

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
First name(s)		0



GCSE

3300U50-1



Z22-3300U50-1-R1

TUESDAY, 24 MAY 2022 – MORNING

MATHEMATICS
UNIT 1: NON-CALCULATOR
HIGHER TIER

1 hour 35 minutes

ADDITIONAL MATERIALS

The use of a calculator is not permitted in this examination.
A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the additional page at the back of the booklet. Question numbers must be given for all work written on the additional page.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 1, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	6	
2.	5	
3.	6	
4.	6	
5.	4	
6.	4	
7.	2	
8.	2	
9.	2	
10.	3	
11.	4	
12.	2	
13.	6	
14.	2	
15.	5	
16.	2	
17.	5	
18.	4	
Total	70	

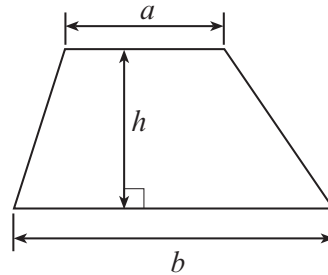
3300U501
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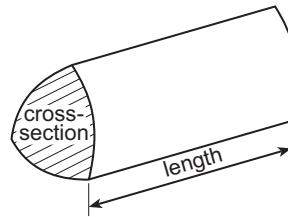
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Formula List – Higher Tier

Area of trapezium $= \frac{1}{2} (a + b)h$

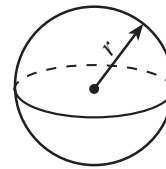


Volume of prism = area of cross-section $\times$ length



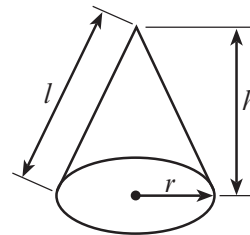
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

Curved surface area of cone $= \pi r l$

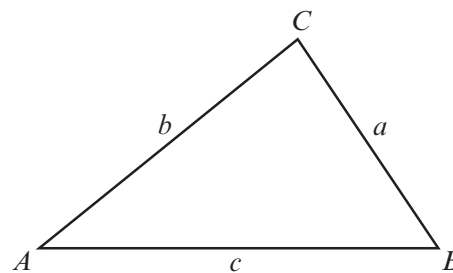


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

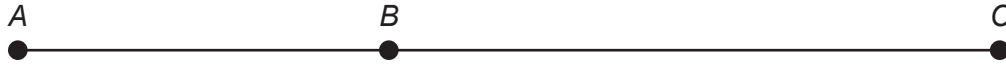


Diagram not
drawn to scale

ABC is a straight road, where the ratio $AB : BC = 3 : 4$.
AC = 56 km.

Calculate the length of BC.

Give your answer in **miles**.

You must show all your working.

[4 + 2 OCW]

$AB : BC = 3 : 4$, so the road is split into $3 + 4 = 7$ equal parts.

One part = $56 \text{ km} \div 7 = 8 \text{ km}$

BC is 4 parts: $BC = 4 \times 8 \text{ km} = 32 \text{ km}$

Converting to miles, using $8 \text{ km} = 5 \text{ miles}$:

$32 \text{ km} \div 8 = 4$

$4 \times 5 \text{ miles} = 20 \text{ miles}$

So the length of BC is 20 miles.

Length of BC = 20 miles



2. The table below shows some of the values of $y = x^2 + x - 4$ for values of x from -3 to 3 .

x	-3	-2	-1	0	1	2	3
$y = x^2 + x - 4$	2	-2	-4	-4	-2	2	8

- (a) Complete the table by finding the values of y for $x = -1$ and for $x = 1$.

[2]

$$x = -1: y = (-1)^2 + (-1) - 4 = 1 - 1 - 4 = -4$$

$$x = 1: y = 1^2 + 1 - 4 = 1 + 1 - 4 = -2$$

- (b) On the graph paper opposite, draw the graph of $y = x^2 + x - 4$ for values of x from -3 to 3 .

[2]

