Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

2300U10-1

S24-2300U10-1

WEDNESDAY, 15 MAY 2024 – MORNING

MATHEMATICS – AS unit 1
PURE MATHEMATICS A

2 hours 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	4	
2	3	
3	3	
4	3	
5	4	
6	7	
7	11	
8	4	
9	9	
10	6	
11	4	
12	10	
13	8	
14	8	
15	7	
16	10	
17	7	
18	12	
Total	120	

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. Given that $y = 12\sqrt{x} - \frac{27}{x} + 4$, find the value of $\frac{dy}{dx}$ when $x = 9$. [4]

$y = 12x^{1/2} - 27x^{-1} + 4$

$\frac{dy}{dx} = 12 \times \frac{1}{2}x^{-1/2} - 27 \times (-1)x^{-2}$

$\frac{dy}{dx} = 6x^{-1/2} + 27x^{-2} = \frac{6}{\sqrt{x}} + \frac{27}{x^2}$

When $x = 9$: $\frac{dy}{dx} = \frac{6}{3} + \frac{27}{81} = 2 + \frac{1}{3} = \frac{7}{3}$

2. Find all values of θ in the range $0° < \theta < 180°$ that satisfy the equation

$$2\sin 2\theta = 1 .$$ [3]

$\sin 2\theta = \frac{1}{2}$, $\quad 0° < 2\theta < 360°$

$2\theta = 30° \quad$ or $\quad 2\theta = 180° - 30° = 150°$

$\theta = 15°, \quad 75°$

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3. Find $\int\left(5x^{\frac{1}{4}}+3x^{-2}-2\right)\mathrm{d}x$. [3]

$\int \left(5x^{1/4} + 3x^{-2} - 2\right) dx = \frac{5x^{5/4}}{5/4} + \frac{3x^{-1}}{-1} - 2x + C$

$= 4x^{5/4} - 3x^{-1} - 2x + C$

4. Given that n is an integer such that $1 \leqslant n \leqslant 6$, use proof by exhaustion to show that n^2-2 is not divisible by 3. [3]

$n = 1$: $n^2 - 2 = 1 - 2 = -1$, not divisible by 3

$n = 2$: $n^2 - 2 = 4 - 2 = 2$, not divisible by 3

$n = 3$: $n^2 - 2 = 9 - 2 = 7$, not divisible by 3

$n = 4$: $n^2 - 2 = 16 - 2 = 14$, not divisible by 3

$n = 5$: $n^2 - 2 = 25 - 2 = 23$, not divisible by 3

$n = 6$: $n^2 - 2 = 36 - 2 = 34$, not divisible by 3

All cases checked, so $n^2 - 2$ is not divisible by 3 for any integer n with $1 \leqslant n \leqslant 6$.

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5. A triangle ABC has sides $AB = 6\,\text{cm}$, $BC = 11\,\text{cm}$ and $AC = 13\,\text{cm}$. Calculate the area of the triangle. [4]

Let α = angle BAC (opposite BC). Cosine rule:

$11^2 = 6^2 + 13^2 - 2 \times 6 \times 13 \times \cos\alpha$

$121 = 205 - 156\cos\alpha$

$\cos\alpha = \frac{84}{156} = \frac{7}{13}$

$\alpha = 57.421...°$

Area $= \frac{1}{2} \times AB \times AC \times \sin\alpha = \frac{1}{2} \times 6 \times 13 \times \sin 57.421...°$

$= 32.863...$

$= 32.9\ \text{cm}^2$ (3 s.f.)

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6. (a) Find the exact value of x that satisfies the equation

$$\frac{7x^{\frac{5}{4}}}{x^{\frac{1}{2}}} = \sqrt{147}\,.$$ [4]

$7x^{5/4 - 1/2} = \sqrt{147}$

$7x^{3/4} = \sqrt{(49 \times 3)} = 7\sqrt{3}$

$x^{3/4} = \sqrt{3} = 3^{1/2}$

$x = (3^{1/2})^{4/3}$

$x = 3^{2/3} \quad (= \sqrt[3]{9})$

(b) Show that $\dfrac{(8x-18)}{(2\sqrt{x}-3)}$, where $x \neq \dfrac{9}{4}$, may be written as $2(2\sqrt{x}+3)$. [3]

