Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

S24-2300U20-1

2300U20-1

THURSDAY, 23 MAY 2024 – AFTERNOON

MATHEMATICS – AS unit 2
APPLIED MATHEMATICS A

1 hour 45 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**3**	
2	**10**	
3	**8**	
4	**11**	
5	**8**	
6	**4**	
7	**4**	
8	**7**	
9	**9**	
10	**11**	
Total	**75**	

2300U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 75.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

JUN242300U20101

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

Section A: Statistics

1. An exercise gym opens at 6:00 a.m. every day. The manager decides to use a questionnaire to gather the opinions of the gym members. The first 30 members arriving at the gym on a particular morning are asked to complete the questionnaire.

(a) What is the intended population in this context? [1]

The members of the gym.

(b) What type of sampling is this? [1]

Opportunity (convenience) sampling.

(c) How could the sampling process be improved? [1]

Use random sampling of all gym members, e.g. ask members at different times of the day and on different days, and ask more people.

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

2. A baker sells 3·5 birthday cakes per hour on average.

(a) State, in context, **two** assumptions you would have to make in order to model the number of birthday cakes sold using a Poisson distribution. [1]

1. Cakes are sold singly (one at a time) and independently of each other.

2. Cakes are sold at random, at a constant average rate of 3.5 per hour.

(b) Using a Poisson distribution and showing your calculation, find the probability that exactly 2 birthday cakes are sold in a randomly selected 1-hour period. [2]

Let W be the number of cakes sold in a 1-hour period.

$W \sim Po(3.5)$

$P(W = 2) = \frac{e^{-3.5} \times 3.5^2}{2!}$

$= 0.184958...$

$= 0.1850$ (4 d.p.)

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(c) Calculate the probability that, during a randomly selected 3-hour period, the baker sells more than 10 birthday cakes. [3]

Let X be the number of cakes sold in a 3-hour period.

$X \sim Po(3 \times 3.5) = Po(10.5)$

$P(X > 10) = 1 - P(X \leq 10)$

$= 1 - 0.5207$

$= 0.4793$

(d) The baker sells a birthday cake at 9:30 a.m. Calculate the probability that the baker will sell the next birthday cake before 10:00 a.m. [3]

Let Y be the number of cakes sold in a half-hour period (9:30 to 10:00).

$Y \sim Po(0.5 \times 3.5) = Po(1.75)$

The next cake is sold before 10:00 a.m. if $Y \geq 1$.

$P(Y \geq 1) = 1 - P(Y = 0)$

$= 1 - e^{-1.75}$

$= 1 - 0.1738$

$= 0.8262$

2300U201
05

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(e) Select one of the assumptions in part (a) and comment on its reasonableness. [1]

Cakes sold singly: not reasonable, as some customers might buy more than one birthday cake at a time (e.g. for twins or for a large party), so cakes are not always sold individually.

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

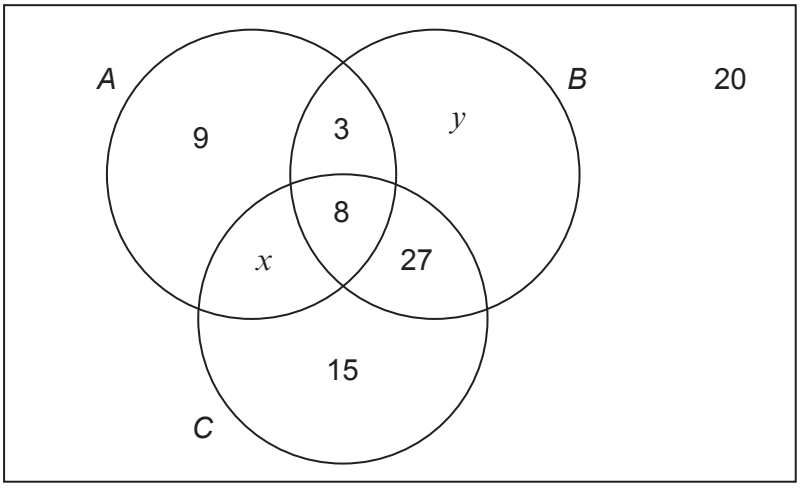
PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

3. The following Venn diagram shows the participation of 100 students in three activities, *A*, *B*, and *C*, which represent athletics, baseball and climbing respectively.



For these 100 students, participation in athletics and participation in climbing are independent events.