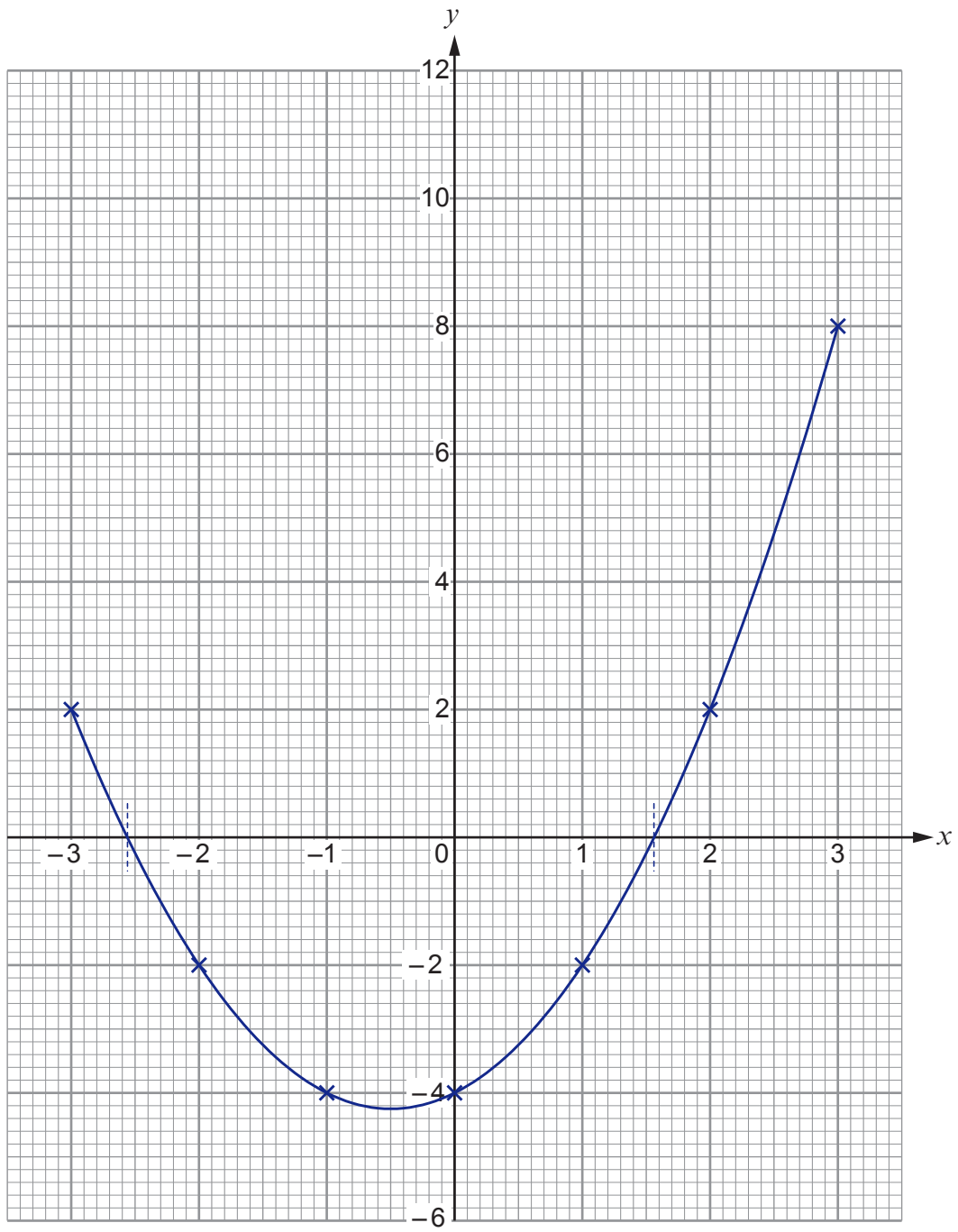
- (c) Use your graph to solve the equation $x^2 + x - 4 = 0$.
Give your answers correct to 1 decimal place.

[1]

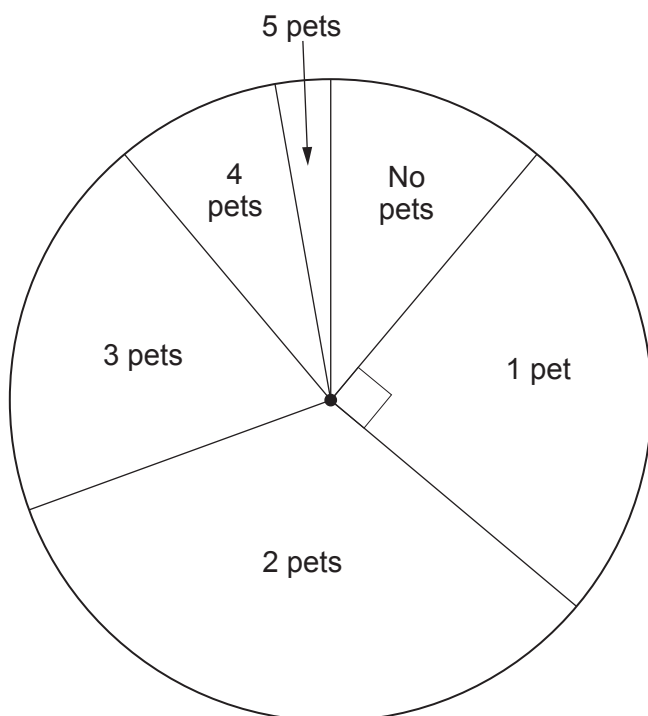
Read off where the curve crosses the x-axis ($y = 0$).

$$x = -2.6 \quad \text{or} \quad x = 1.6$$

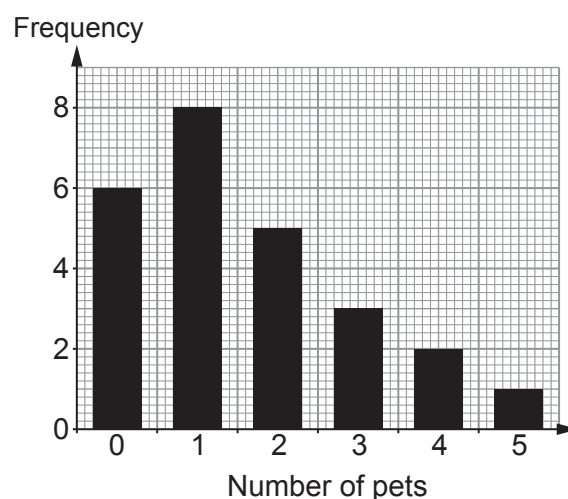




3. The children in year 5 and year 6 in a primary school took part in a survey. The children were asked, "How many pets do you have?" The results are shown in the pie chart and bar chart below. No child in either year had more than 5 pets.



