

Surname <i>StemUp Model Solution</i>	Centre Number	Candidate Number
Other Names		0



GCSE – NEW

3300U60-1



MATHEMATICS
UNIT 2: CALCULATOR-ALLOWED
HIGHER TIER

TUESDAY, 20 JUNE 2017 – AFTERNOON

1 hour 45 minutes

ADDITIONAL MATERIALS

A calculator will be required for this paper.

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

If you run out of space, use the continuation page(s) at the back of the booklet, taking care to number the question(s) correctly.

Take π as 3.14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

In question 11, the assessment will take into account the quality of your linguistic and mathematical organisation, communication and accuracy in writing.

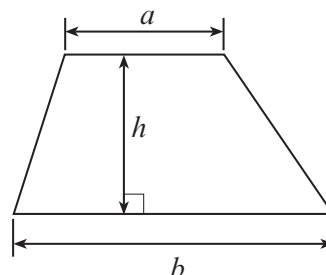
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	5	
3.	4	
4.	3	
5.	3	
6.	3	
7.	5	
8.	5	
9.	2	
10.	2	
11.	7	
12.	5	
13.	5	
14.	3	
15.	3	
16.	5	
17.	1	
18.	5	
19.	2	
20.	4	
21.	4	
Total	80	



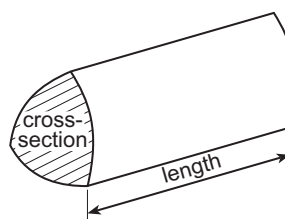
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Formula List - Higher Tier

Area of trapezium $= \frac{1}{2}(a + b)h$

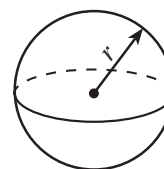


Volume of prism = area of cross-section $\times$ length



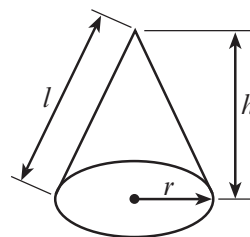
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

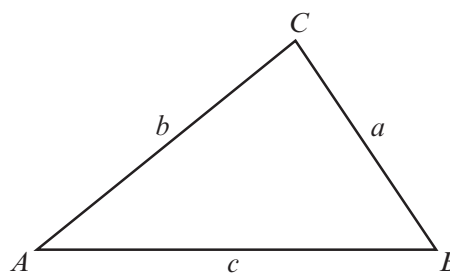


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2}ab \sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Annual Equivalent Rate (AER)

AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.



1. (a) Calculate $\sqrt{8.5^3 + (4.5 - 0.76)^2}$, correct to 3 significant figures. [2]

$$\sqrt{(614.125 + 13.9876)} = \sqrt{628.1126} = 25.06217...$$

$$= 25.1 \text{ (3 s.f.)}$$

- (b) Calculate the reciprocal of -0.07 , correct to 1 decimal place. [2]

$$1 \div (-0.07) = -14.285714...$$

$$= -14.3 \text{ (1 d.p.)}$$

2. Show that the triangle below is **not** a right-angled triangle. [5]

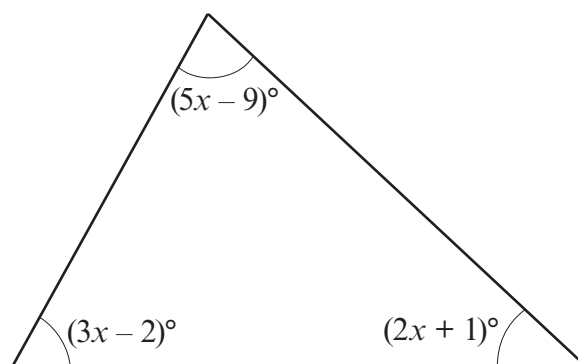


Diagram not drawn to scale

Angles in a triangle add up to 180° :

$$3x - 2 + 2x + 1 + 5x - 9 = 180$$

$$10x - 10 = 180$$

$$10x = 190$$

$$x = 19$$

$$3x - 2 = 3 \times 19 - 2 = 55^\circ$$

$$2x + 1 = 2 \times 19 + 1 = 39^\circ$$

$$5x - 9 = 5 \times 19 - 9 = 86^\circ$$

The angles are 55° , 39° and 86° . None of them is 90° ,

so the triangle is not a right-angled triangle.