(a) Show that $x = 10$ and find the value of y. [5]

$P(A) = \frac{9+3+8+x}{100} = \frac{20+x}{100}$, $P(C) = \frac{x+8+27+15}{100} = \frac{50+x}{100}$

A and C independent $\Rightarrow$ $P(A \cap C) = P(A) \times P(C)$

$\frac{20+x}{100} \times \frac{50+x}{100} = \frac{8+x}{100}$

$(20+x)(50+x) = 100(8+x)$

$1000 + 70x + x^2 = 800 + 100x$

$x^2 - 30x + 200 = 0$

$(x-10)(x-20) = 0$

$x = 10$ or $x = 20$

Total: $9 + 3 + 8 + x + y + 27 + 15 + 20 = 100 \Rightarrow x + y = 18$

If $x = 20$ then $y = -2$, which is impossible (y cannot be negative),

so reject $x = 20$. $\therefore$ $x = 10$

$y = 18 - 10 = 8$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) Two students are selected at random, one after the other without replacement. Find the probability that the first student does athletics and the second student does only climbing. [3]

P(first athletics and second only climbing)

$= \frac{30}{100} \times \frac{15}{99}$

$= \frac{450}{9900}$

$= \frac{1}{22}$ (= 0.04545...)

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

4. A company produces sweets of varying colours. The company claims that the proportion of blue sweets is 13·6%. A consumer believes that the true proportion is less than this. In order to test this belief, the consumer collects a random sample of 80 sweets.

(a) State suitable hypotheses for the test. [1]

H_0: $p = 0.136$

H_1: $p < 0.136$, where p is the proportion of blue sweets.

(b) (i) Determine the critical region if the test is to be carried out at a significance level as close as possible to, but not exceeding, 5%.

(ii) Given that there are 6 blue sweets in the sample of 80, complete the significance test. [5]

(i) Let X be the number of blue sweets in the sample of 80.

Under H_0, $X \sim B(80, 0.136)$

$P(X \leq 5) = 0.0309 < 0.05$

$P(X \leq 6) = 0.0690 > 0.05$

Critical region is $X \leq 5$

(ii) 6 is not in the critical region ($P(X \leq 6) = 0.0690 > 0.05$),

so there is insufficient evidence to reject H_0.

There is insufficient evidence to suggest that the proportion of blue sweets is less than the company's claim of 13.6%.

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(c) Suppose the proportion of blue sweets claimed by the company is correct. The consumer conducts the sampling and testing process on a further 20 occasions, using the sample size of 80 each time. What is the expected number of these occasions on which the consumer would reach the incorrect conclusion? [2]

The incorrect conclusion is rejecting H_0 when it is true.

$P(\text{reject } H_0) = P(X \leq 5) = 0.0309$

Expected number of occasions $= 20 \times 0.0309$

$= 0.618$

(d) Now suppose that the proportion of blue sweets is 7%. Find the probability of a Type II error. Interpret your answer in context. [3]

Type II error: $X \geq 6$ (H_0 not rejected) when $X \sim B(80, 0.07)$

$P(\text{Type II error}) = P(X \geq 6 \mid X \sim B(80, 0.07))$

$= 1 - P(X \leq 5)$

$= 1 - 0.50817...$

$= 0.4918$

There is a 49.2% chance of concluding that the proportion of blue sweets has not reduced (failing to reject the company's claim of 13.6%), when in fact it has fallen to 7%.

© WJEC CBAC Ltd.

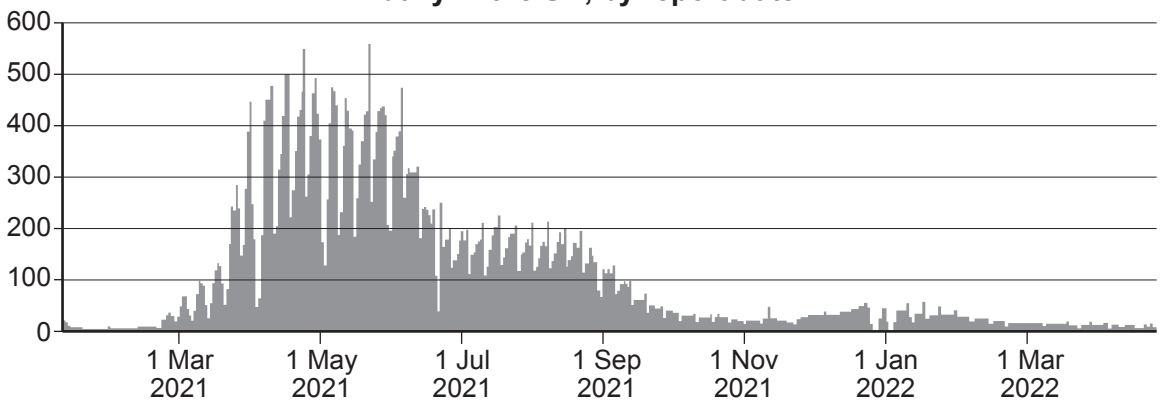
stemup.co.uk

Examiner only

5. In March 2020, the coronavirus pandemic caused major disruption to the lives of individuals across the world. A newspaper published the following graph from the gov.uk website, along with an article which included the following excerpt.

