GCE AS/A LEVEL

2300U20-1

S23-2300U20-1

THURSDAY, 25 MAY 2023 – AFTERNOON

MATHEMATICS – AS unit 2
APPLIED MATHEMATICS A

1 hour 45 minutes

2300U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0 1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 75.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Additional Formulae for 2023

Laws of Logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$

$$k \log_a x \equiv \log_a \left(x^k\right)$$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y \, \mathrm{d}x$

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

Section A: Statistics

0 1 The events A and B are such that $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}2$ and $P(A \cap B) = 0{\cdot}1$.

a) Find $P(A \cup B)$. [2]

The event C is such that $P(C) = 0{\cdot}45$. The events B and C are independent.

b) Find $P(B \cap C)$. [2]

The events A and C are mutually exclusive.

c) Draw a Venn diagram to show the events A, B and C. Include the probability for each distinct region on your diagram. [4]

d) Find $P((B \cup C)')$. [2]

0 2 A card is selected at random from a pack of 20 cards numbered 1 to 20. What distribution models the number on the card? Give a reason for your answer. [2]

0 3 The probability distribution of a discrete random variable X is given below.

x	4	8	16
$P(X = x)$	0·3	0·5	0·2

Two independent values of X are generated. Find the probability that the first value of X is less than or equal to the second value of X. [4]

TURN OVER

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0 4 Sources of the radioactive element caesium each emit radioactive particles at an average rate of 4 per second. The numbers of emissions are modelled by a Poisson distribution.

a) Find the probability that, in a 3-second interval, one such source will emit at least 10 radioactive particles, giving your answer correct to 4 decimal places. [3]

b) A random sample of 9 of these sources is taken. Calculate the probability that at most 5 of them will emit at least 10 radioactive particles in a 3-second interval. [3]

0 5 A doctor knows that 34% of his patients currently take too little exercise. He believes he can reduce this proportion by putting patients through a behaviour change programme.

To test his belief, he takes a random sample of 156 of his patients and puts each of them through the programme.

It is found that 40 of these patients take too little exercise following the behaviour change programme.

a) The doctor initially considers using a two-tailed test. Explain why this is not appropriate. [1]

b) The doctor then decides to use a one-tailed test.

i) Explain what is meant by a *p*-value in the context of this question. [1]

ii) Test the doctor's belief at the 1% level of significance, stating clearly the conclusion that the doctor should reach. [6]

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0 6 Poppi and Clare are investigating Welsh language skills in Wales. Poppi randomly selects a sample of 20 different students from her school and asks them to state their Welsh language skills according to the categories in the 2011 census. She numbers the students in her school from 001 to 578. She uses a random number generator to select her sample. The first 6 numbers generated are shown below.

195 752 023 195 432 271

a) Write down the numbers of the first 4 students in Poppi's sample. [1]

Clare selects a random sample of 20 students from a different school.

b) Give two reasons why Clare's conclusion about Welsh language skills in Wales may be different from Poppi's conclusion. [2]

The data collected from the 2011 census for two regions in Wales are shown below.

		No skills in Welsh	Understand spoken Welsh only	Speak Welsh	Other skills in Welsh
Region A	Number of people	111 232	5517	13 103	4858
	Percentage	82·6	4·1	9·7	3·6
Region B	Number of people	102 153	4083	13 189	2744
	Percentage	83·7	3·3	10·8	2·2

On looking at these data, Poppi concludes that people in Region B have better Welsh language skills than people in Region A.

c) Without carrying out any calculations, use data from the table

i) to argue for Poppi's conclusion,

ii) to argue against Poppi's conclusion. [2]

d) Seven 4-year-old children who "Speak Welsh" were asked to concentrate on a task. The times, in minutes, until they lost concentration are given below.