$\dfrac{8x - 18}{2\sqrt{x} - 3} = \dfrac{(8x - 18)(2\sqrt{x} + 3)}{(2\sqrt{x} - 3)(2\sqrt{x} + 3)}$

$= \dfrac{(8x - 18)(2\sqrt{x} + 3)}{4x - 9}$

$= \dfrac{2(4x - 9)(2\sqrt{x} + 3)}{4x - 9}$

$= 2(2\sqrt{x} + 3)$ as required $(4x - 9 \neq 0$ as $x \neq \dfrac{9}{4})$

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7. (a) The line L_1 passes through the points $A(-3, 0)$ and $B(1, 4)$. Determine the equation of L_1. [3]

Gradient of $L_1 = \frac{4-0}{1-(-3)} = \frac{4}{4} = 1$

$y - 0 = 1(x - (-3))$

$y = x + 3$

(b) The line L_2 has equation $y = 3x - 3$.

(i) Given that L_1 and L_2 intersect at the point C, find the coordinates of C.

(ii) The line L_2 crosses the x-axis at the point D. Show that the coordinates of D are (1, 0). [4]

(i) $x + 3 = 3x - 3$

$6 = 2x \Rightarrow x = 3$

$y = 3 + 3 = 6$

C is the point $(3, 6)$

(ii) At D, $y = 0$: $0 = 3x - 3$

$3x = 3 \Rightarrow x = 1$

so D is the point $(1, 0)$ as required

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(c) Calculate the area of triangle *ACD*. [2]

A(−3, 0) and D(1, 0) both lie on the x-axis: AD = 1 − (−3) = 4

Height of triangle = y-coordinate of C = 6

Area of ACD = $\frac{1}{2}$ × 4 × 6

= 12

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(d) Determine the angle *ACD*. [2]

L_1 makes an angle of $\tan^{-1}(1) = 45°$ with the x-axis

L_2 makes an angle of $\tan^{-1}(3) = 71.565...°$ with the x-axis

Angle $ACD = 71.565...° - 45°$

$= 26.565...°$

$= 26.57°$

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8. Prove that $x-10<x^2-5x$ for all real values of x. [4]

$x - 10 < x^2 - 5x \quad \Leftrightarrow \quad x^2 - 6x + 10 > 0$

$x^2 - 6x + 10 = (x - 3)^2 - 9 + 10$

$= (x - 3)^2 + 1$

$(x - 3)^2 \geq 0$ for all real x, so $(x - 3)^2 + 1 \geq 1 > 0$

(the minimum value of $x^2 - 6x + 10$ is 1, when $x = 3$)

Hence $x^2 - 6x + 10 > 0$, so $x - 10 < x^2 - 5x$ for all real x.

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9. (a) Write down the binomial expansion of $(2-x)^6$ up to and including the term in x^2. [3]

$(2-x)^6 = 2^6 + 6 \times 2^5 \times (-x) + \frac{6 \times 5}{2 \times 1} \times 2^4 \times (-x)^2 + \ldots$

$= 64 - 192x + 240x^2 - \ldots$

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(b) Given that

$$(1+ax)(2-x)^6 \equiv 64+bx+336x^2+\ldots\ ,$$

find the values of the constants a, b. [6]

$(1 + ax)(2 - x)^6 = (1 + ax)(64 - 192x + 240x^2 - \ldots)$

$= 64 - 192x + 240x^2 + 64ax - 192ax^2 + \ldots$

$= 64 + (64a - 192)x + (240 - 192a)x^2 + \ldots$

$\equiv 64 + bx + 336x^2 + \ldots$

Coefficients of x^2: $240 - 192a = 336$

$-192a = 96$

$a = -\frac{1}{2}$

Coefficients of x: $b = 64a - 192$

$b = 64 \times \left(-\frac{1}{2}\right) - 192 = -32 - 192$

$b = -224$

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10. Water is being emptied out of a sink. The depth of water, y cm, at time t seconds, may be modelled by

$$y = t^2 - 14t + 49 \qquad 0 \leqslant t \leqslant 7.$$

(a) Find the value of t when the depth of water is 25 cm. [3]

$t^2 - 14t + 49 = 25$

$t^2 - 14t + 24 = 0$

$(t - 2)(t - 12) = 0$

$t = 2$ or $t = 12$

$t = 12$ is rejected since $0 \leqslant t \leqslant 7$, so $t = 2$

(b) Find the rate of decrease of the depth of water when $t = 3$. [3]