"The daily number of vaccines administered continues to fall. In order to get control of the virus, we need the number of people receiving a second dose of the vaccine to keep rocketing. The fear is it will start to drop off soon, which will leave many people still unprotected."

Number of people (1000s) who received 2nd dose vaccinations daily in the UK, by report date



(a) By referring to the graph, explain how the quote could be misleading. [1]

The graph shows the daily number of second doses falling because most of the population had already received their second dose, so the fall is inevitable rather than a bad thing; the quote makes it sound alarming.

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

The daily numbers of second dose vaccines, in thousands, over the period April 1st 2021 to May 31st 2021 are shown in the table below.

Daily number of 2nd dose vaccines (1000s)	**Midpoint** x	**Frequency** f	**Percentage**
$0 \leqslant v < 100$	50	2	3·3
$100 \leqslant v < 200$	150	8	13·1
$200 \leqslant v < 300$	250	10	16·4
$300 \leqslant v < 400$	350	13	21·3
$400 \leqslant v < 500$	450	26	42·6
$500 \leqslant v < 600$	550	2	3·3
Total		61	100

(b) (i) Calculate estimates of the mean and standard deviation for the daily number of second dose vaccines given over this period. You may use $\sum x^2 f = 8\,272\,500$. [4]

(ii) Comment on the skewness of these data. [1]

(i) $\Sigma fx = 2\times50 + 8\times150 + 10\times250 + 13\times350 + 26\times450 + 2\times550 = 21150$

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{21150}{61} = 346.721...$

Mean = 346.721 thousand = 346 721 vaccines per day

$\sigma = \sqrt{\left(\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2\right)} = \sqrt{\left(\frac{8272500}{61} - \left(\frac{21150}{61}\right)^2\right)}$

$= \sqrt{15399.086...} = 124.093...$ thousand

Standard deviation = 124 093 vaccines per day

(ii) The data are negatively skewed (skewed to the left).

2300U201
13

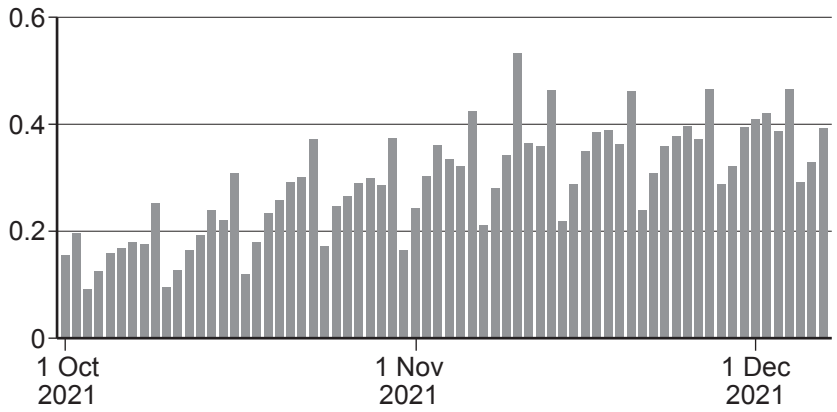
© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

A second graph in the article shows the number of people receiving a third dose of the vaccine. This graph has a repeated pattern of rising then falling. An extract is shown below.

Number of people (millions) who received 3rd dose vaccinations daily in the UK, by report date



(c) Give a possible reason for the pattern observed in this graph. [1]

There seem to be lower bars on a weekly cycle: fewer people get vaccinated at weekends (e.g. Sundays) when some vaccine centres are closed, and more people get vaccinated on weekdays.