7·2 5·4 7·4 4·6 13·2 8·4 7·7

i) Calculate the mean and standard deviation for these data. [3]

ii) For a similar group of 4-year-olds with "No skills in Welsh", the mean concentration time is 6·5 minutes and the standard deviation is 3·5 minutes. Compare the concentration times for these two groups of 4-year-olds. [2]

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Section B: Mechanics

0 7 A person is standing in a lift which is descending with a constant acceleration of $\frac{g}{10}\text{ms}^{-2}$. The lift is supported by a single cable.

a) The total mass of the lift and the person is 1200 kg. Find the tension in the lift cable. [3]

b) The mass of the person is M kg. Given that the magnitude of the reaction of the floor of the lift on the person is $63g$ N, find the value of M. [3]

0 8 A particle *P* moves in a straight line such that its displacement, s metres, at time t seconds, relative to a fixed point is given by

$$s = 4t^2 - t^3 \qquad \text{for } t \geqslant 0.$$

Find the speed of *P* when it returns to its starting position. [5]

0 9 Three forces $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$ are such that

$$\mathbf{F} = (2\mathbf{i} - 13\mathbf{j})\text{N}, \qquad \mathbf{G} = (-6\mathbf{i} + 8\mathbf{j})\text{N}, \qquad \mathbf{H} = (12\mathbf{i} + k\mathbf{j})\text{N},$$

where k is a constant. The resultant of the three forces, $\mathbf{R}$, has magnitude 17 N.

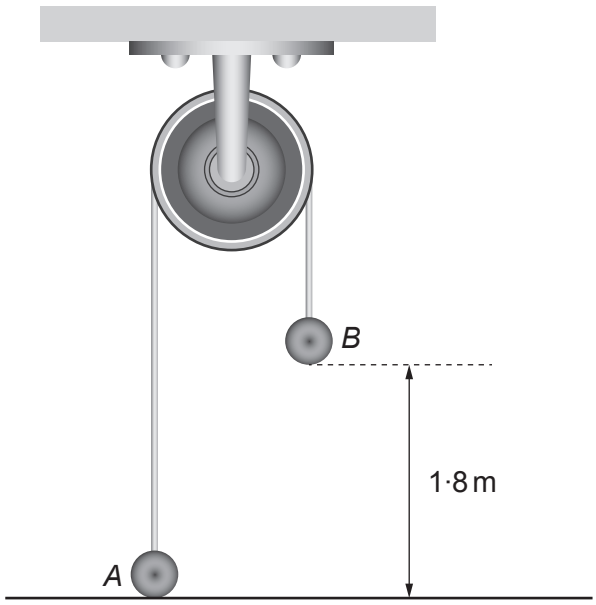
a) Find the two possible values of the constant k. [4]

b) For the larger of the two values of k, calculate the angle that $\mathbf{R}$ makes with the $\mathbf{i}$ vector. [2]

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1 0 The diagram below shows two objects *A* and *B* connected by a light inextensible string passing over a smooth fixed pulley. Object *A* has mass 4·5 kg and object *B* has mass 7·5 kg. Initially, object *A* is held on a horizontal surface so that object *B* hangs freely with the string taut. The distance of object *B* above the horizontal surface is 1·8 m.



a) Write down the tension in the string connecting the two objects. [1]

b) Object *A* is then released.

i) Calculate the magnitude of the acceleration of *A*.

ii) Hence, find the time taken for the objects to be at the same height above the horizontal surface.