3. A solution to the equation

$$x^3 - 2x - 45 = 0$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working.

[4]

x	$x^3 - 2x - 45$	
3	$27 - 6 - 45 = -24$	too low
4	$64 - 8 - 45 = 11$	too high
3.5	$42.875 - 7 - 45 = -9.125$	too low
3.7	$50.653 - 7.4 - 45 = -1.747$	too low
3.8	$54.872 - 7.6 - 45 = 2.272$	too high
3.75	$52.734... - 7.5 - 45 = 0.234...$	too high

3.75 gives a positive value and 3.7 a negative value,

so the solution is between 3.7 and 3.75.

$x = 3.7$ (1 d.p.)



4. A right-angled triangle LMN is shown below.
 $LN = 16.9$ cm and $LM = 6.5$ cm.

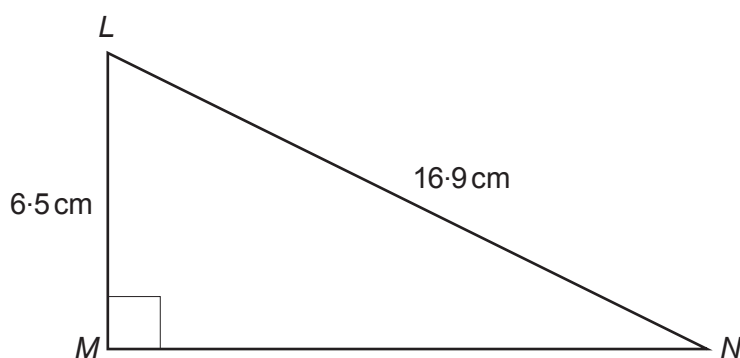


Diagram not drawn to scale

Calculate the length MN .

[3]

$$16.9^2 = 6.5^2 + MN^2$$

$$MN^2 = 16.9^2 - 6.5^2 = 285.61 - 42.25$$

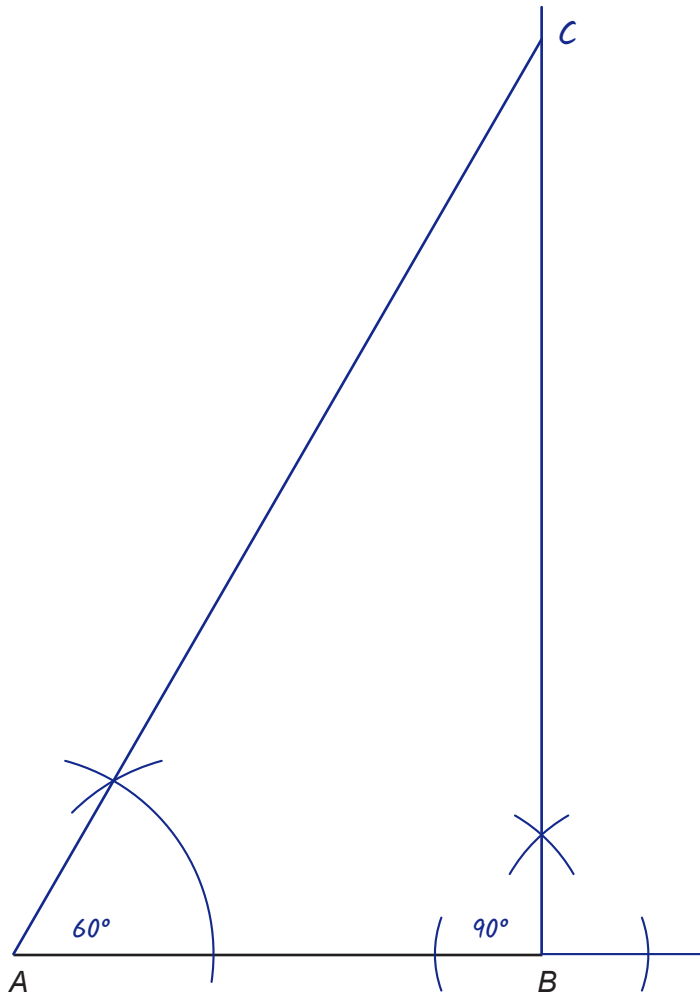
$$MN^2 = 243.36$$

$$MN = \sqrt{243.36} = 15.6 \text{ cm}$$



5. Construct an accurate drawing of triangle ABC , where $AB = 7$ cm, $\hat{A}BC = 90^\circ$ and $\hat{B}AC = 60^\circ$.
Use only a ruler and a pair of compasses.
The side AB has been drawn for you.
You must show your construction arcs.

