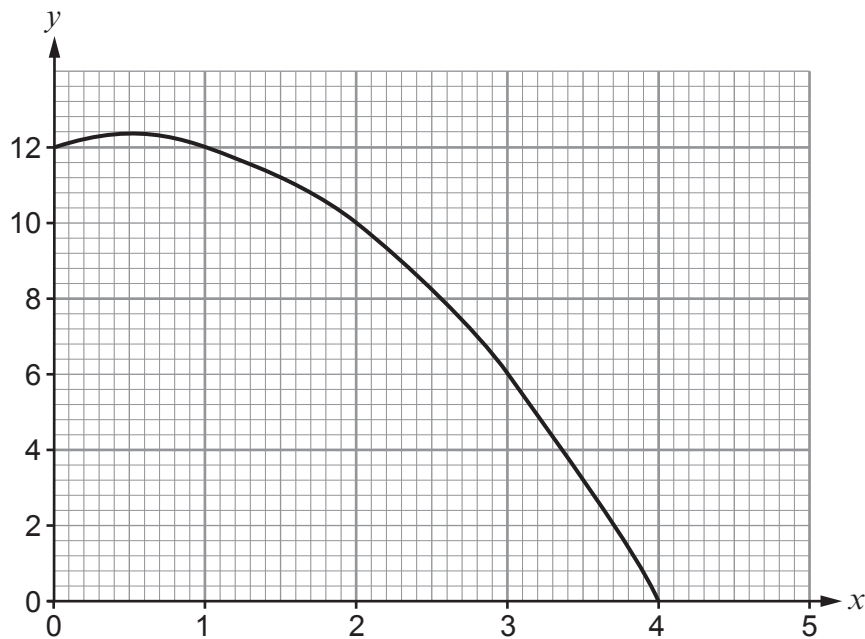
$$P(\text{even on one spinner}) = \frac{2}{5}$$

$$P(\text{all three even}) = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$

$$= \frac{8}{125}$$



14. The graph of $y = 12 + x - x^2$, for values of x from $x = 0$ to $x = 4$, is drawn below.



Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$ and $x = 4$, to estimate the area of the region bounded by the curve, the positive x -axis and the positive y -axis. [3]

$$x \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$$y \quad 12 \quad 12 \quad 10 \quad 6 \quad 0$$

$$h = 1$$

$$\text{Area} \approx \frac{1}{2} \times h \times [y_0 + y_4 + 2(y_1 + y_2 + y_3)]$$

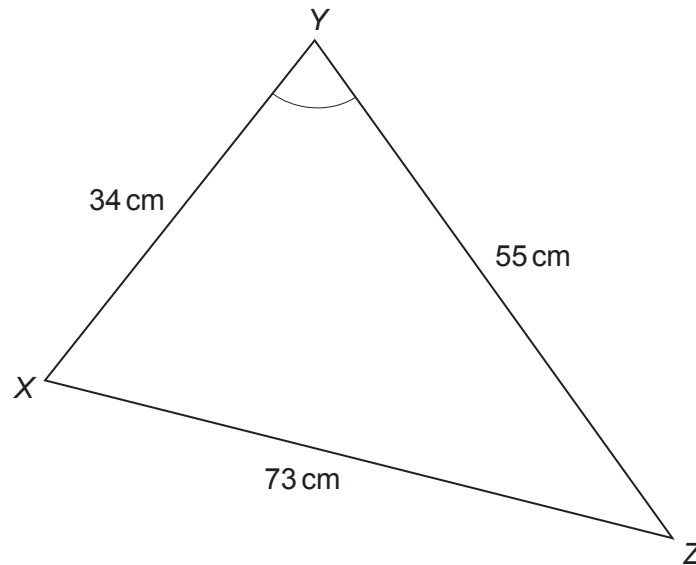
$$= \frac{1}{2} \times 1 \times [12 + 0 + 2(12 + 10 + 6)]$$

$$= \frac{1}{2} \times 68$$

$$= 34 \text{ square units}$$



15.

Diagram not
drawn to scaleCalculate the size of $\hat{XYZ}$.

[3]

$$\cos XYZ = \frac{34^2 + 55^2 - 73^2}{2 \times 34 \times 55}$$

$$= \frac{1156 + 3025 - 5329}{3740} = \frac{-1148}{3740}$$

$$= -0.3069518717...$$

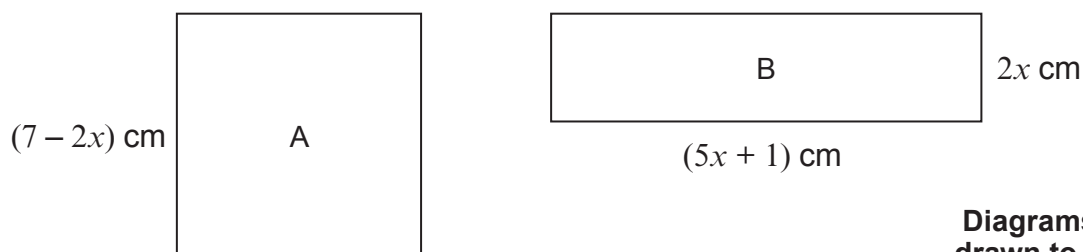
$$XYZ = \cos^{-1}(-0.3069518717...)$$

$$= 107.8756318...^\circ$$

$$\hat{XYZ} = 107.9^\circ$$



16. Shape A is a square and Shape B is a rectangle, as shown below.



Diagrams not
drawn to scale

The area of square A is equal to the area of rectangle B.

Form an equation in x and hence calculate the value of x .

Give your answer correct to 1 decimal place.

You must show all your working.

[6]

$$\text{Area of A} = (7 - 2x)^2, \text{ area of B} = 2x(5x + 1)$$

$$2x(5x + 1) = (7 - 2x)^2$$

$$10x^2 + 2x = 49 - 14x - 14x + 4x^2$$

$$10x^2 + 2x = 49 - 28x + 4x^2$$

$$6x^2 + 30x - 49 = 0$$

$$a = 6, b = 30, c = -49$$

$$x = \frac{-30 \pm \sqrt{30^2 - 4 \times 6 \times (-49)}}{2 \times 6}$$

$$x = \frac{-30 \pm \sqrt{2076}}{12}$$

$$x = 1.296928583... \text{ or } x = -6.296928583...$$

$$x \text{ cannot be negative (} 2x \text{ is a length), so } x = 1.3 \text{ (1 d.p.)}$$

$$x = 1.3$$



17. Solid A and Solid B are **similar**.

Solid A has a volume of 8000 cm^3 and a height of 30 cm.
Solid B has a volume of 4913 cm^3 .

Calculate the height of Solid B.

[3]

$$\text{Volume scale factor} = 4913 \div 8000 = 0.614125$$

$$\text{Linear scale factor} = \sqrt[3]{0.614125} = 0.85$$

$$\text{Height of Solid B} = 30 \times 0.85$$

$$= 25.5 \text{ cm}$$

Height of Solid B = ... 25.5 cm

END OF PAPER



[illegible]

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