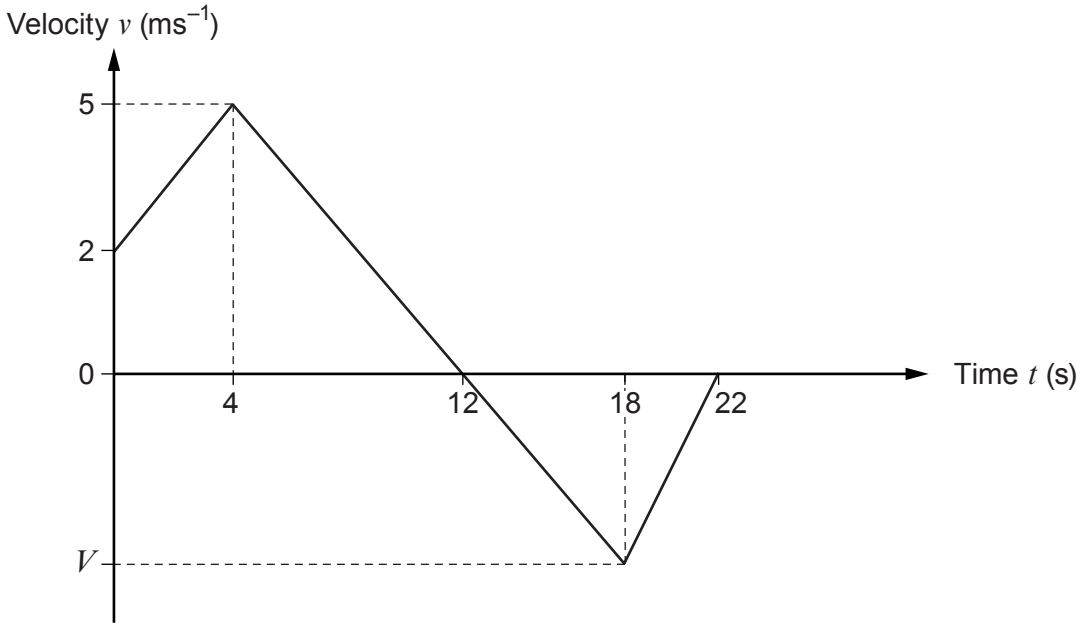
iii) In reality, there will be air resistance acting on the objects as they move. State how this fact will affect your answer to (ii). [9]

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1 1 A particle, which is initially at the point *A*, travels in a straight line during the interval $0 \leqslant t \leqslant 22$, where t is the time in seconds. The diagram below shows a velocity-time graph for the motion of the particle. The velocity of the particle when $t = 18$ is V ms^{-1}.



a) Find the acceleration of the particle when $t = 1$. [2]

b) Calculate the distance travelled by the particle during the interval $0 \leqslant t \leqslant 12$. [2]

c) When $t = 22$, the displacement of the particle from *A* is 14 m. Determine the value of V. [3]

d) Write down the time interval during which the particle is moving towards *A* and its speed is increasing. [1]