Year 5



Year 6

There are 36 children in year 5.

One child is chosen at random from all the children in year 5 and year 6. What is the probability that this child has no more than 1 pet?

[6]

$$\text{Year 5: each child} = 360^\circ \div 36 = 10^\circ$$

$$\text{No pets angle} = 40^\circ, \text{ so no pets} = 40^\circ \div 10^\circ = 4 \text{ children}$$

$$\text{1 pet angle} = 90^\circ \text{ (right angle)}, \text{ so 1 pet} = 90^\circ \div 10^\circ = 9 \text{ children}$$

$$\text{Year 6 (from the bar chart): no pets} = 6, \text{ 1 pet} = 8$$

$$\text{Year 6 total} = 6 + 8 + 5 + 3 + 2 + 1 = 25 \text{ children}$$

$$\text{Total in year 5 and year 6} = 36 + 25 = 61 \text{ children}$$

$$\text{No more than 1 pet (0 or 1 pet)} = 4 + 9 + 6 + 8 = 27 \text{ children}$$

$$P(\text{no more than 1 pet}) = \frac{27}{61}$$



4. A and B are independent events.
The probability of event A occurring is 0.6.
The probability of event A **and** event B occurring is 0.48.

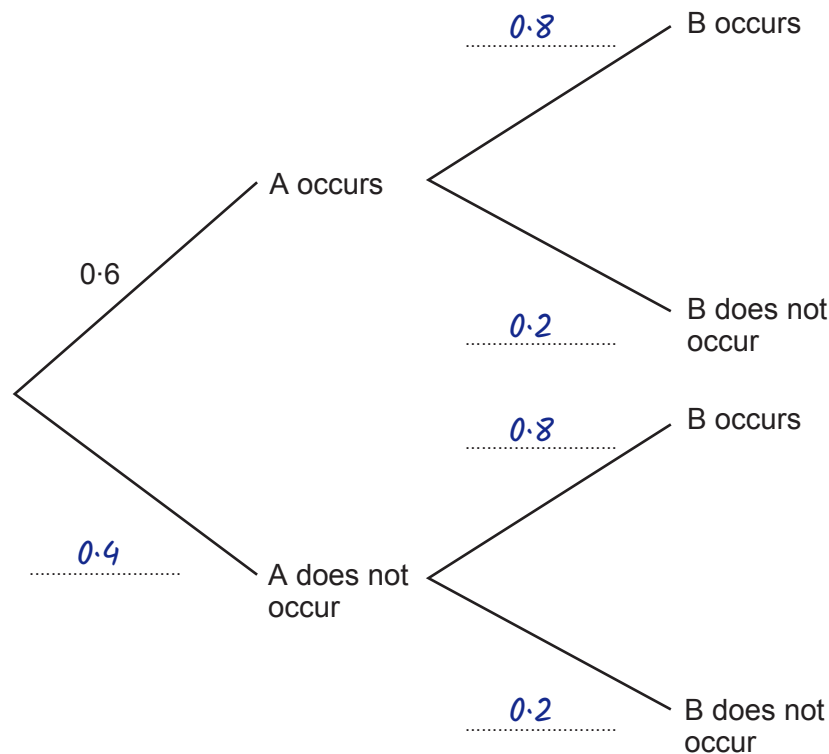
(a) Complete the tree diagram.

[4]

$$\text{Independent, so } P(A \text{ and } B) = P(A) \times P(B)$$

$$0.6 \times P(B) = 0.48$$

$$P(B) = 0.48 \div 0.6 = 0.8, \quad P(B \text{ does not occur}) = 1 - 0.8 = 0.2$$



- (b) Calculate the probability of neither event A nor event B occurring.