$\frac{dy}{dt} = 2t - 14$

When $t = 3$: $\frac{dy}{dt} = 2 \times 3 - 14 = -8$

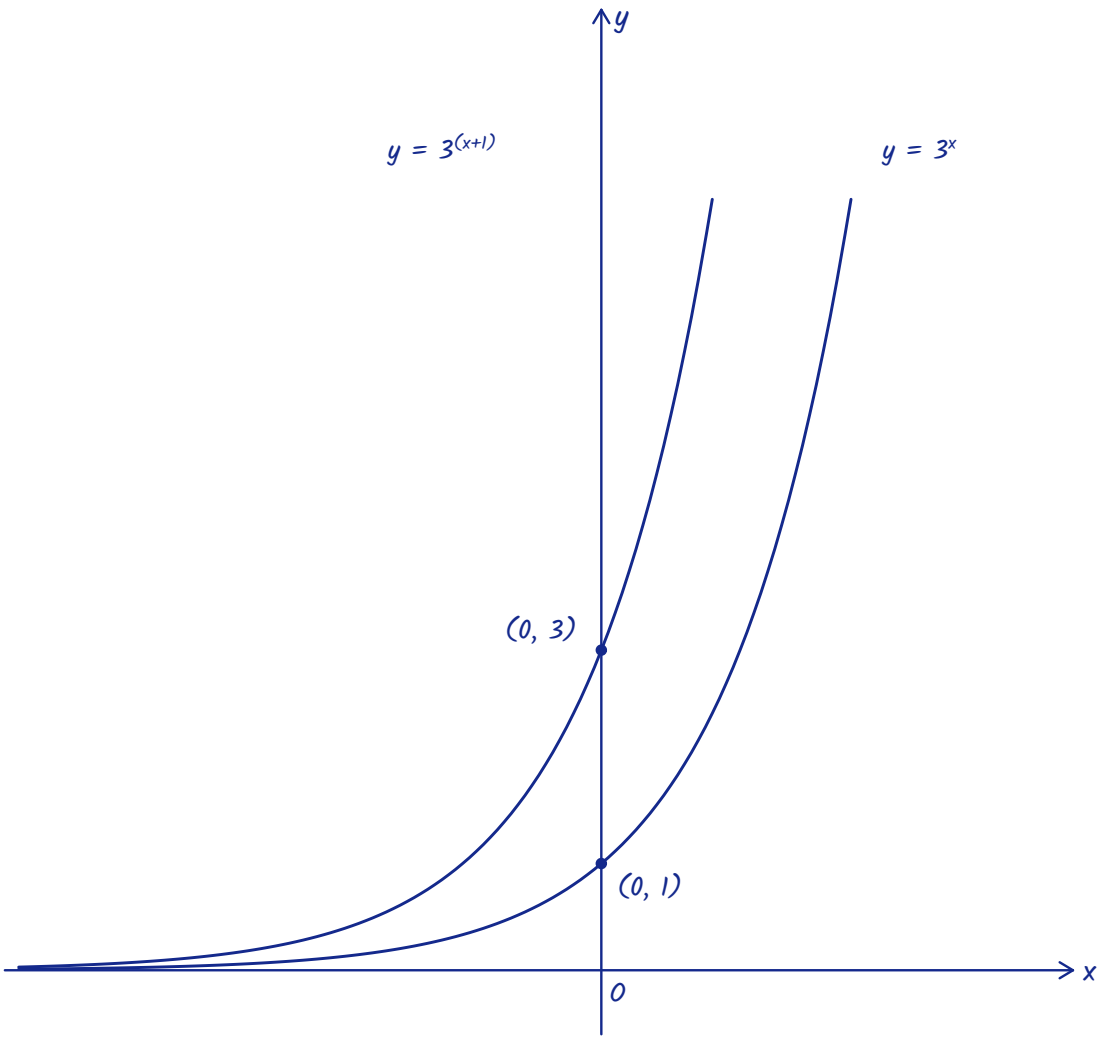
Rate of decrease of the depth = 8 cm s^{-1}

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11. (a) Sketch the graph of $y = 3^x$. Clearly label the coordinates of the point where the graph crosses the y-axis. [2]



The graph crosses the y-axis at (0, 1), since $3^0 = 1$

(b) On the **same set of axes**, sketch the graph of $y = 3^{(x+1)}$, clearly labelling the coordinates of the point where the graph crosses the y-axis. [2]

$y = 3^{(x+1)} = 3 \times 3^x$; crosses the y-axis at (0, 3)

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12. A curve C has equation $y = -x^3 + 12x - 20$.