[3]



6. Calculate the length of the side QR in the triangle PQR shown below.

[3]

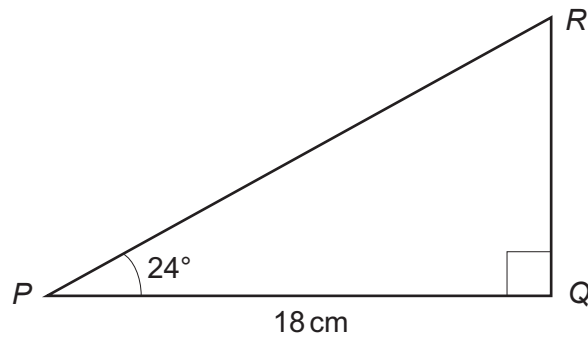


Diagram not drawn to scale

$$\tan 24^\circ = QR / 18$$

$$QR = 18 \times \tan 24^\circ$$

$$QR = 8.014116...$$

$$QR = 8.01 \text{ cm}$$



7. 100 boxes each contain 10 balls.

45 of the boxes are labelled A.
They each contain 7 black balls and 3 white balls.

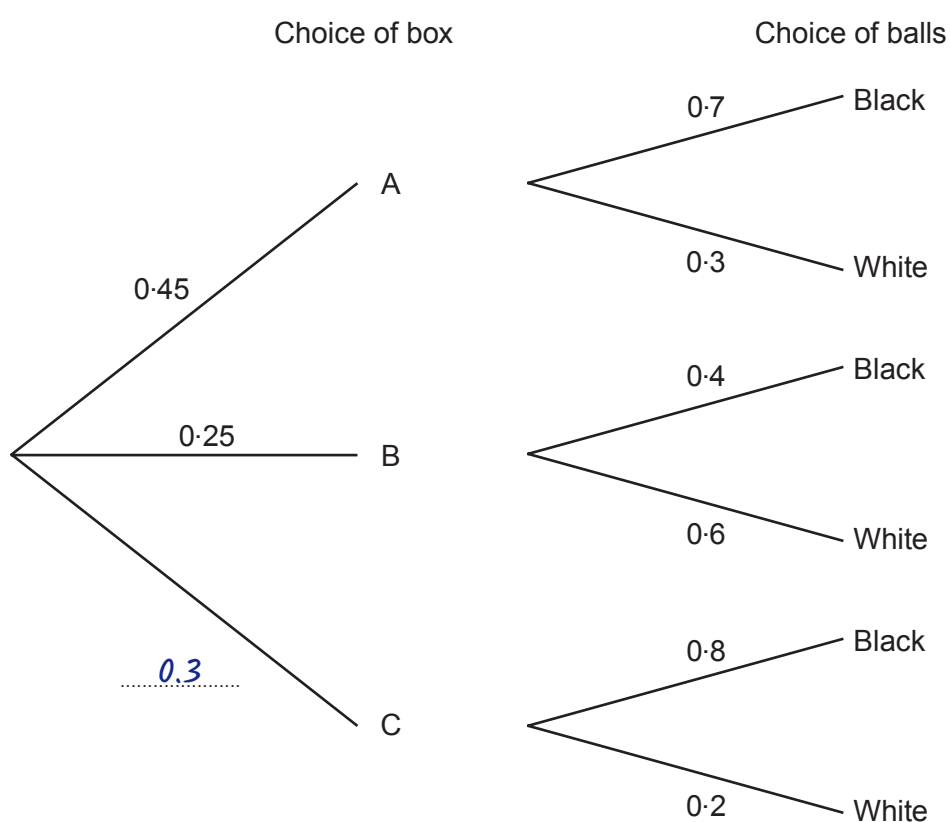
25 of the boxes are labelled B.
They each contain 4 black balls and 6 white balls.

The rest of the boxes are labelled C.
They each contain 8 black balls and 2 white balls.

In a game, a player chooses a box at random, and then chooses a ball at random from that box.