END OF PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.3 + 0.2 - 0.1$

$P(A \cup B) = 0.4$

1 (b) B and C independent: $P(B \cap C) = P(B) \times P(C)$

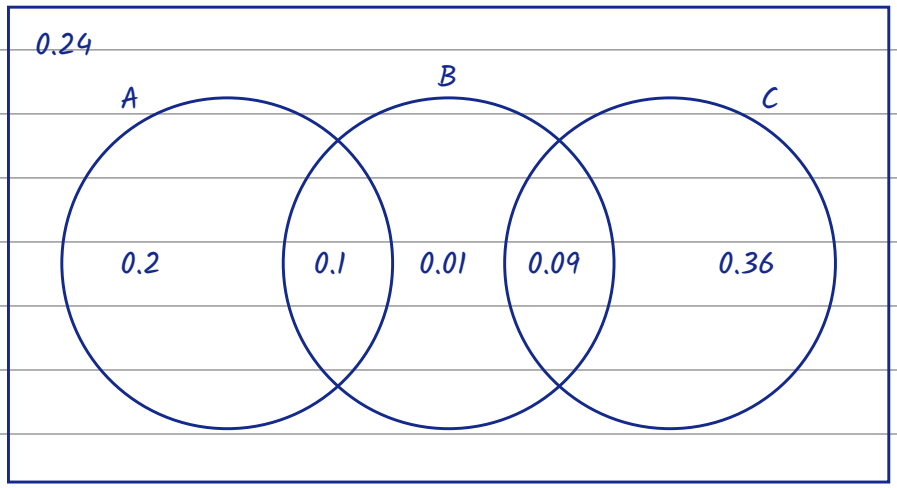
$P(B \cap C) = 0.2 \times 0.45$

$P(B \cap C) = 0.09$

1 (c) $P(A \cap C) = 0$ (mutually exclusive). B only $= 0.2 - 0.1 - 0.09 = 0.01$

A only $= 0.3 - 0.1 = 0.2$, C only $= 0.45 - 0.09 = 0.36$,

outside $= 1 - (0.2 + 0.1 + 0.01 + 0.09 + 0.36) = 0.24$



1 (d) From the Venn diagram, $P((B \cup C)') = 0.2 + 0.24$

$P((B \cup C)') = 0.44$

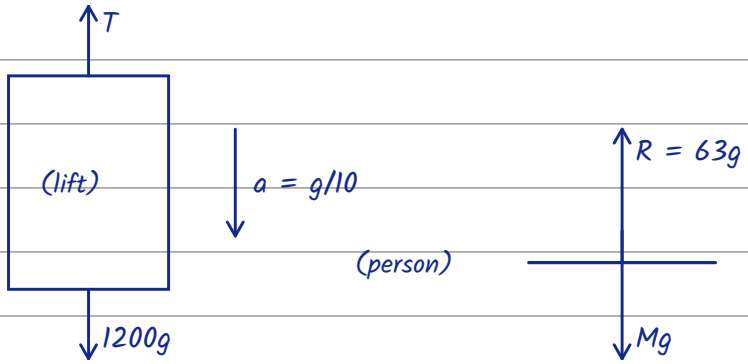
Question number	
2	A discrete uniform distribution. Because each card has the same probability (1/20) of being selected.
3	$P(X_1 \leq X_2) = P(4, 4) + P(4, 8) + P(4, 16) + P(8, 8) + P(8, 16) + P(16, 16)$ $= 0.3^2 + 0.3 \times 0.5 + 0.3 \times 0.2 + 0.5^2 + 0.5 \times 0.2 + 0.2^2$ $= 0.09 + 0.15 + 0.06 + 0.25 + 0.1 + 0.04$ $= 0.69$
4 (a)	Let X be the number of emissions in a 3-second interval. $X \sim Po(4 \times 3) = Po(12)$ $P(X \geqslant 10) = 1 - P(X \leqslant 9)$ $= 1 - 0.2424$ $= 0.7576$ (4 d.p.)
4 (b)	Let Y be the number of the 9 sources that emit at least 10 particles in a 3-second interval. $Y \sim B(9, 0.7576)$ $P(Y \leqslant 5) = 0.15180... = 0.1518$
5 (a)	The doctor is only interested in a reduction in the proportion of his patients who take too little exercise, so the test should be one-tailed.
5 (b)	(i) The p-value is the probability that 40 or fewer of the 156 patients take too little exercise after the programme, given that the proportion is still 0.34 (i.e. given H_0 is true). (ii) Let p be the proportion of patients who take too little exercise. H_0: $p = 0.34$ H_1: $p < 0.34$ Let X be the number of patients in the sample who take too little exercise. Under H_0, $X \sim B(156, 0.34)$

Question number	
5 (b) cont.	p-value = $P(X \leq 40) = 0.015488...$ Since $0.0155 > 0.01$, there is insufficient evidence to reject H_0. There is insufficient evidence at the 1% level to suggest that the behaviour change programme has reduced the proportion of patients who take too little exercise.
6 (a)	195, 023, 432, 271
6 (b)	Clare's school may be in a different area of Wales (e.g. one school may be Welsh medium and the other English medium). Different random samples can lead to different conclusions (the samples are small, only 20 students each).
6 (c)	(i) 10.8% of people in Region B speak Welsh compared with only 9.7% in Region A. (ii) 4.1% of people in Region A understand spoken Welsh only compared with 3.3% in Region B (and 3.6% have other skills in Welsh in A compared with 2.2% in B).
6 (d)	(i) $\Sigma x = 7.2 + 5.4 + 7.4 + 4.6 + 13.2 + 8.4 + 7.7 = 53.9$ Mean $= \bar{x} = 53.9/7 = 7.7$ minutes $\Sigma x^2 = 461.01$ $\sigma = \sqrt{(\Sigma x^2/n - \bar{x}^2)} = \sqrt{(461.01/7 - 7.7^2)}$ $\sigma = 2.5629... = 2.56$ minutes (ii) On average, the children who speak Welsh concentrated for longer (mean 7.7 min) than the children with no skills in Welsh (mean 6.5 min). The concentration times of the children with no skills in Welsh are more variable (less consistent), since $3.5 > 2.56$.