(a) Find the coordinates of the stationary points of C and determine their nature. [7]

$\frac{dy}{dx} = -3x^2 + 12$

Stationary points: $\frac{dy}{dx} = 0 \Rightarrow -3x^2 + 12 = 0$

$x^2 = 4 \Rightarrow x = 2$ or $x = -2$

$x = 2$: $y = -8 + 24 - 20 = -4$

$x = -2$: $y = 8 - 24 - 20 = -36$

$\frac{d^2y}{dx^2} = -6x$

$x = 2$: $\frac{d^2y}{dx^2} = -12 < 0$, so $(2, -4)$ is a maximum point

$x = -2$: $\frac{d^2y}{dx^2} = 12 > 0$, so $(-2, -36)$ is a minimum point

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(b) Determine the range of values of x for which the curve is decreasing.
Give your answer in set notation. [3]

Decreasing when $\frac{dy}{dx} < 0$: $-3x^2 + 12 < 0$

$x^2 > 4 \Rightarrow x < -2$ or $x > 2$

$\{x : x < -2\} \cup \{x : x > 2\}$

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13. The position vectors of the points *A* and *B*, relative to a fixed origin *O*, are given by

$$\mathbf{a} = 4\mathbf{i} + 7\mathbf{j}, \qquad \mathbf{b} = \mathbf{i} + 3\mathbf{j},$$

respectively.

(a) Find the vector **AB**. [2]

$AB = b - a = (i + 3j) - (4i + 7j)$

$AB = -3i - 4j$

(b) Determine the distance between the points *A* and *B*. [2]

$AB = \sqrt{((-3)^2 + (-4)^2)} = \sqrt{25}$

Distance $AB = 5$

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(c) The position vector of the point C is given by $\mathbf{c} = -2\mathbf{i} + 5\mathbf{j}$. The point D is such that the distance between C and D is equal to the distance between A and B, and CD is parallel to AB. Find the possible position vectors of the point D. [4]

CD is parallel to AB and |CD| = |AB|, so CD = AB or CD = −AB

d = c + AB = (−2i + 5j) + (−3i − 4j) = −5i + j

or

d = c − AB = (−2i + 5j) − (−3i − 4j) = i + 9j

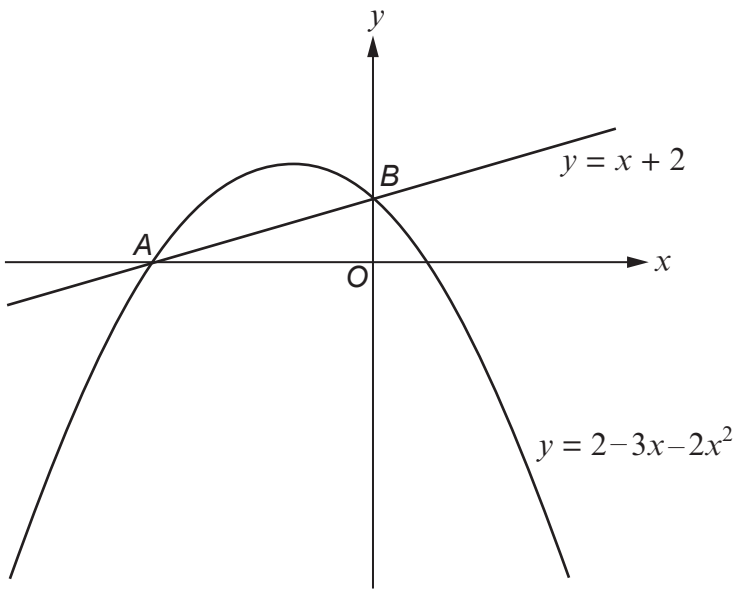
Possible position vectors of D: −5i + j or i + 9j

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14. The diagram below shows a sketch of the curve *C* with equation $y = 2 - 3x - 2x^2$ and the line *L* with equation $y = x + 2$. The curve and the line intersect the coordinate axes at the points *A* and *B*.