- (a) Complete the tree diagram shown below.

[1]



- (b) What is the probability that a player will select a black ball?

[3]

$$P(\text{black}) = 0.45 \times 0.7 + 0.25 \times 0.4 + 0.3 \times 0.8$$

$$= 0.315 + 0.1 + 0.24$$

$$= 0.655$$



- (c) If a large number of people played the game, approximately what fraction of them would you expect to choose a white ball?
Circle your answer. [1]

$\frac{1}{10}$

$\frac{1}{5}$

$\frac{1}{4}$

$\frac{1}{3}$

$\frac{1}{2}$

$$P(\text{white}) = 1 - 0.655 = 0.345 \approx 1/3$$

8. (a) Factorise $x^3 - 5x$. [1]

$$x(x^2 - 5)$$

- (b) Expand and simplify $(2x - 3)(x + 4)$. [2]

$$2x^2 + 8x - 3x - 12$$

$$= 2x^2 + 5x - 12$$

- (c) Factorise $x^2 - 3x - 28$. [2]

$$(x - 7)(x + 4)$$



9. (a) Circle the equation of a straight line that is parallel to the line $3y = 2x + 6$. [1]

$$3y = 2x + 7$$

$$2y = 3x + 6$$

$$3y = -2x + 6$$

$$-3y = 2x + 6$$

$$2y = -3x + 6$$

Parallel lines have the same gradient, 2/3

- (b) Circle the equation of a straight line that is perpendicular to the line $y = 5x - 3$. [1]

$$y = \frac{x}{5} + 3$$

$$y = 5x + 3$$

$$y = 5x + \frac{1}{3}$$

$$y = -5x + 3$$

$$y = \frac{-x}{5} + 3$$

Gradient of perpendicular = -1/5



10. Points A , B and C lie on the circumference of a circle, centre O .
 $\angle ACB = 37^\circ$.

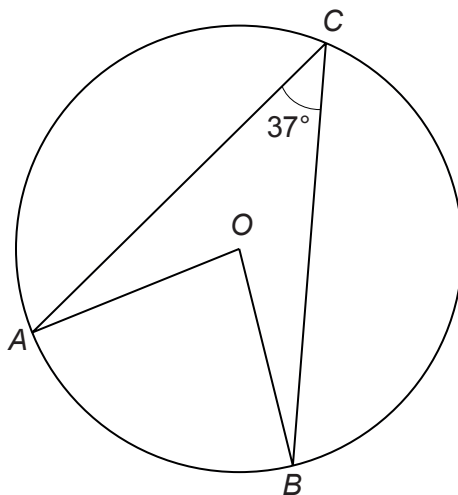


Diagram not drawn to scale

Calculate the size of the **reflex** angle $\angle AOB$.

[2]

*Angle AOB (obtuse) = $2 \times 37^\circ = 74^\circ$ (angle at centre is twice
the angle at the circumference)*

Reflex angle AOB = $360^\circ - 74^\circ = 286^\circ$



11. In this question, you will be assessed on the quality of your organisation, communication and accuracy in writing.

The area of triangle ABD , shown in the diagram below, is 35 cm^2 .

$AD = 5 \text{ cm}$ and $BC = 32 \text{ cm}$.

D is on the line AC , and BD is perpendicular to AC .

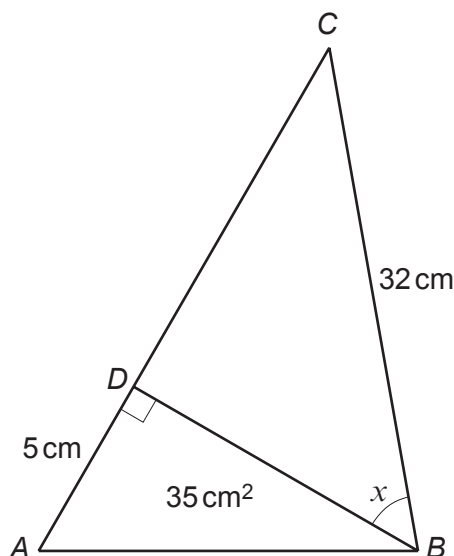


Diagram not drawn to scale

Calculate the size of angle x .
You must show all your working.