[2]

$$P(\text{neither}) = P(A \text{ does not occur}) \times P(B \text{ does not occur})$$

$$= 0.4 \times 0.2$$

$$= 0.08$$



5. In the diagram:
- AB and ED are parallel
 - triangles ABC and DEC are similar.

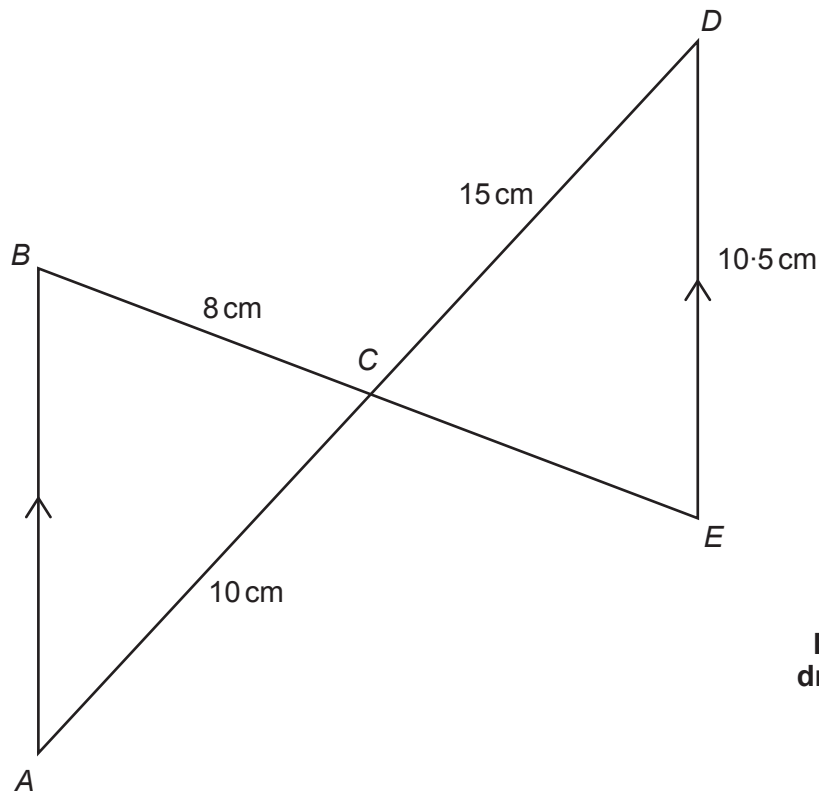


Diagram not
drawn to scale

- (a) Calculate the length of CE .

[2]

$$\text{Scale factor} = \frac{DC}{AC} = \frac{15}{10} = 1.5$$

$$CE \text{ corresponds to } BC: CE = 8 \times 1.5$$

$$CE = 12 \text{ cm}$$

- (b) Calculate the length of AB .

[2]

$$AB \text{ corresponds to } DE: AB = 10.5 \div 1.5$$

$$= 10.5 \div 1.5$$

$$AB = 7 \text{ cm}$$



6. Solve the following simultaneous equations using an algebraic (not graphical) method. You must show all your working. [4]

$$2x + 3y = 29$$

$$5x - 4y = -8$$

$$2x + 3y = 29 \quad (1)$$

$$5x - 4y = -8 \quad (2)$$

$$(1) \times 4: \quad 8x + 12y = 116$$

$$(2) \times 3: \quad 15x - 12y = -24$$

$$\text{Add:} \quad 23x = 92$$

$$x = 4$$

Substitute $x = 4$ into (1):

$$2(4) + 3y = 29$$

$$8 + 3y = 29$$

$$3y = 21$$

$$y = 7$$

$$\text{Check in (2):} \quad 5(4) - 4(7) = 20 - 28 = -8$$

$$x = 4, \quad y = 7$$



7. Circle the correct answer for each of the following statements.

(a) 7.2 m^3 is equal to

[1]

720 cm^3

72000 cm^3

$7.2 \times 10^5 \text{ cm}^3$

$7.2 \times 10^3 \text{ cm}^3$

$7.2 \times 10^6 \text{ cm}^3$

(b) $36^{\frac{1}{2}}$ is equal to

[1]

18

6

$\frac{1}{18}$

$\frac{1}{6}$

$\frac{1}{36}$

8. Find the value of $\frac{30000}{1.5 \times 10^5}$.

Write your answer as a decimal.

[2]