(a) Write down the coordinates of *A* and *B*. [2]

A: y = x + 2 meets the x-axis at x = −2, A(−2, 0)

B: x = 0, y = 2, B(0, 2)

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(b) Calculate the area enclosed by C and L. [6]

Area under $C = \int_{-2}^{0} (2 - 3x - 2x^2)\, dx$

$= \left[2x - \frac{3}{2}x^2 - \frac{2}{3}x^3 \right]_{-2}^{0}$

$= 0 - \left(-4 - 6 + \frac{16}{3} \right)$

$= \frac{14}{3}$

Area of triangle AOB under $L = \frac{1}{2} \times 2 \times 2 = 2$

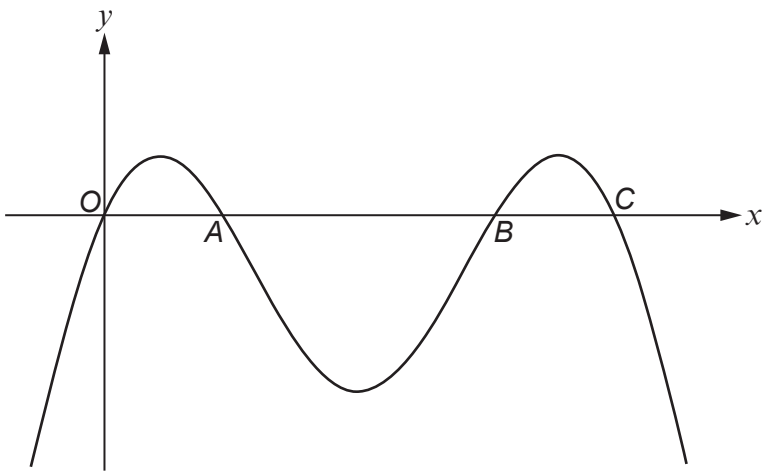
Area enclosed by C and $L = \frac{14}{3} - 2 = \frac{8}{3}$

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15. The diagram shows a sketch of part of the curve with equation $y = 2\sin x + 3\cos^2 x - 3$.
The curve crosses the x-axis at the points O, A, B and C.



Find the value of x at each of the points A, B and C. [7]

$2\sin x + 3\cos^2 x - 3 = 0$

$2\sin x + 3(1 - \sin^2 x) - 3 = 0$

$2\sin x - 3\sin^2 x = 0 \quad \Rightarrow \quad 3\sin^2 x - 2\sin x = 0$

$\sin x(3\sin x - 2) = 0$

$\sin x = 0 \quad$ or $\quad \sin x = \frac{2}{3}$

$\sin x = \frac{2}{3}$: $\quad x = 41.810...° \quad$ or $\quad x = 180° - 41.810...° = 138.189...°$

$\sin x = 0$: $\quad x = 0° \ (\text{at } O) \quad$ or $\quad x = 180°$

A: $\quad x = 41.81°$

B: $\quad x = 138.19°$

C: $\quad x = 180°$

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16. (a) Find the range of values of k for which the quadratic equation $x^2 - kx + 4 = 0$ has no real roots. [4]

No real roots ⇒ discriminant < 0

$b^2 - 4ac = (-k)^2 - 4 \times 1 \times 4 = k^2 - 16$

$k^2 - 16 < 0 \Rightarrow (k - 4)(k + 4) < 0$

Critical values: $k = -4$, $k = 4$

$-4 < k < 4$

(b) Determine the coordinates of the points of intersection of the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$. [4]

$x^2 - 3x + 4 = x + 16$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2$ or $x = 6$