[5 + 2 OCW]

Triangle ABD is right-angled at D , so area = $\frac{1}{2} \times AD \times BD$.

$$\frac{1}{2} \times 5 \times BD = 35$$

$$2.5 \times BD = 35, \text{ so } BD = 35 \div 2.5 = 14 \text{ cm.}$$

Triangle BCD is right-angled at D , with hypotenuse $BC = 32 \text{ cm}$

and $BD = 14 \text{ cm}$ adjacent to angle x .

$$\cos x = \text{adjacent} / \text{hypotenuse} = BD / BC = 14 / 32 = 0.4375$$

$$x = \cos^{-1} 0.4375$$

$$x = 64.0555...^\circ$$

So angle $x = 64^\circ$ (to the nearest degree).



12. Make c the subject of the following formula.
Give your answer in its simplest form.

[5]

$$c - 5 = \frac{3c - 7}{d}$$

$$d(c - 5) = 3c - 7$$

$$dc - 5d = 3c - 7$$

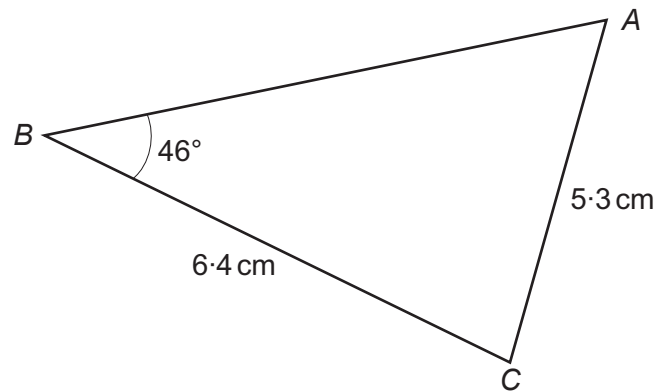
$$dc - 3c = 5d - 7$$

$$c(d - 3) = 5d - 7$$

$$c = \frac{5d - 7}{d - 3}$$



13.

*Diagram not drawn to scale*

By first calculating the size of $\hat{BAC}$, calculate the area of triangle ABC.
You must show all your working.

[5]

$$\text{Sine rule: } \sin BAC / 6.4 = \sin 46^\circ / 5.3$$

$$\sin BAC = 6.4 \times \sin 46^\circ \div 5.3 = 0.868636...$$

$$BAC = \sin^{-1} 0.868636... = 60.3006...^\circ$$

$$ACB = 180^\circ - 46^\circ - 60.3006...^\circ = 73.6993...^\circ$$

$$\text{Area} = \frac{1}{2} \times 5.3 \times 6.4 \times \sin 73.6993...^\circ$$

$$= 16.27824...$$

$$\text{Area} = 16.3 \text{ cm}^2$$



14. The region between two rectangles is shaded, as shown in the diagram below. All of the measurements shown are given **correct to the nearest cm**.

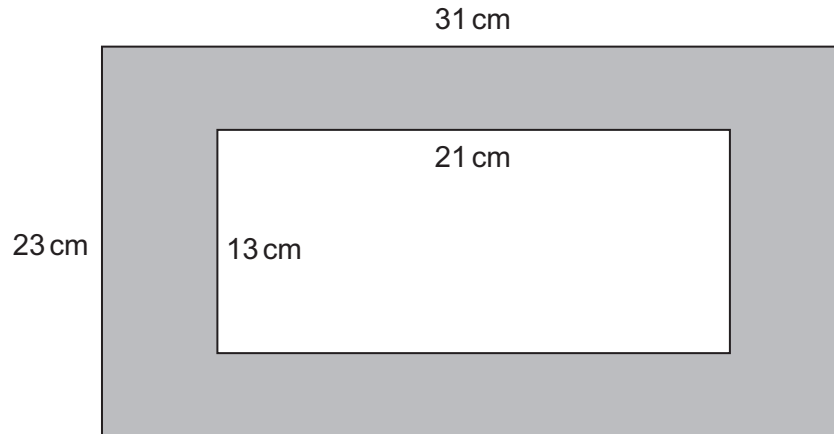


Diagram not drawn to scale

Calculate the greatest possible area of the shaded region.