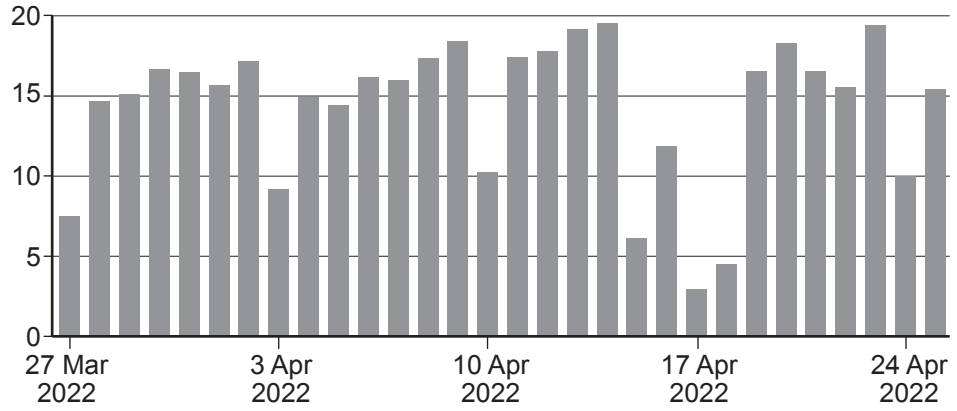
© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

Another extract shows the number of people who received the third dose of the vaccine between 27th March 2022 and 25th April 2022.

Number of people (1000s) who received 3rd dose vaccinations daily in the UK, by report date



(d) State, with a reason, whether or not you think the data for April 15th to April 18th are incorrect. [1]

It may not be incorrect: 15th to 18th April 2022 was the Easter bank holiday weekend, so fewer vaccination centres would have been open and fewer people would have been vaccinated on those days.

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

Section B: Mechanics

6. A ship *S* is moving with constant velocity $(4\mathbf{i} - 7\mathbf{j})\text{ms}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors due east and due north respectively.

Find the speed and direction of *S*, giving the direction as a three-figure bearing, correct to the nearest degree. [4]

Speed $= |4i - 7j| = \sqrt{(4^2 + (-7)^2)}$

$= \sqrt{65}$

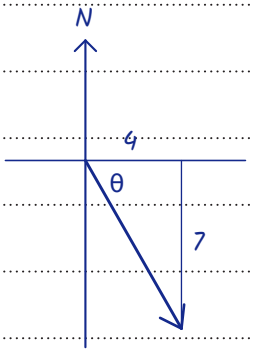
$= 8.062\ldots = 8.06\ m\ s^{-1}$

$\tan\theta = \frac{7}{4} \Rightarrow \theta = \tan^{-1}(\frac{7}{4}) = 60.255\ldots°$ below the east direction

Bearing $= 90° + 60.255\ldots°$

$= 150.255\ldots°$

Direction: bearing 150°

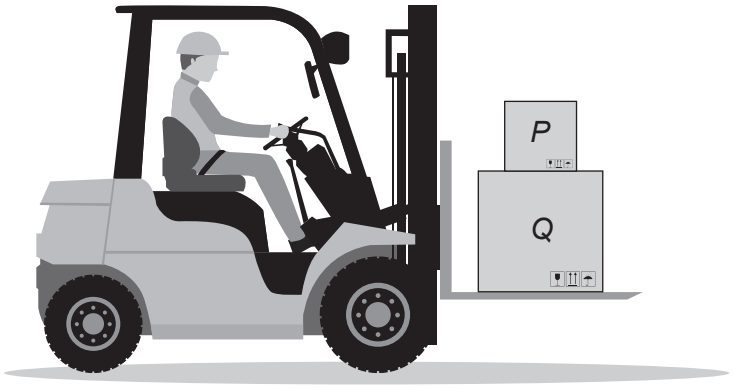


© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

7. The diagram below shows a forklift truck being used to raise two boxes, *P* and *Q*, vertically. Box *Q* rests on horizontal forks and box *P* rests on top of box *Q*. Box *P* has mass 25 kg and box *Q* has mass 55 kg.



(a) When the boxes are moving upwards with uniform acceleration, the reaction of the horizontal forks on box *Q* is 820 N.
Calculate the magnitude of the acceleration. [3]

N2L applied to both boxes together (mass 80 kg), upwards positive:

$R - 80g = 80a$

$820 - 80 \times 9.8 = 80a$

$820 - 784 = 80a$

$36 = 80a$

$a = 0.45\ m\ s^{-2}$

(b) Calculate the reaction of box *Q* on box *P* when they are moving vertically upwards with constant speed. [1]

Constant speed $\Rightarrow a = 0$, so for box P: $R - 25g = 0$

$R = 25 \times 9.8 = 245\ N$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