$x = -2$: $y = -2 + 16 = 14$, $x = 6$: $y = 6 + 16 = 22$

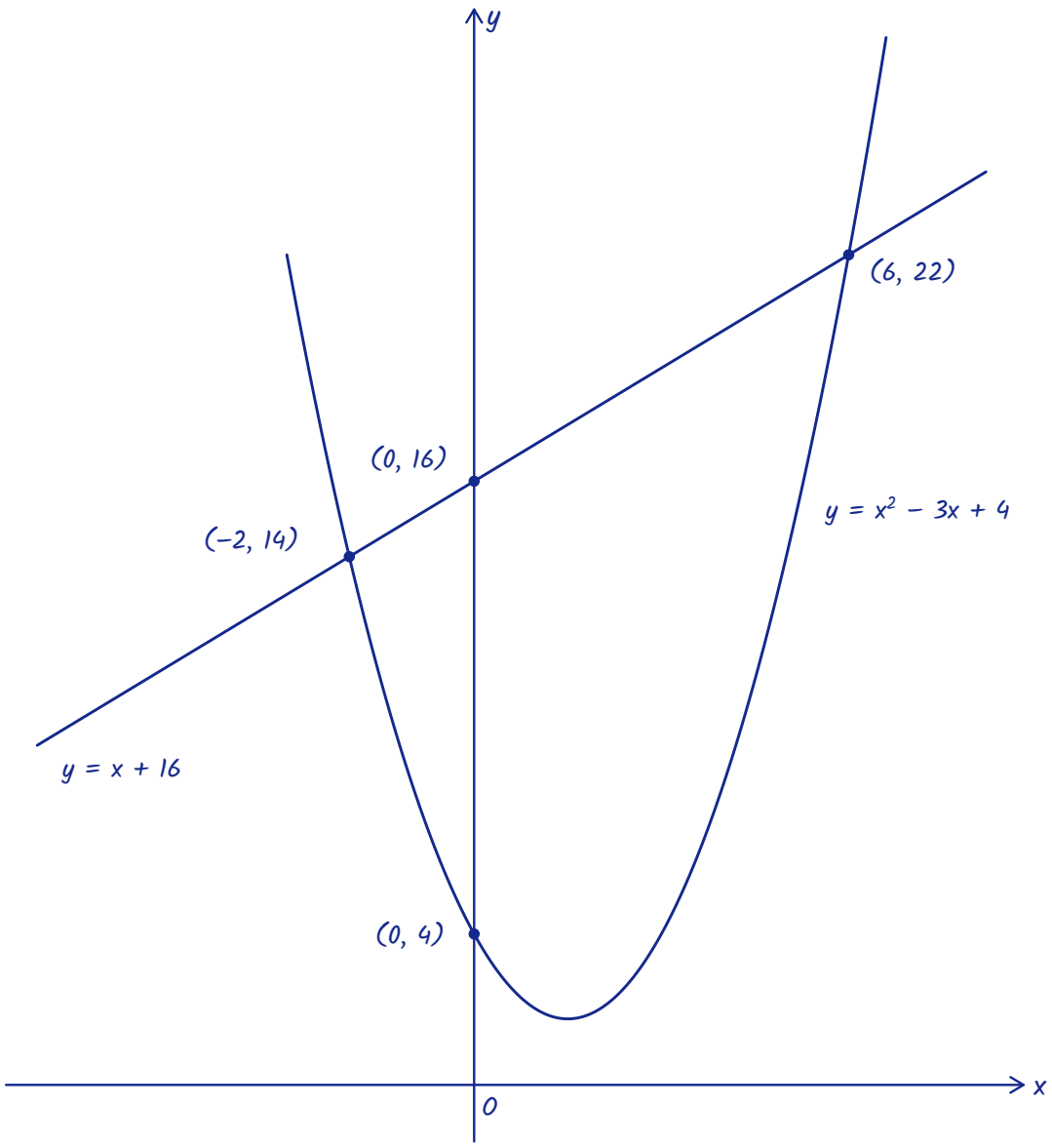
Points of intersection: $(-2, 14)$ and $(6, 22)$

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(c) Using the information obtained in parts (a) and (b), sketch the graphs of $y = x^2 - 3x + 4$ and $y = x + 16$ on the same set of axes. [2]