[3]

Greatest area = largest outer – smallest inner

Outer: $31.5 \times 23.5 = 740.25$

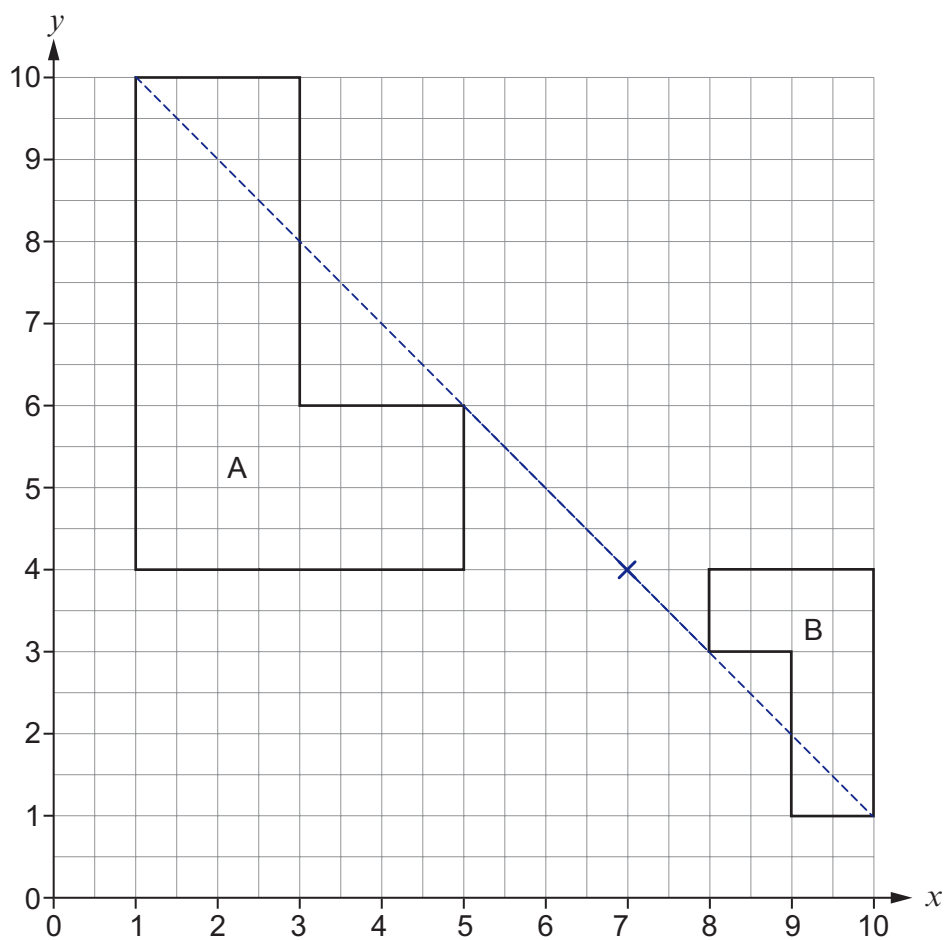
Inner: $20.5 \times 12.5 = 256.25$

Shaded area = $740.25 - 256.25 = 484 \text{ cm}^2$



15. Describe fully a **single** transformation that transforms shape A onto shape B.

[3]



Enlargement, scale factor $-\frac{1}{2}$, centre (7, 4)



16. The table below shows the three-day rain forecast for Monday, Tuesday and Wednesday in Eglwyswrw.

Day	Probability of rain
Monday	80%
Tuesday	80%
Wednesday	80%

For these three days,

- (a) calculate the probability that it will rain on all three days.

[2]

$$P(\text{rain all three days}) = 0.8 \times 0.8 \times 0.8$$

$$= 0.8^3 = 0.512$$

- (b) calculate the probability that it will rain on exactly 2 consecutive days.

[3]

Rain, Rain, Dry or Dry, Rain, Rain

$$= 0.8 \times 0.8 \times 0.2 + 0.2 \times 0.8 \times 0.8$$

$$= 2 \times 0.8^2 \times 0.2 = 2 \times 0.128$$

$$= 0.256$$



17. Circle the expression that is equivalent to $w^{-\frac{3}{5}}$.

[1]

$$-(\sqrt[3]{w})^5$$

$$-\frac{3}{5}w$$

$$-(\sqrt[5]{w})^3$$

$$\frac{1}{(\sqrt[5]{w})^3}$$

$$\frac{1}{(\sqrt[3]{w})^5}$$

$$w^{-3/5} = 1 / w^{3/5} = 1 / (\sqrt[5]{w})^3$$

18. Solve the equation $x = \frac{7}{5x-3}$.

Give your answers correct to 2 decimal places.

