

Surname	<i>StemUp Model Solution</i>
Other Names	

Centre Number

Candidate Number
0



GCSE

3300U60-1



MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

THURSDAY, 7 JUNE 2018 – MORNING

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page at the back of the booklet. Question numbers must be given for all work written on the continuation page.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 3, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

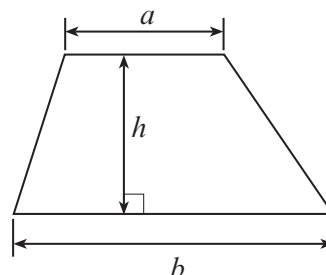
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	3	
3.	6	
4.	4	
5.	3	
6.	5	
7.	6	
8.	5	
9.	6	
10.	2	
11.	3	
12.	4	
13.	3	
14.	2	
15.	4	
16.	4	
17.	5	
18.	4	
19.	7	
Total	80	



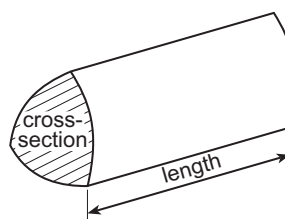
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Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

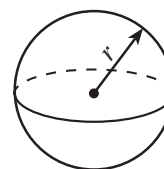


Volume of prism = area of cross-section $\times$ length



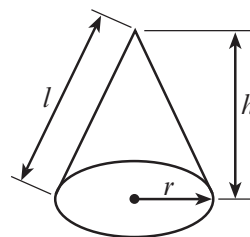
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

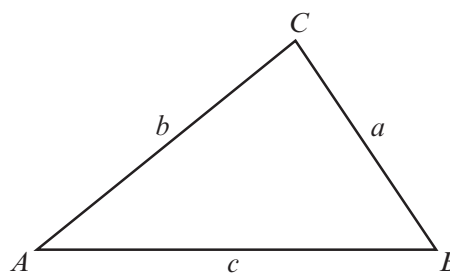


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) Calculate $\frac{145.3}{(12.4 - 9.8)^3}$, giving your answer correct to 3 significant figures. [2]

$$(12.4 - 9.8)^3 = 2.6^3 = 17.576$$

$$145.3 \div 17.576 = 8.266954939... = 8.27 \text{ (3 s.f.)}$$

- (b) Calculate the reciprocal of 47, giving your answer correct to 4 decimal places. [2]

$$1 \div 47 = 0.021276595...$$

$$\text{Reciprocal} = 0.0213 \text{ (4 d.p.)}$$

2. Circle the correct answer in each of the following.

- (a) Which of the following values **cannot** be an external angle of a regular polygon? [1]

10°

18°

30°

48°

72°

$$360 \div 48 = 7.5, \text{ not a whole number of sides}$$

- (b) An arrow on a spinner is facing north.
It is turned clockwise through an angle of 1530°.
In which direction will the arrow now be facing? [1]

North

East

South

West

None of these

$$1530^\circ = 4 \times 360^\circ + 90^\circ, \text{ so it turns } 90^\circ \text{ clockwise from north}$$

- (c) Point A is on a bearing of 100° from point B.
What is the bearing of point B from point A? [1]

260°

100°

280°

180°

80°

$$100^\circ + 180^\circ = 280^\circ$$



3. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

A solution of the equation

$$x^3 - 7x - 51 = 0$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place.

You must show all your working.

[4 + 2 OCW]

Let $f(x) = x^3 - 7x - 51$. I want the value of x where $f(x) = 0$.

I substitute values of x and check whether $f(x)$ is positive or negative.

x	$x^3 - 7x - 51$	Result
4	$64 - 28 - 51 = -15$	negative, too low
5	$125 - 35 - 51 = 39$	positive, too high
4.5	$91.125 - 31.5 - 51 = 8.625$	positive, too high
4.3	$79.507 - 30.1 - 51 = -1.593$	negative, too low
4.4	$85.184 - 30.8 - 51 = 3.384$	positive, too high
4.35	$82.312875 - 30.45 - 51 = 0.862875$	positive, too high

$f(4.3)$ is negative and $f(4.35)$ is positive, so the solution lies

between 4.3 and 4.35. Every value in this interval rounds to 4.3.