Question number

6 (d) cont. However, the samples are small, so these comparisons should not be relied on too heavily.

7 (a)



N2L for the lift and person, downwards positive:

$1200g - T = 1200a, \quad a = g/10$

$1200g - T = 1200 \times g/10 = 120g$

$T = 1080g = 10584\ N$

7 (b) N2L for the person only, downwards positive:

$Mg - R = Ma$

$Mg - 63g = M \times g/10$

$9.8M - 617.4 = 0.98M \quad \Rightarrow \quad (9/10)M = 63$

$M = 70$

8 Starting position at $t = 0$, $s = 0$. Returns when $s = 0$:

$4t^2 - t^3 = 0 \quad \Rightarrow \quad t^2(4 - t) = 0$

$t = 4$ ($t = 0$ is the start)

$v = ds/dt = 8t - 3t^2$

At $t = 4$: $v = 8(4) - 3(4)^2 = 32 - 48 = -16$

Speed $= 16\ ms^{-1}$

9 (a) $R = F + G + H = (2i - 13j) + (-6i + 8j) + (12i + kj)$

$R = 8i + (k - 5)j$

Question number

9 (a) cont.

$|R| = 17$: $\sqrt{(8^2 + (k-5)^2)} = 17$

$64 + (k-5)^2 = 289 \Rightarrow (k-5)^2 = 225$

$k - 5 = \pm 15$

$k = -10$ or $k = 20$

9 (b)

For $k = 20$: $R = 8i + 15j$

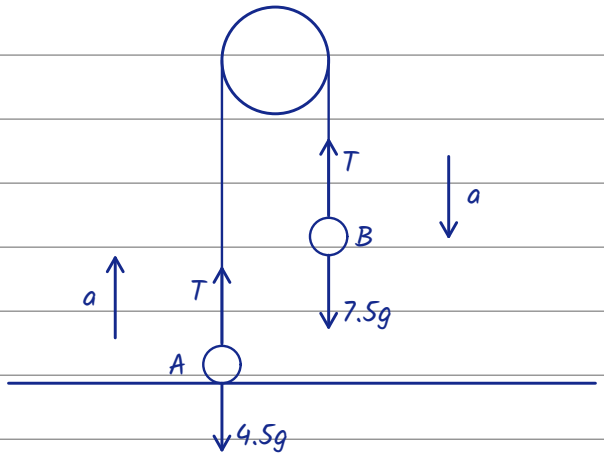
Let α be the angle R makes with i: $\tan\alpha = 15/8$

$\alpha = 61.927...° = 61.9°$

10 (a)

B is in equilibrium, so $T = 7.5g = 73.5$ N

10 (b)



(i) N2L for A (upwards): $T - 4.5g = 4.5a$

N2L for B (downwards): $7.5g - T = 7.5a$

Adding to eliminate T: $3g = 12a$

$a = g/4 = 2.45\ ms^{-2}$

(ii) Same height when A has risen 0.9 m and B has fallen 0.9 m.

$s = ut + \frac{1}{2}at^2$, $s = 0.9$, $u = 0$, $a = 2.45$

$0.9 = \frac{1}{2} \times 2.45 \times t^2 \Rightarrow t^2 = 0.73469...$

$t = 6/7 = 0.857$ s $(0.85714...)$

Question number	
10 (b) cont.	(iii) Air resistance would reduce the acceleration, so the time in (ii) would be greater.
11 (a)	Acceleration at $t = 1$ = gradient of the first line segment $= (5 - 2)/(4 - 0)$ $= 3/4 = 0.75\ ms^{-2}$
11 (b)	Distance = area under graph from $t = 0$ to $t = 12$ $= \frac{1}{2}(2 + 5)(4) + \frac{1}{2}(8)(5)$ $= 14 + 20 = 34\ m$
11 (c)	Area below the t-axis $= 34 - 14 = 20$ $\frac{1}{2} \times (22 - 12) \times (-V) = 20 \quad \Rightarrow \quad -5V = 20$ $V = -4$
11 (d)	$12 < t < 18$