8. A particle, of mass 4 kg, moves in a straight line under the action of a single force FN, whose magnitude at time t seconds is given by

$$F = 12\sqrt{t} - 32 \quad \text{for} \quad t \geqslant 0.$$

(a) Find the acceleration of the particle when $t = 9$. [2]

N2L: $4a = 12\sqrt{t} - 32 \Rightarrow a = 3\sqrt{t} - 8$

When $t = 9$: $a = 3\sqrt{9} - 8 = 9 - 8$

$a = 1 \text{ m s}^{-2}$

(b) Given that the particle has velocity –1 ms^{-1} when $t = 4$, find an expression for the velocity of the particle at t s. [3]

$v = \int a\,dt = \int (3t^{1/2} - 8)\,dt$

$v = \frac{3}{3/2} t^{3/2} - 8t + C = 2t^{3/2} - 8t + C$

When $t = 4$, $v = -1$: $-1 = 2(4)^{3/2} - 8(4) + C$

$-1 = 16 - 32 + C \Rightarrow C = 15$

$v = 2t^{3/2} - 8t + 15$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(c) Determine whether the speed of the particle is increasing or decreasing when $t = 9$. [2]

When $t = 9$: $v = 2(9)^{3/2} - 8(9) + 15 = 54 - 72 + 15 = -3$

When $t = 9$, $a = 1 > 0$ (from (a)) and $v = -3 < 0$,

so v is increasing towards zero, i.e. $|v|$ is getting smaller.

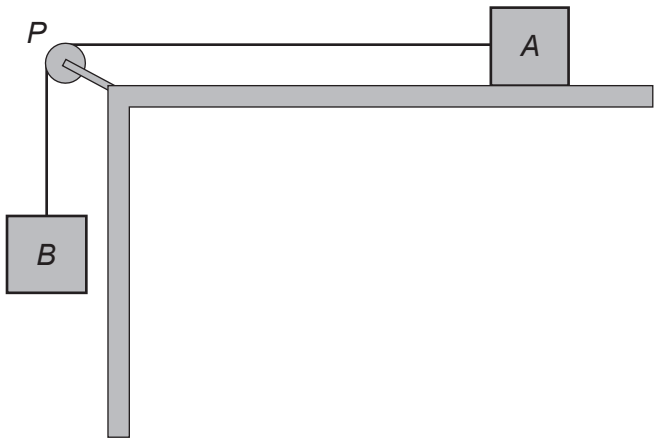
$\therefore$ The speed is decreasing.

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

9. The diagram below shows an object *A*, of mass $2m$ kg, lying on a horizontal table. It is connected to another object *B*, of mass m kg, by a light inextensible string, which passes over a smooth pulley *P*, fixed at the edge of the table. Initially, object *A* is held at rest so that object *B* hangs freely with the string taut.



Object *A* is then released.

(a) When object *B* has moved downwards a vertical distance of 0·4 m, its speed is 1·2 ms^{-1}. Use a formula for motion in a straight line with constant acceleration to show that the magnitude of the acceleration of *B* is 1·8 ms^{-2}. [2]

$v^2 = u^2 + 2as$, with $u = 0$, $v = 1.2$, $s = 0.4$

$1.2^2 = 0^2 + 2a(0.4)$

$1.44 = 0.8a$

$a = \frac{1.44}{0.8} = 1.8 \text{ m s}^{-2}$ as required

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) During the motion, object *A* experiences a constant resistive force of 22 N. Find the value of m and hence determine the tension in the string. [6]

Apply N2L to A and B separately ($a = 1.8$ m s^{-2}):

A (horizontally): $T - 22 = 2m \times 1.8 \Rightarrow T - 22 = 3.6m$

B (vertically down): $mg - T = m \times 1.8 \Rightarrow 9.8m - T = 1.8m$

$T = 8m$

Eliminating T: $8m - 22 = 3.6m$

$4.4m = 22$

$m = 5$

$T = 8 \times 5 = 40$ N

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(c) What assumption did the word 'inextensible' in the description of the string enable you to make in your solution? [1]

The inextensible string enables me to assume that the magnitude of the acceleration of A and B is equal (and so is their speed).

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

10. A car, starting from rest at a point *A*, travels along a straight horizontal road towards a point *B*. The distance between points *A* and *B* is 1·9 km. Initially, the car accelerates uniformly for 12 seconds until it reaches a speed of 26 ms^{-1}. The car continues at 26 ms^{-1} for 1 minute, before decelerating at a constant rate of 0·75 ms^{-2} until it passes the point *B*.