$$30\,000 = 3 \times 10^4$$

$$3 \times 10^4 \div (1.5 \times 10^5) = (3 \div 1.5) \times 10^{4-5}$$

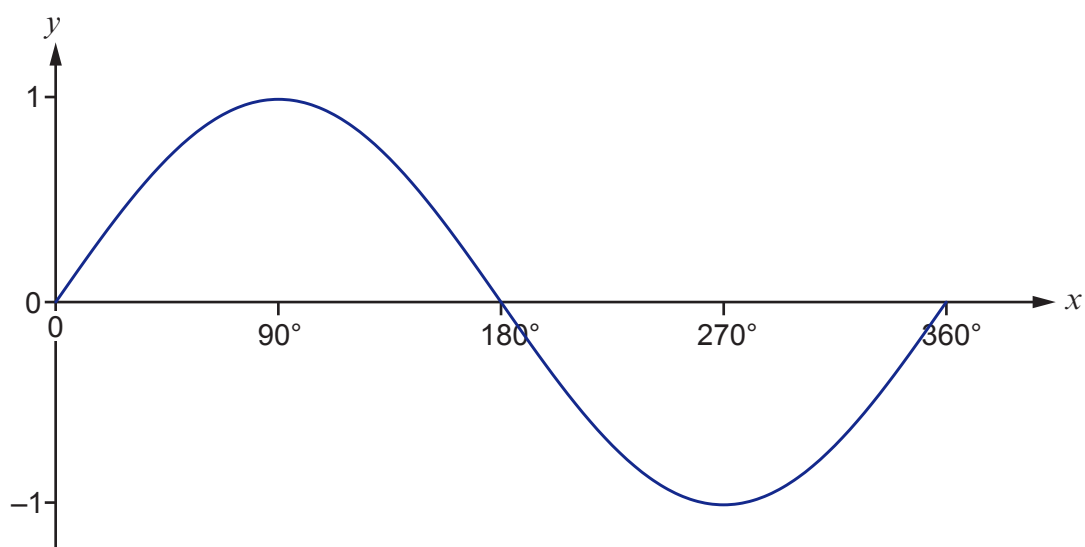
$$= 2 \times 10^{-1}$$

$$= 0.2$$

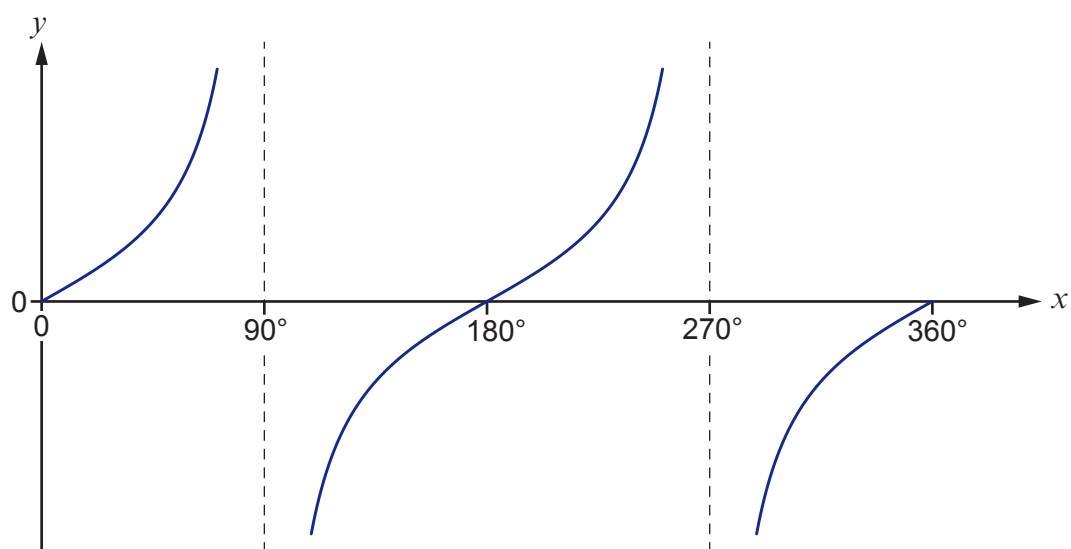
Answer: 0.2



9. (a) Sketch the curve $y = \sin x$, for values of x in the range $x = 0^\circ$ to $x = 360^\circ$. [1]



- (b) Sketch the curve $y = \tan x$, for values of x in the range $x = 0^\circ$ to $x = 360^\circ$. [1]



10. Rearrange the following formula to make x the subject.

$$5x + 4 = t - yx$$

You must show all your working.

[3]

$$5x + 4 = t - yx$$

$$5x + yx = t - 4$$

$$x(5 + y) = t - 4$$

$$x = \frac{t - 4}{5 + y}$$



11. The wavelength of a sound wave, W metres, is **inversely proportional** to its frequency, f hertz (Hz).

Richard is measuring the wavelengths of different sound waves.
The wavelength is 0.5 m when the frequency is 1200 Hz.

What is the frequency when the wavelength is 10 m?

[4]