[5]

$$x(5x-3) = 7$$

$$5x^2 - 3x = 7$$

$$5x^2 - 3x - 7 = 0 \quad a = 5, b = -3, c = -7$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-7)}}{2 \times 5}$$

$$x = \frac{3 \pm \sqrt{149}}{10}$$

$$x = (3 + 12.2065...)/10 = 1.520655... \quad \text{or} \quad x = (3 - 12.2065...)/10 = -0.920655...$$

$$x = 1.52 \quad \text{or} \quad x = -0.92$$



19. (a) Give one example to show that the square of an irrational number is **not** always rational. [1]

$1 + \sqrt{3}$ is irrational.

$$(1 + \sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$$

$4 + 2\sqrt{3}$ is irrational, so the square is not rational.

Number = $1 + \sqrt{3}$

Square of the number = $4 + 2\sqrt{3}$

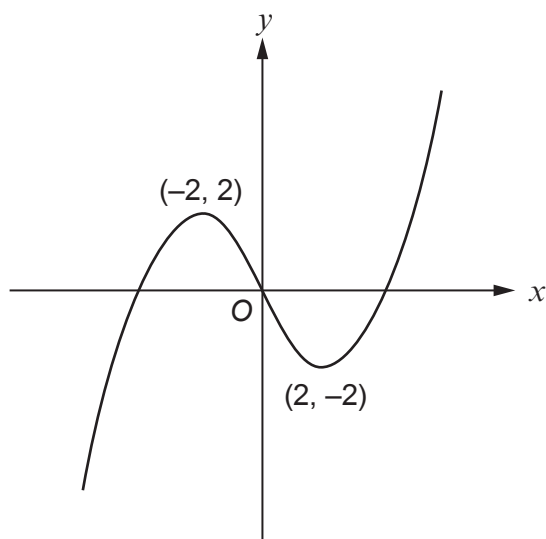
- (b) Find two **different** irrational numbers to make the answer to the calculation below rational. Complete the calculation by filling in the three boxes. [1]