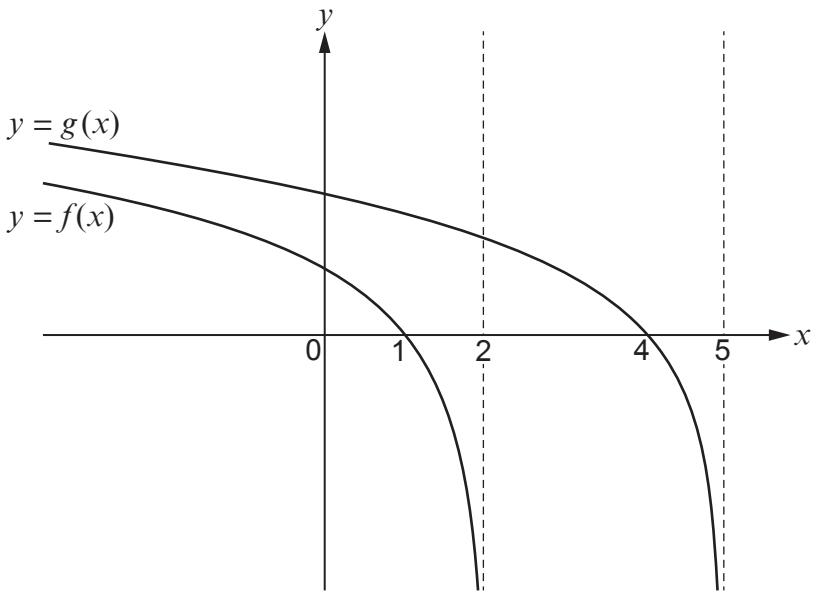


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17. A function f is defined by $f(x) = \log_{10}(2-x)$. Another function g is defined by $g(x) = \log_{10}(5-x)$. The diagram below shows a sketch of the graphs of $y = f(x)$ and $y = g(x)$.



(a) The point $(c, 1)$ lies on $y = f(x)$. Find the value of c. [2]

$1 = \log_{10}(2 - c)$

$2 - c = 10^1 = 10$

$c = -8$

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(b) A point P lies on $y = f(x)$ and has x-coordinate α. Another point Q lies on $y = g(x)$ and also has x-coordinate α. The distance between P and Q is 1·2 units. Find the value of α, giving your answer correct to three decimal places. [5]

$P(\alpha, \log_{10}(2 - \alpha))$, $Q(\alpha, \log_{10}(5 - \alpha))$, Q is above P:

$\log_{10}(5 - \alpha) - \log_{10}(2 - \alpha) = 1.2$

$\log_{10}\left(\frac{5 - \alpha}{2 - \alpha}\right) = 1.2$

$\frac{5 - \alpha}{2 - \alpha} = 10^{1.2}$ $(= 15.8489...)$

$5 - \alpha = 10^{1.2}(2 - \alpha)$

$\alpha(10^{1.2} - 1) = 2 \times 10^{1.2} - 5$

$\alpha = \frac{2 \times 10^{1.2} - 5}{10^{1.2} - 1} = \frac{26.6978...}{14.8489...} = 1.79797...$

$\alpha = 1.798$ (3 d.p.)

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18. (a) A circle C has centre (–3, –1) and radius $\sqrt{5}$. Show that the equation of C can be written as $x^2 + y^2 + 6x + 2y + 5 = 0$. [2]

$(x - (-3))^2 + (y - (-1))^2 = (\sqrt{5})^2$

$(x + 3)^2 + (y + 1)^2 = 5$

$x^2 + 6x + 9 + y^2 + 2y + 1 = 5$

$x^2 + y^2 + 6x + 2y + 5 = 0$ as required

(b) (i) Find the equations of the tangents to C that pass through the origin O. [6]

(ii) Determine the coordinates of the points where the tangents touch the circle. [4]

(i) Tangents through O have equations $y = mx$

Substitute: $x^2 + m^2x^2 + 6x + 2mx + 5 = 0$

$(1 + m^2)x^2 + 2(3 + m)x + 5 = 0$

Tangent $\Rightarrow$ repeated root $\Rightarrow$ discriminant = 0:

$4(3 + m)^2 - 4 \times 5 \times (1 + m^2) = 0$

$9 + 6m + m^2 - 5 - 5m^2 = 0$

$-4m^2 + 6m + 4 = 0 \Rightarrow 2m^2 - 3m - 2 = 0$

$(2m + 1)(m - 2) = 0$

$m = -\frac{1}{2}$ or $m = 2$

Tangents: $y = -\frac{1}{2}x$ and $y = 2x$

(ii) $m = -\frac{1}{2}$: $\left(1 + \frac{1}{4}\right)x^2 + 2\left(3 - \frac{1}{2}\right)x + 5 = 0$

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$\frac{5}{4}x^2 + 5x + 5 = 0 \quad \Rightarrow \quad x^2 + 4x + 4 = 0$

$(x + 2)^2 = 0 \quad \Rightarrow \quad x = -2$

$y = -\frac{1}{2} \times (-2) = 1$: point $(-2, 1)$

$m = 2$: $5x^2 + 10x + 5 = 0 \quad \Rightarrow \quad x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0 \quad \Rightarrow \quad x = -1$

$y = 2 \times (-1) = -2$: point $(-1, -2)$

Points of contact: $(-2, 1)$ and $(-1, -2)$

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