$$W \propto \frac{1}{f}, \text{ so } W = \frac{k}{f}$$

$$0.5 = \frac{k}{1200}$$

$$k = 0.5 \times 1200 = 600$$

$$W = \frac{600}{f}$$

$$\text{When } W = 10: 10 = \frac{600}{f}$$

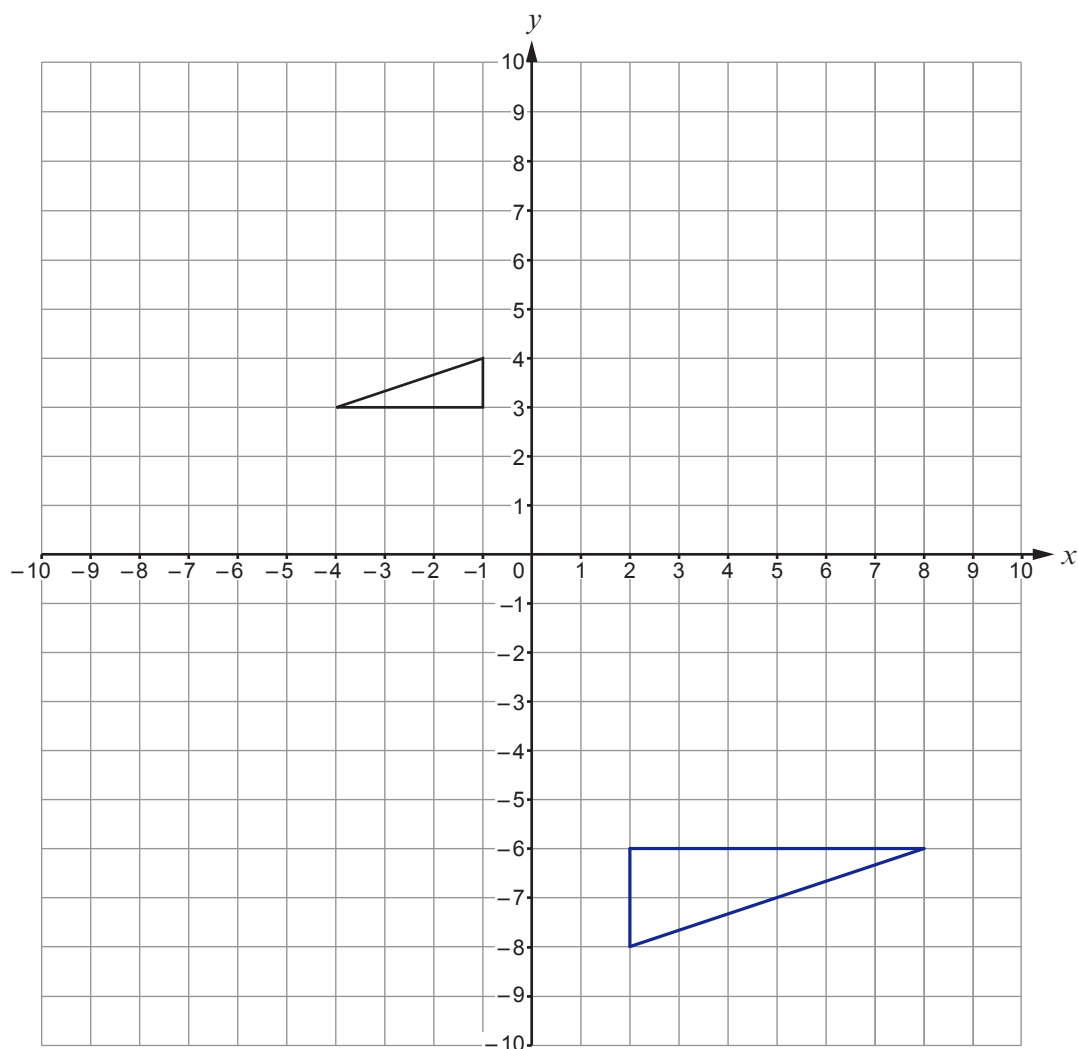
$$f = 600 \div 10 = 60$$

The frequency is 60 Hz.



12. Enlarge the triangle below by a scale factor of -2 .
Use the origin as the centre of enlargement.

[2]



$$(-4, 3) \rightarrow (8, -6), \quad (-1, 3) \rightarrow (2, -6), \quad (-1, 4) \rightarrow (2, -8)$$



13. Marian is competing in a race.
The race is $(6x + 5)$ miles long.
She completes the race in x hours.
Her average speed during the race is $(2x + 3)$ miles per hour.

Calculate how long Marian takes to complete the race.
You must use an algebraic method (not trial and improvement).

[6]

$$\text{Time} = \text{distance} \div \text{speed}$$

$$x = \frac{6x + 5}{2x + 3}$$

$$x(2x + 3) = 6x + 5$$

$$2x^2 + 3x = 6x + 5$$

$$2x^2 + 3x - 6x - 5 = 0$$

$$2x^2 - 3x - 5 = 0$$

$$(2x - 5)(x + 1) = 0$$

$$2x - 5 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = 2.5 \quad \text{or} \quad x = -1$$

Time cannot be negative, so $x = 2.5$

Check: $6(2.5) + 5 = 20$ miles, speed = $2(2.5) + 3 = 8$ mph, $20 \div 8 = 2.5$

Marian takes 2.5 hours (2 hours 30 minutes).



14. Find the value of $125^{-\frac{1}{3}}$.

Simplify your answer.

[2]

$$125^{1/3} = \sqrt[3]{125} = 5 \quad (\text{since } 5 \times 5 \times 5 = 125)$$

$$125^{-1/3} = \frac{1}{125^{1/3}} = \frac{1}{5}$$



15. Simplify the following.

$$\frac{\sqrt{800}}{(\sqrt{2})^3} + (3 - \sqrt{7})^2$$

State whether the answer is rational or irrational.