$$\boxed{\sqrt{2}} \times \boxed{\sqrt{8}} = \boxed{4}$$

$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$, which is rational



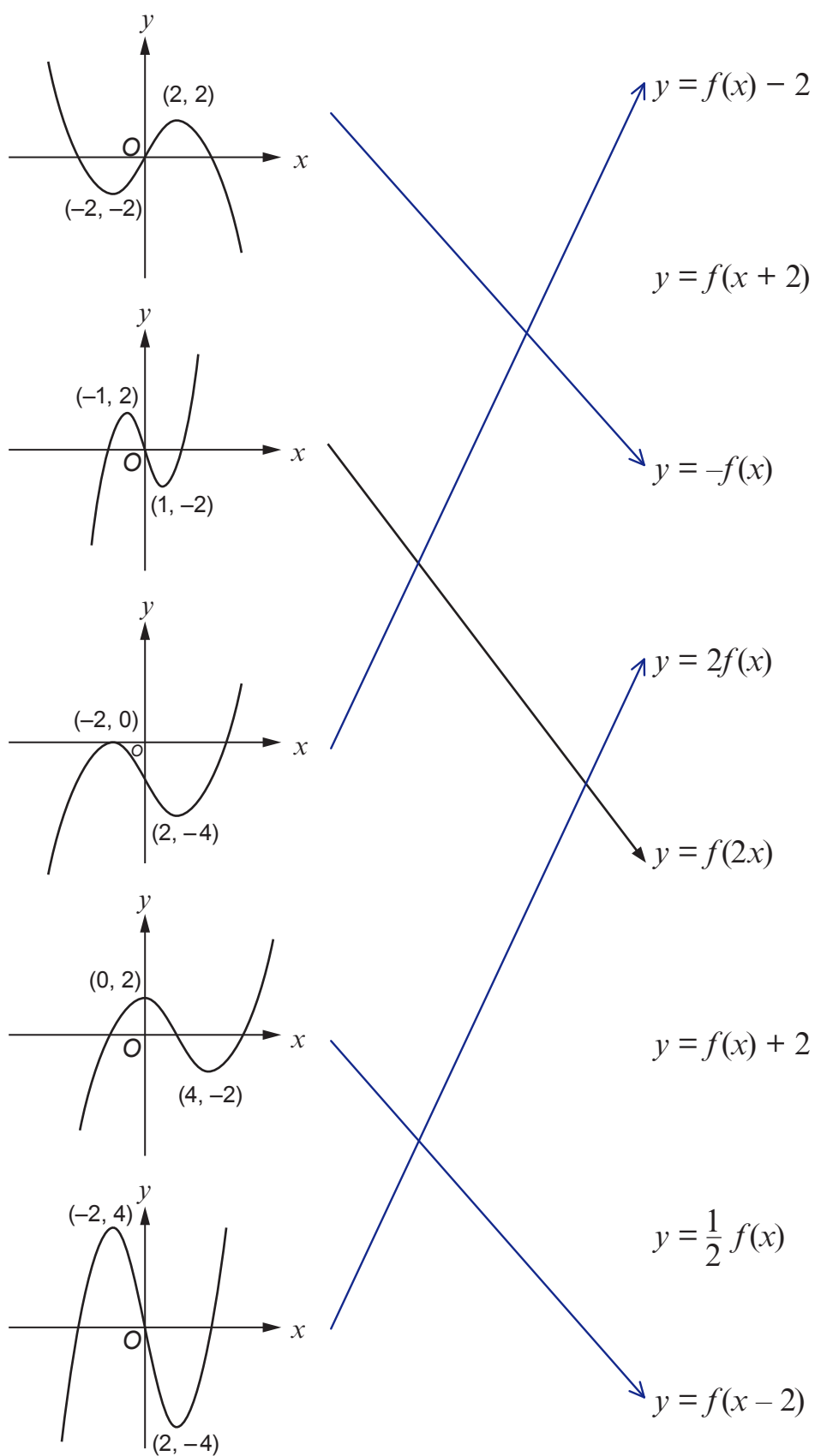
20. A sketch of the graph $y = f(x)$ is shown below.
Two specific points are shown on the graph. They are called a maximum point and a minimum point.
The maximum point shown is $(-2, 2)$ and the minimum point shown is $(2, -2)$.



The graphs on the opposite page are transformations of $y = f(x)$.
Draw a line connecting each graph to the equation describing the transformation.
One has been done for you.

[4]





21. The cube below has an internal diagonal of length 20 cm. Each edge of the cube is of length x cm.

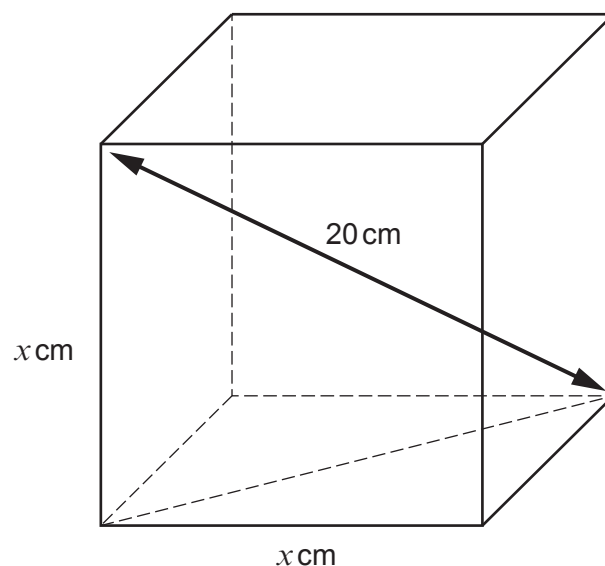


Diagram not drawn to scale

Calculate the value of x .
You must use an algebraic method and show all your working.

[4]

$$\text{Face diagonal}^2: d^2 = x^2 + x^2 = 2x^2$$

$$\text{Internal diagonal: } d^2 + x^2 = 20^2$$

$$2x^2 + x^2 = 400$$

$$3x^2 = 400$$

$$x^2 = 400/3 = 133.33...$$

$$x = \sqrt{400/3} = 11.54700...$$

$$x = 11.5 \text{ cm}$$

END OF PAPER



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