Solution: $x = 4.3$ (correct to 1 decimal place)



4. (a) The highest common factor (HCF) of 30 and 75 is the square root of a number.
What is the number? [2]

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 75: 1, 3, 5, 15, 25, 75

HCF = 15, and $15 = \sqrt{225}$ ($15^2 = 15 \times 15 = 225$)

The number is 225

- (b) The cube root of 32.768 is $33\frac{1}{3}\%$ of a number.
What is the number? [2]

$\sqrt[3]{32.768} = 3.2$

$33\frac{1}{3}\% = \frac{1}{3}$, so $\frac{1}{3}$ of the number = 3.2

Number = $3.2 \times 3 = 9.6$

The number is 9.6



5. PQR is a right-angled triangle, as shown below.
 $PQ = 1.41 \text{ m}$ and $PR = 0.89 \text{ m}$.

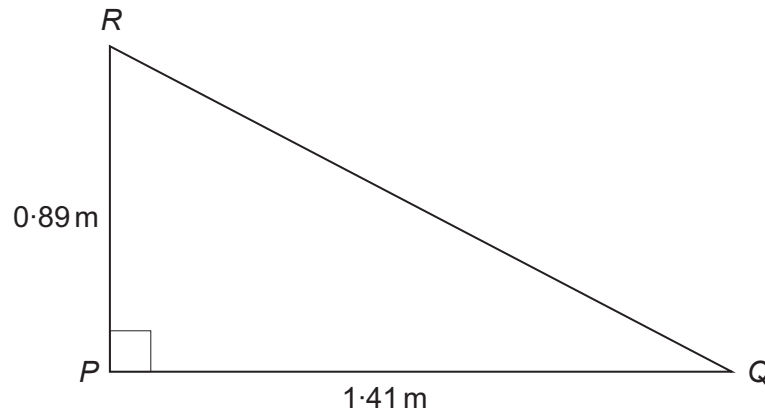


Diagram not drawn to scale

Calculate the length of QR .

[3]

Pythagoras: $QR^2 = PQ^2 + PR^2$

$QR^2 = 1.41^2 + 0.89^2 = 1.9881 + 0.7921$

$QR^2 = 2.7802$

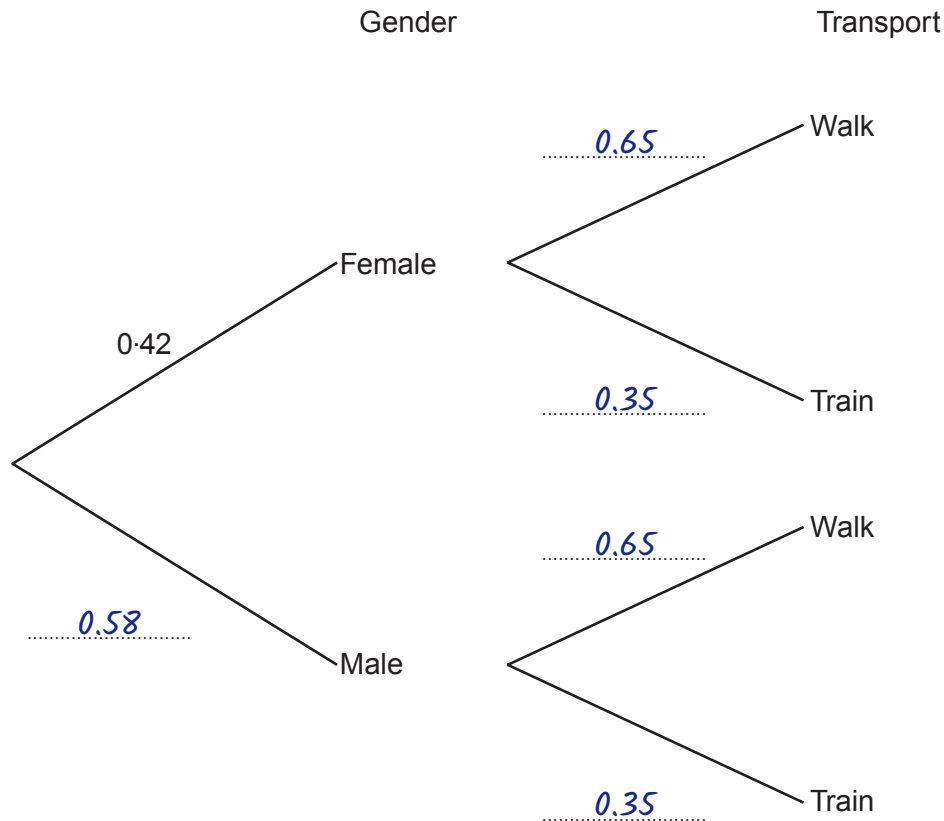
$QR = \sqrt{2.7802} = 1.667393175...$

$QR = 1.67 \text{ m}$



- The decision to walk or take the train is independent of gender.

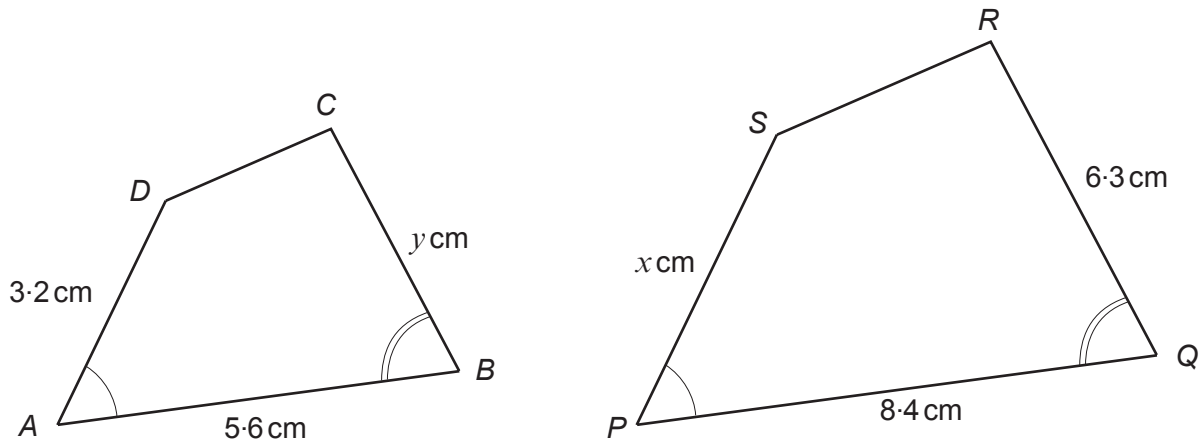
[3]



- [2]

$$= 0.147$$

7. The diagrams below show two similar shapes, $ABCD$ and $PQRS$.



Diagrams not drawn to scale

- (a) Calculate the value of x .

[2]

$$\text{Scale factor} = PQ \div AB = 8.4 \div 5.6 = 1.5$$

$$x = 3.2 \times 1.5$$

$$x = 4.8$$

- (b) Calculate the value of y .

[2]

$$y = 6.3 \div 1.5$$

$$y = 4.2$$



- (c) Explain clearly why the following statement cannot be true.

[2]

'The length of CD is 3.9 cm and the length of RS is 6.5 cm'.

CD and RS are corresponding sides, so $RS \div CD$ must equal

the scale factor $8.4 \div 5.6 = 1.5$.

But $6.5 \div 3.9 = 1.666\dots$, which is not 1.5.

(If $CD = 3.9$ cm, then $RS = 3.9 \times 1.5 = 5.85$ cm, not 6.5 cm.)

*So the shapes would not be similar, and the statement
cannot be true.*



8. A rectangle of length 12 cm and width $(2x - y)$ cm has an area of 72 cm^2 .

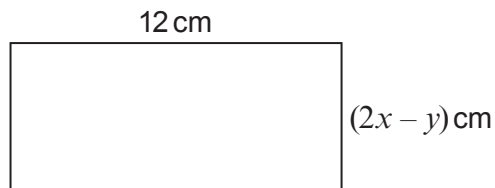


Diagram not drawn to scale

$KLMN$ is a kite where $KL = 3x \text{ cm}$ and $LM = y \text{ cm}$.

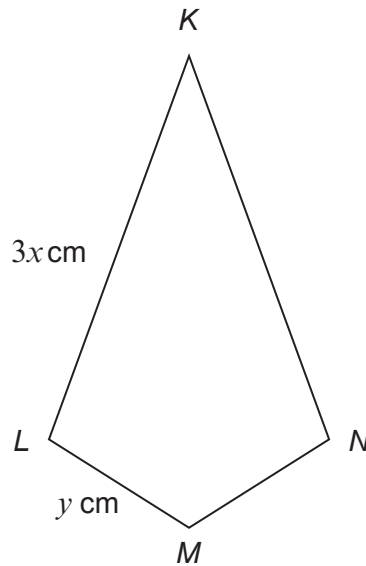


Diagram not drawn to scale

The perimeter of the kite $KLMN = 33 \text{ cm}$.

Calculate the values of x and y .

You must show all your working.

Do not use a trial and improvement method.

[5]

Area: $12(2x - y) = 72$, so $2x - y = 6$ (1)

Perimeter: $3x + 3x + y + y = 33$, so $6x + 2y = 33$

so $3x + y = 16.5$ (2)

Add (1) and (2): $5x = 22.5$

$x = 4.5$



Substitute $x = 4.5$ into (2): $3 \times 4.5 + y = 16.5$

$$13.5 + y = 16.5$$

$$y = 3$$

Check in (1): $2 \times 4.5 - 3 = 9 - 3 = 6$, correct

$$x = 4.5 \text{ and } y = 3$$



9. ABC and CDE are two right-angled triangles.

In triangle ABC , $AB = 6.5$ cm and $BC = 10.4$ cm.

In triangle CDE , $CE = 9.4$ cm.

$$\hat{BCE} = 22^\circ.$$

$$\hat{ACB} = x^\circ.$$

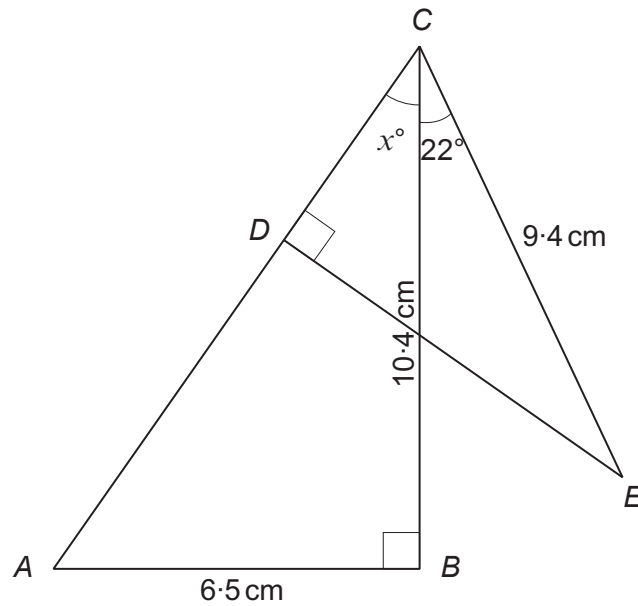


Diagram not drawn to scale

- (a) Calculate the value of x .

[3]

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{6.5}{10.4} = 0.625$$

$$x = \tan^{-1}(0.625) = 32.00538321\dots$$

$$x = 32$$



(b) Hence find the length of DE .

[3]

$\text{Angle } DCE = 22^\circ + 32^\circ = 54^\circ$ (D lies on AC)

In right-angled triangle CDE , $CE = 9.4$ cm is the hypotenuse.

$$\sin 54^\circ = \frac{DE}{9.4}$$

$$DE = 9.4 \times \sin 54^\circ = 7.604759747...$$

$$DE = 7.6 \text{ cm}$$



10. Factorise $4m^2 - 289$.

[2]

$$4m^2 - 289 = (2m)^2 - 17^2$$

$$= (2m + 17)(2m - 17)$$

11. Calculate the volume of a pyramid with a base area of $13\,200\text{ cm}^2$ and a perpendicular height of 460 cm .
Give your answer in m^3 .

[3]

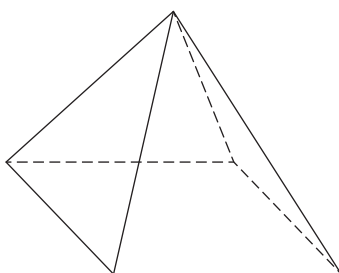


Diagram not drawn to scale

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$= 13200 \times 460 \div 3$$

$$= 2024000\text{ cm}^3$$

$$1\text{ m}^3 = 100 \times 100 \times 100\text{ cm}^3 = 1000000\text{ cm}^3$$

$$2024000 \div 1000000 = 2.024$$

$$\text{Volume} = \underline{2.024} \text{ m}^3$$



12. Five quadratic equations are listed below.
Draw a line connecting each equation to its solution.
One has been completed for you.

[4]

Equation	Solution
$x^2 - 4 = 0$	$x = 1, x = -\frac{3}{2}$
	$x = 2, x = -2$
	$x = 1, x = \frac{3}{2}$
$x(2x + 3) = 0$	$x = \frac{4}{9}$
	$x = -1, x = -\frac{2}{3}$
	$x = -\frac{2}{3}, x = \frac{2}{3}$
$(x - 1)(2x - 3) = 0$	$x = \frac{3}{2}, x = -\frac{3}{2}$
	$x = 1, x = -\frac{2}{3}$
	$x = -\frac{9}{4}$
$(2x - 3)(2x + 3) = 0$	$x = 0, x = \frac{2}{3}$
	$x = \frac{81}{16}$
	$x = 0, x = -\frac{3}{2}$
$(4x + 9)^2 = 0$	$x = \frac{3}{2}$
	$x = -\frac{9}{4}, x = 0$

.....

.....

.....

.....

.....



13. The values $a = 27$, $b = 1.9$ and $c = 0.81$ are each correct to 2 significant figures.

Use the formula $d = \frac{a-b}{c}$ to calculate the **least** value of d .

You must show all your working.

[3]

Bounds: $26.5 \leq a < 27.5$, $1.85 \leq b < 1.95$, $0.805 \leq c < 0.815$

Least d uses the least a , the greatest b and the greatest c .

Least $d = (26.5 - 1.95) \div 0.815$

$= 24.55 \div 0.815$

$= 30.12269939...$

Least value of $d = 30.12$ (2 d.p.)



14. A and B are points on a circle with centre O .
Calculate the length of the arc AB shown below.

[2]

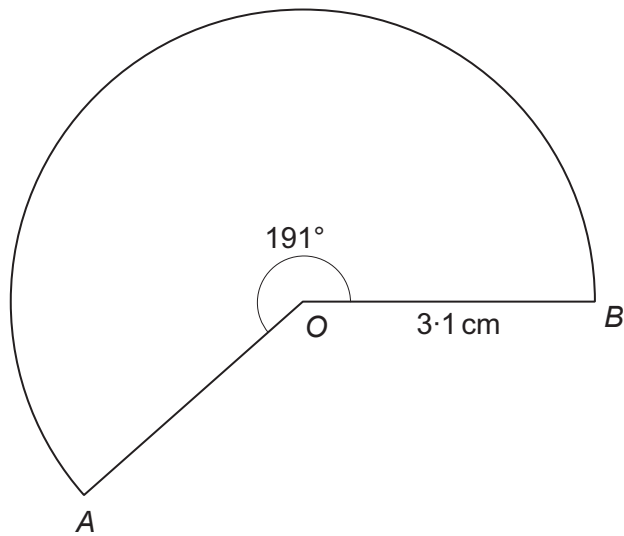


Diagram not drawn to scale

$$\text{Arc } AB = \frac{191}{360} \times 2 \times \pi \times 3.1$$

$$= 10.33409450\dots$$

$$\text{Arc } AB = 10.3 \text{ cm}$$



15. Express the following as a single fraction in its simplest form.

[4]

$$\frac{2}{3x-5} - \frac{7}{11x-13}$$

$$\frac{2}{3x-5} - \frac{7}{11x-13} = \frac{2(11x-13) - 7(3x-5)}{(3x-5)(11x-13)}$$

$$= \frac{22x - 26 - 21x + 35}{(3x-5)(11x-13)}$$

$$= \frac{x+9}{(3x-5)(11x-13)}$$



16. A bag contains 200 beads.
Some of the beads are red.
A bead is selected at random.
Its colour is recorded and then the bead is **replaced**.
A second bead is selected at random and its colour is also recorded.

The probability that two red beads are selected is 0.1369.
Calculate the number of red beads in the bag.

[4]

Let n be the number of red beads, so $P(\text{red}) = n \div 200$.

$$P(\text{red, red}) = \frac{n}{200} \times \frac{n}{200} = 0.1369$$

$$n^2 = 0.1369 \times 200^2 = 0.1369 \times 40000 = 5476$$

$$n = \sqrt{5476} = 74$$

There are 74 red beads in the bag.



17. Solve the equation $(2x + 5)(3x - 11) = 7$.
Give your answers correct to 2 decimal places.

[5]

$$(2x + 5)(3x - 11) = 7$$

$$6x^2 - 22x + 15x - 55 = 7$$

$$6x^2 - 7x - 62 = 0$$

$$a = 6, b = -7, c = -62$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 6 \times (-62)}}{2 \times 6} = \frac{7 \pm \sqrt{1537}}{12}$$

$$x = (7 + 39.20459...) \div 12 = 3.850382631...$$

$$x = (7 - 39.20459...) \div 12 = -2.683715964...$$

$$x = 3.85 \text{ or } x = -2.68$$



18. Make c the subject of the following formula.

[4]

$$\sqrt{gc^2 - v} = c$$

Square both sides: $gc^2 - v = c^2$

$$gc^2 - c^2 = v$$

$$c^2(g - 1) = v$$

$$c^2 = \frac{v}{g - 1}$$

$$c = \pm \sqrt{\frac{v}{g - 1}}$$



19. BC is the tangent to the circle at point E , as shown below.

$EC = 8\text{ cm}$, $AC = 11\text{ cm}$ and $\hat{DCE} = 31^\circ$.

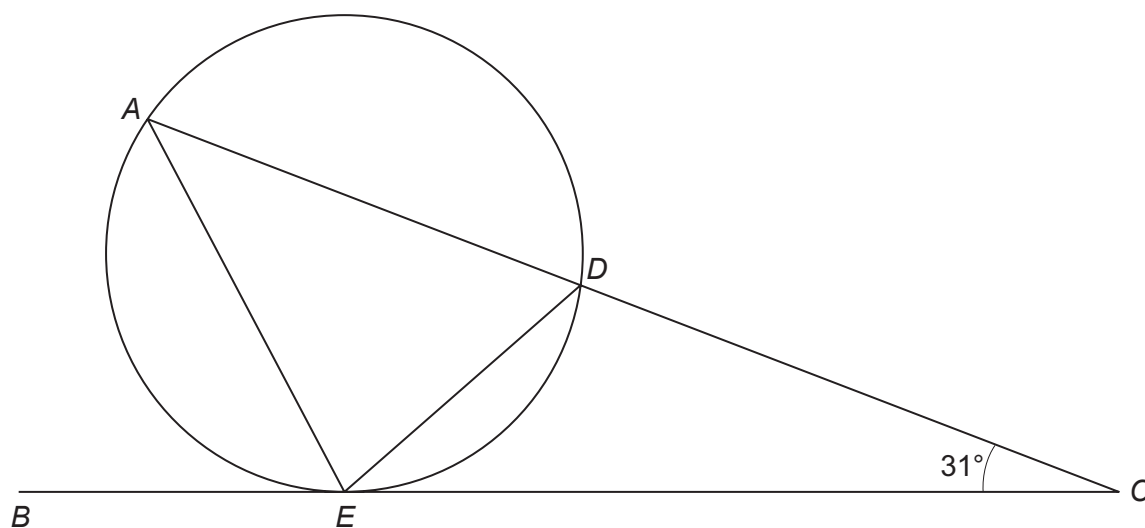


Diagram not drawn to scale

- (a) Calculate the length of AE .

[3]

Cosine rule in triangle ACE:

$$AE^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 31^\circ$$

$$AE^2 = 185 - 176 \cos 31^\circ = 34.13855508...$$

$$AE = \sqrt{34.13855508...} = 5.842820815...$$

$$AE = 5.84\text{ cm}$$



(b) Calculate the size of $\hat{CED}$.

[4]

Sine rule in triangle ACE:

$$\frac{\sin CAE}{8} = \frac{\sin 31^\circ}{5.842820815...}$$

$$\sin CAE = 8 \times \sin 31^\circ \div 5.842820815... = 0.705190991...$$

$$\text{Angle } CAE = \sin^{-1}(0.705190991...) = 44.84497597...^\circ$$

*Angle CED = angle CAE = angle EAD (alternate segment theorem:
the angle between a tangent and a chord is equal to the
angle in the alternate segment).*

$$\text{Angle } CED = 44.8^\circ$$

END OF PAPER



[illegible]