[5]

The answer is
rational

☐

The answer is
irrational

☒

$$\sqrt{800} = \sqrt{400 \times 2} = 20\sqrt{2}$$

$$(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$$

$$\sqrt{800} \div (\sqrt{2})^3 = 20\sqrt{2} \div 2\sqrt{2} = 10$$

$$(3 - \sqrt{7})^2 = (3 - \sqrt{7})(3 - \sqrt{7})$$

$$= 9 - 3\sqrt{7} - 3\sqrt{7} + 7$$

$$= 16 - 6\sqrt{7}$$

$$\text{Total} = 10 + 16 - 6\sqrt{7}$$

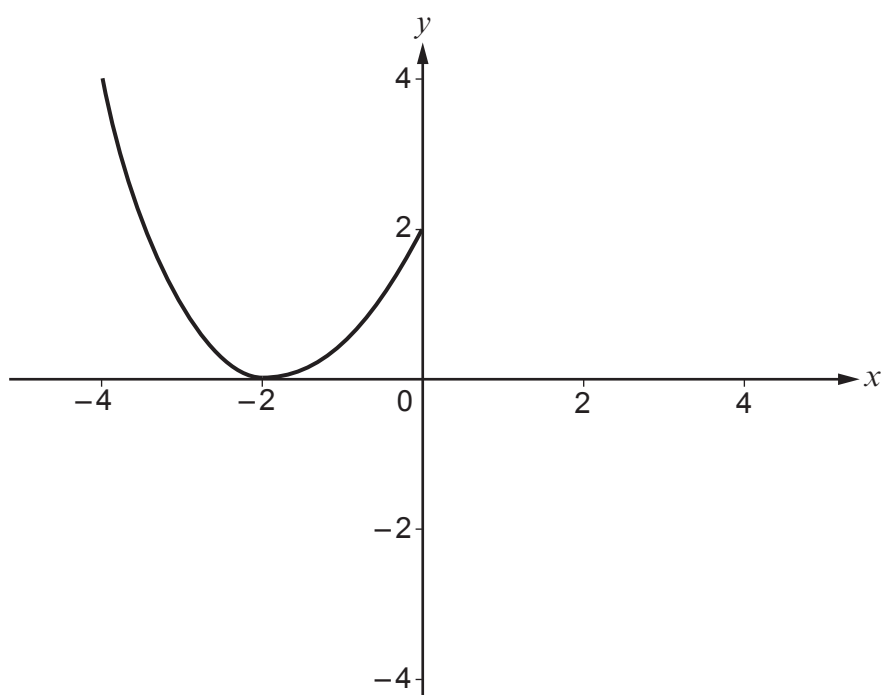
$$= 26 - 6\sqrt{7}$$

$\sqrt{7}$ is irrational, so $26 - 6\sqrt{7}$ is irrational.

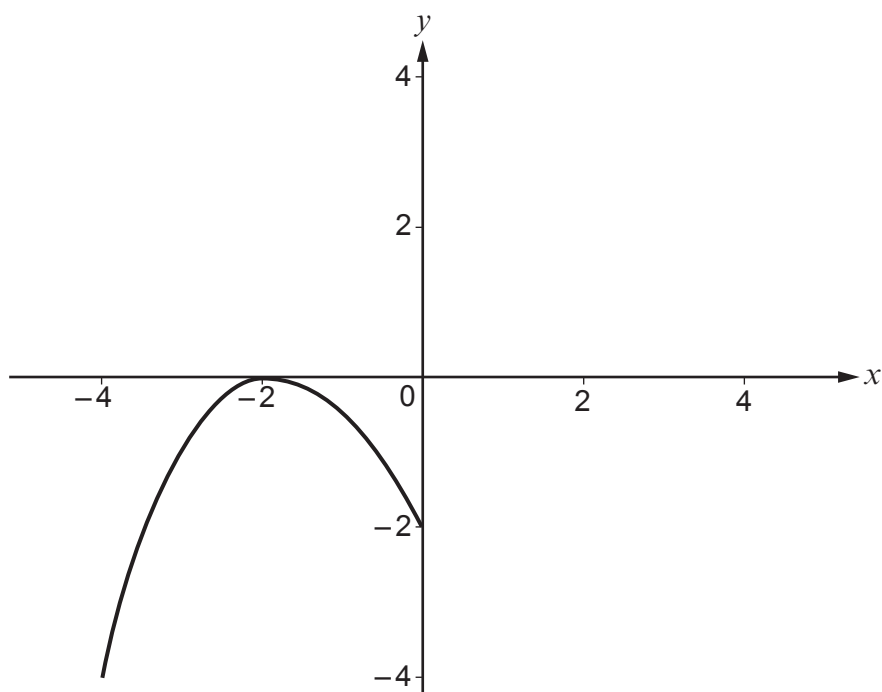
Answer: $26 - 6\sqrt{7}$ (irrational)



16. (a) The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



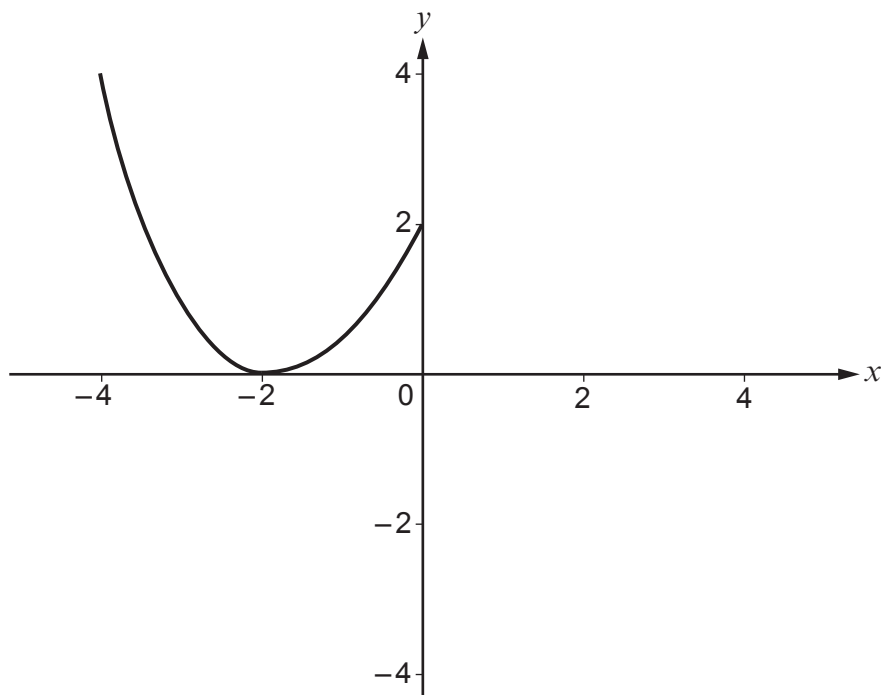
Using function notation, complete the equation of the transformed curve.

[1]

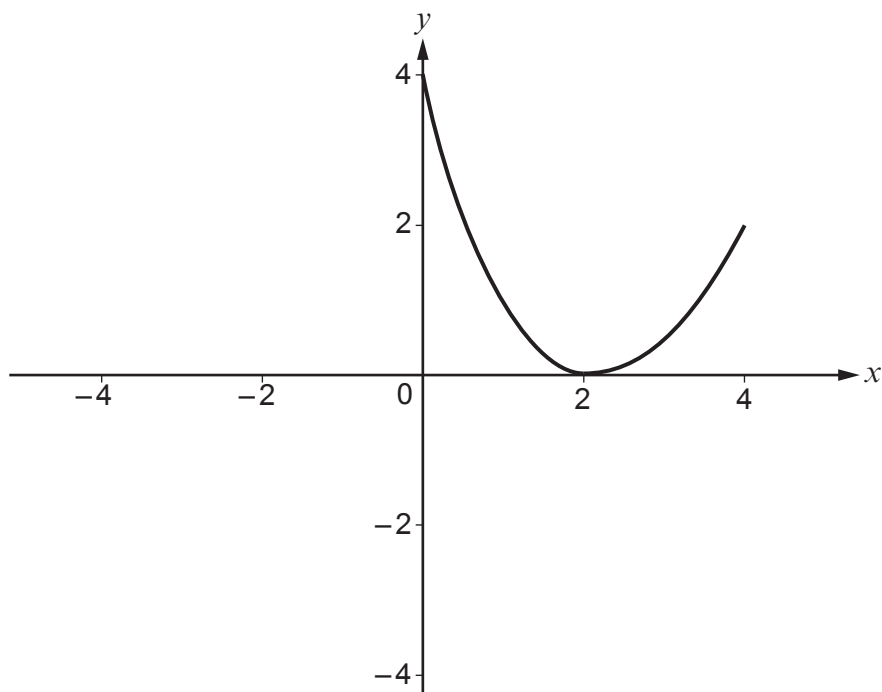
$y = \dots -f(x) \dots$



- (b) The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



Using function notation, complete the equation of the transformed curve.

[1]

$y = \dots\dots\dots f(x - 4) \dots\dots\dots$



17. A bag contains 10 balls.

5 of the balls are blue, 4 of the balls are red and 1 ball is green.

Three balls are chosen at random, one at a time, without replacement.

- (a) Calculate the probability that the first ball is blue, the second ball is red and the third ball is green.
You must show all your working. [2]

$$P(\text{blue, red, green}) = \frac{5}{10} \times \frac{4}{9} \times \frac{1}{8}$$

$$= \frac{20}{720}$$

$$= \frac{1}{36}$$

- (b) Calculate the probability that at least one blue ball is chosen.
You must show all your working. [3]

$$P(\text{at least one blue}) = 1 - P(\text{no blue})$$

There are 5 balls that are not blue (4 red, 1 green).

$$P(\text{no blue}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720}$$

$$P(\text{at least one blue}) = 1 - \frac{60}{720}$$

$$= \frac{660}{720} = \frac{11}{12}$$



18. Simplify $\frac{6x-15}{4x^2-25}$.

[4]

$$6x - 15 = 3(2x - 5)$$

$$4x^2 - 25 = (2x - 5)(2x + 5) \quad (\text{difference of two squares})$$

$$\frac{6x - 15}{4x^2 - 25} = \frac{3(2x - 5)}{(2x - 5)(2x + 5)}$$

$$= \frac{3}{2x + 5}$$

END OF PAPER



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