(a) Show that the car travels 156 m while it is accelerating. [2]

Distance while accelerating: $s = \frac{1}{2}(u + v)t$, $u = 0$, $v = 26$, $t = 12$

$s = \frac{1}{2}(0 + 26) \times 12$

$s = 156$ m as required

(b) (i) Work out the distance travelled by the car while travelling at a constant speed. [1]

(ii) Hence calculate the length of time for which the car is decelerating until it passes the point *B*. [5]

(i) Distance at constant speed $= 26 \times 60 = 1560$ m

(ii) Distance while decelerating $= 1900 - 1560 - 156 = 184$ m

$s = ut + \frac{1}{2}at^2$, $s = 184$, $u = 26$, $a = -0.75$

$184 = 26t + \frac{1}{2}(-0.75)t^2$

$184 = 26t - 0.375t^2$

$0.375t^2 - 26t + 184 = 0$

$0.75t^2 - 52t + 368 = 0$

$(t - 8)(0.75t - 46) = 0$

$t = 8$ or $t = \frac{184}{3} = 61.33...$

$t = \frac{184}{3}$ is rejected: the car would have stopped ($v = 0$) at

$t = \frac{26}{0.75} = 34.7$ s, before then, so it is impossible.

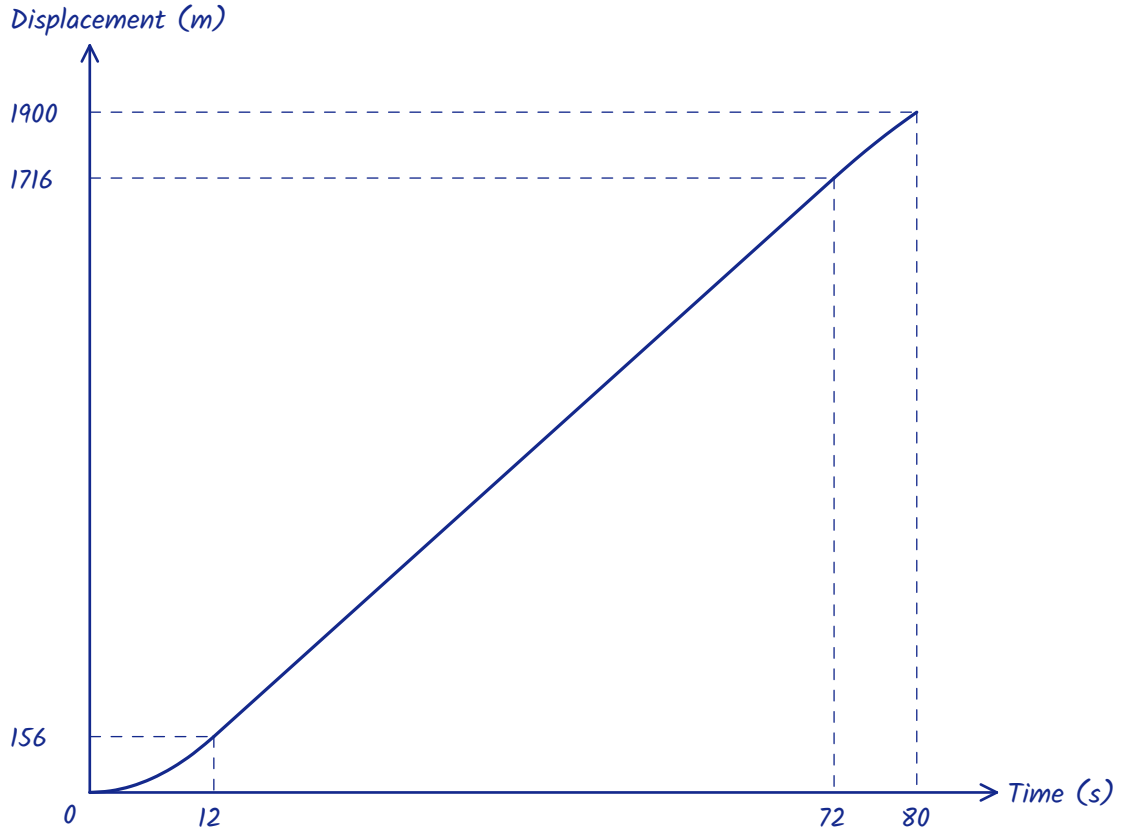
© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

∴ Time decelerating = 8 s

(c) Sketch a displacement-time graph for the motion of the car between *A* and *B*. [3]



END OF PAPER

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk