GCE AS/A LEVEL

2300U20-1

Z22-2300U20-1

WEDNESDAY, 8 JUNE 2022 – AFTERNOON

MATHEMATICS – AS unit 2
APPLIED MATHEMATICS A

1 hour 45 minutes

2300U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid. You may use a pencil for graphs and diagrams only.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, for example, **0 1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 75.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

Section A: Statistics

0 1 The events A and B are such that

$$P(A) = 0{\cdot}3, \qquad P(B) = 0{\cdot}6, \qquad P(A \cup B) = 0{\cdot}82.$$

a) Determine whether or not A and B are independent. [4]

b) Find the probability that exactly one of A and B occurs. [3]

0 2 The probability distribution for X, the lifetime of a light bulb, in hours, is given below.

X	$256 \leqslant x < 259$	$259 \leqslant x < 262$	$262 \leqslant x < 265$	$265 \leqslant x < 267$	$267 \leqslant x < 300$
Probability	0·05	0·25	0·45	0·20	0·05

a) Suppose that a random sample of 40 light bulbs is tested, and a histogram is drawn of their lifetimes. Calculate the expected height of the bar for the interval $262 \leqslant x < 265$. [2]

b) Now suppose that the last two intervals are changed to $265 \leqslant x < 268$ and $268 \leqslant x < 300$. Explain why it is not possible to tell what will happen to the expected heights of the last two bars. [1]

c) Celyn collects a different random sample of 40 light bulbs to test. She draws a histogram of their lifetimes and finds that it is different to the histogram referred to in part (a). Should Celyn be concerned that the two histograms are different? [1]

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0 3 In a study, samples of soil were collected during the summer. Soil samples of dimensions 25 cm × 25 cm × 40 cm were collected for analysis. The study found that there were, on average, 11 earthworms per sample.

a) Explain briefly the conditions under which a Poisson distribution could be used to model the number of earthworms per sample. [1]

b) In July, pupils at a primary school are asked to dig a smaller hole, 25 cm × 25 cm × 10 cm, and to count the number of earthworms they find. Calculate the probability that the pupils find exactly 5 earthworms. [3]

c) In the autumn, the average number of earthworms per sample is greater than in the summer. The probability that, in the autumn, there are fewer than 13 earthworms in a soil sample of dimensions 25 cm × 25 cm × 40 cm is close to 36%. Find the mean number of earthworms, to the nearest whole number, per 25 cm × 25 cm × 40 cm soil sample in the autumn. [2]

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0 4 Jessica is studying the relationship between hip girth, h cm, and thigh girth, t cm, for American adults who are physically active. She takes a random sample of 11 people from a very large dataset which she has downloaded into a spreadsheet software package. The results are shown below.

h (cm)	98·6	112·1	97·9	110·2	89·2	111·7	87·0	94·7	100·4	104·0	88·4
t (cm)	48·3	87·2	55·2	68·0	48·5	63·2	49·5	55·7	59·1	64·0	52·4

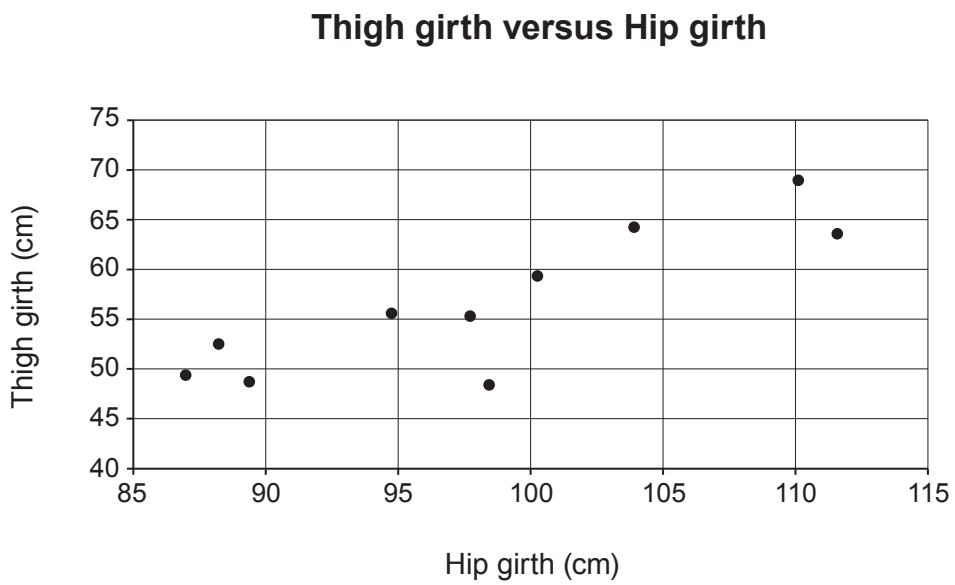
a) Jessica notes that, for the thigh girth data, the lower quartile is 49·5 and the upper quartile is 64·0.

i) Show that 87·2 should be classified as an outlier for t.

ii) Give a reason why Jessica might exclude the outlier.

iii) Give a reason why Jessica might include the outlier. [4]

Jessica decides to exclude the outlier and produces the following scatter diagram.



b) Interpret, in context, the correlation in the data shown in the diagram. [1]

The equation of the regression line of t on h for this sample is

$$t = 0{\cdot}69h - 11{\cdot}26.$$

c) Interpret the gradient of the regression line in this context. [1]

d) Use your knowledge of large data sets and spreadsheet software packages to suggest a way in which Jessica could improve her investigation. [1]

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0 5 A company, *Run4Lyfe*, sponsors an athletic event. The organisers of the event claim that 70% of the participants know the name of the sponsoring company. *Run4Lyfe* is concerned that the proportion, p, of participants knowing the name of the sponsoring company is less than 70%. They decide to survey 60 randomly selected participants to carry out a significance test.

a) State suitable hypotheses for carrying out the test. [1]

b) **i)** Explain what is meant by the critical region for this test.

ii) Determine the critical region if the test is to be carried out at a significance level as close as possible to, but not exceeding, 5%.

iii) Given that 40 participants out of the 60 in the sample know the name of the company, complete the significance test. [6]

c) State, with a reason, how you would advise *Run4Lyfe* with regards to sponsoring the event next year. [1]

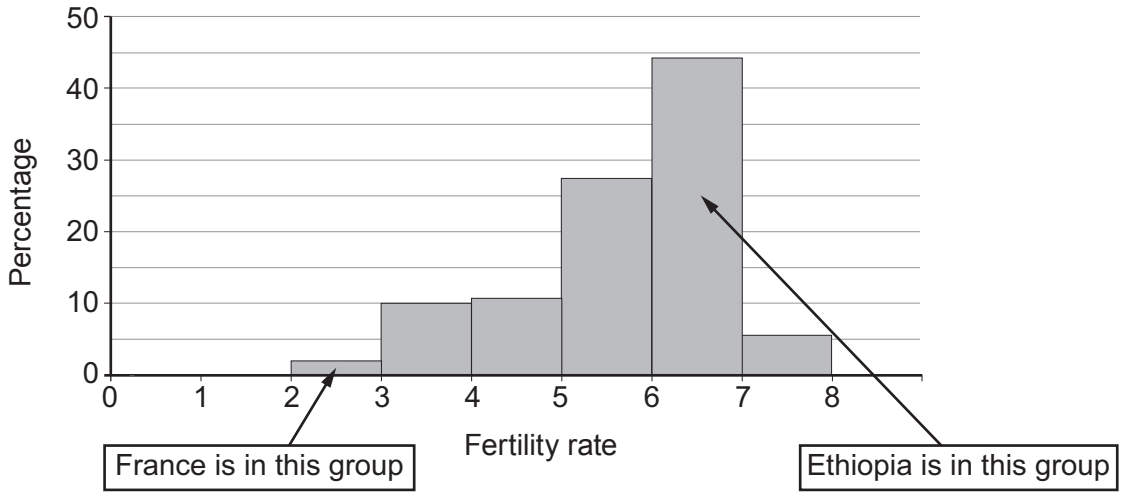
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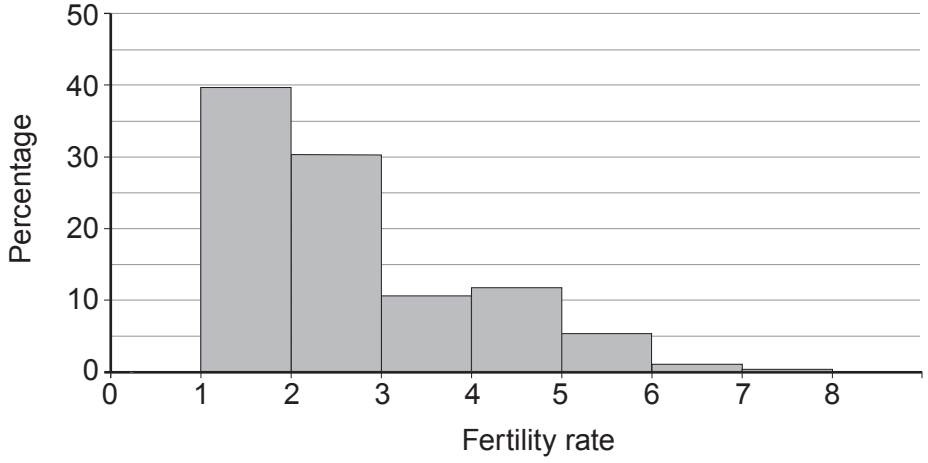
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0 6 The fertility rate for a country is the average number of children that are born to a woman over her lifetime. The graphs and table below show some data on the fertility rates for 197 countries in the years 1914 and 2014.

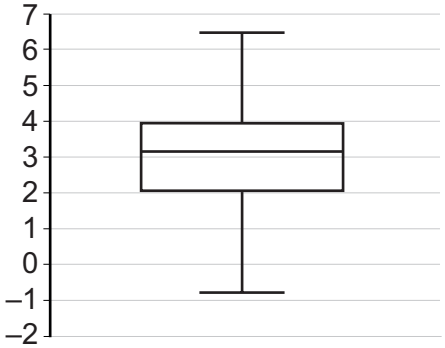
Fertility rates in 1914



Fertility rates in 2014



Decreases in fertility rates from 1914 to 2014



Decreases in fertility rates from 1914 to 2014, n = 197	
Minimum value	–0·71
Lower quartile	2·08
Median	3·19
Upper quartile	3·94
Maximum value	6·49

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a) Comment on the shapes of the distributions of fertility rates for 1914 and 2014. [2]

b) Interpret the minimum value, –0·71, in the boxplot. [1]

You are also given the following information:

Country	Fertility rate for 1914	Fertility rate for 2014
France	Between 2 and 3	1·98
Ethiopia	Between 6 and 7	4·4

c) **i)** Find the best possible estimate for the decrease in the fertility rate from 1914 to 2014 for France.

ii) Find the best possible estimate for the decrease in the fertility rate from 1914 to 2014 for Ethiopia.

iii) Give one possible reason why the answers to **i)** and **ii)** are so different.

iv) Explain why these estimates may not be very accurate. [5]

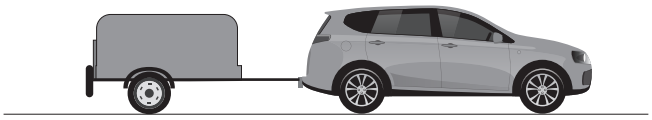
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Section B: Mechanics

0 7 The diagram below shows a vehicle of mass 1300 kg towing a trailer of mass 500 kg by means of a light horizontal tow bar. The vehicle is moving forward along a straight horizontal road such that a constant resistance of magnitude 650 N acts on the vehicle and a constant resistance of magnitude 320 N acts on the trailer. The vehicle's engine produces a constant driving force of F N.



Given that the acceleration of the vehicle and trailer is $0{\cdot}85\,\text{ms}^{-2}$, show that $F = 2500$ and determine the tension in the tow bar. [6]

0 8 An aircraft moves along a straight horizontal runway with a constant acceleration of $1{\cdot}5\,\text{ms}^{-2}$. Points A and B lie on the runway. The aircraft passes A with speed $4\,\text{ms}^{-1}$ and its speed at B must be at least $78\,\text{ms}^{-1}$ if it is to take off successfully.

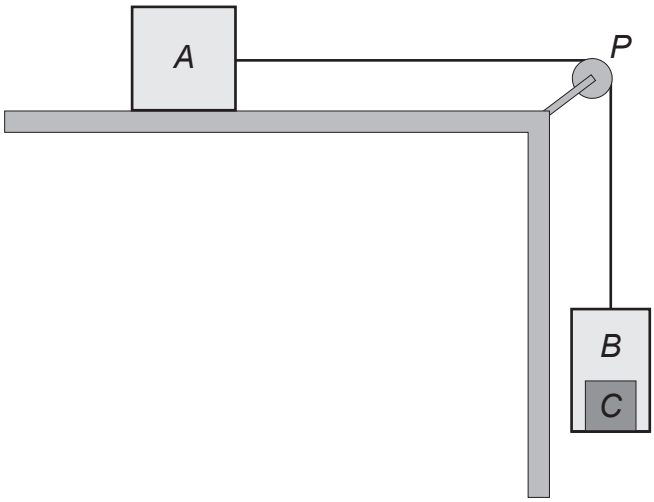
a) Find the speed of the aircraft 8 seconds after it passes A. [2]

b) Determine the minimum value of the distance AB for the aircraft to take off successfully. [3]

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0 9 The diagram below shows an object *A*, of mass 15 kg, lying on a smooth horizontal surface. It is connected to a box *B* by a light inextensible string which passes over a smooth pulley *P*, fixed at the edge of the surface, so that box *B* hangs freely. An object *C* lies on the horizontal floor of box *B* so that the combined mass of *B* and *C* is 10 kg.



Initially, the system is held at rest with the string just taut. A horizontal force of magnitude 150 N is then applied to *A* in the direction *PA* so that box *B* is raised.

a) Find the magnitude of the acceleration of *A* and the tension in the string. [6]

b) Given that object *C* has mass 4 kg, calculate the reaction of the floor of the box on object *C*. [3]

1 0 In this question, $\mathbf{i}$ and $\mathbf{j}$ represent unit vectors due east and due north respectively.

Sarah is going for a walk. She leaves her house and walks directly to the shop. She then walks directly from the shop to the park. Relative to her house:

- the shop has position vector $\left(-\frac{2}{3}\mathbf{j}\right)$ km,
- the park is 2 km away on a bearing of 060°.

a) Show that the position vector of the park relative to the house is $\left(\sqrt{3}\mathbf{i}+\mathbf{j}\right)$ km. [2]

b) Determine the total distance walked by Sarah from her house to the park. [3]

c) By considering a modelling assumption you have made, explain why the answer you found in part (b) may not be the actual distance that Sarah walked. [1]

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1 1 A particle *P* moves along the x-axis so that its velocity v ms^{-1} at time t seconds $(t \geqslant 0)$ is given by

$$v = 3t^2 - 24t + 36.$$

a) Find the values of t when *P* is instantaneously at rest. [2]

b) Calculate the total distance travelled by the particle *P* whilst its velocity is decreasing. [7]

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.82 = 0.3 + 0.6 - P(A \cap B) \quad \Rightarrow \quad P(A \cap B) = 0.08$

$P(A) \times P(B) = 0.3 \times 0.6 = 0.18$

Since $P(A \cap B) = 0.08 \neq 0.18 = P(A) \times P(B)$,

A and B are not independent.

1 (b)

$P(\text{exactly one of A and B}) = P(A \cup B) - P(A \cap B)$

$= 0.82 - 0.08$

$= 0.74$

2 (a)

Expected frequency $= 40 \times 0.45 = 18$, class width $= 265 - 262 = 3$

Height = frequency density $= 40 \times 0.45 \div 3 = 6$

2 (b)

We do not know how the probability is distributed within the two groups

$265 \leq x < 267$ and $267 \leq x < 300$, so we cannot find the new frequencies.

2 (c)

No. Different random samples will lead to different results, so the

lifetimes of the bulbs Celyn collects will be different from those in (a).

She should only be concerned if the differences are very large.

3 (a)

The earthworms need to occur at a uniform (constant) average rate

throughout the soil, and independently (randomly) of one another.

3 (b)

Volume is $\frac{1}{4}$ of the original, so mean $= \frac{11}{4} = 2.75$

X = number of earthworms, $X \sim Po(2.75)$

Question number	
	$P(X = 5) = \frac{e^{-2.75} \times 2.75^5}{5!} = 0.0837861... = 0.0838$ (3 s.f.)
3 (c)	Let $Y \sim Po(\lambda)$. Need $P(Y \leq 12) \approx 0.36$, with $\lambda > 11$. From tables: $\lambda = 13$: $P(Y \leq 12) = 0.4631$, $\lambda = 14$: $P(Y \leq 12) = 0.3585$ 0.3585 is closest to 36%, so the mean is 14 earthworms per sample.
4 (a)(i)	$IQR = 64.0 - 49.5 = 14.5$ Upper limit $= Q_3 + 1.5 \times IQR = 64 + 1.5 \times 14.5 = 85.75$ $87.2 > 85.75$, therefore 87.2 is an outlier.
4 (a)(ii)	The outlier may distort calculations (e.g. skew the regression line) and therefore any inferences made about the population.
4 (a)(iii)	It is only an outlier: it is still a valid measurement from the dataset, so it should be included (to use all of the data).
4 (b)	Positive correlation: the larger the hip girth, the larger the thigh girth, on average.
4 (c)	Each increase of 1 cm in hip girth corresponds to an increase of 0.69 cm in thigh girth, on average.
4 (d)	Use all of the data in the large data set (the software can handle it) instead of a sample of 11, which would lead to more accurate results.
5 (a)	H_0: $p = 0.7$ H_1: $p < 0.7$
5 (b)(i)	The critical region is the range of values of the number of people who know the name of the company that would lead us to reject H_0.

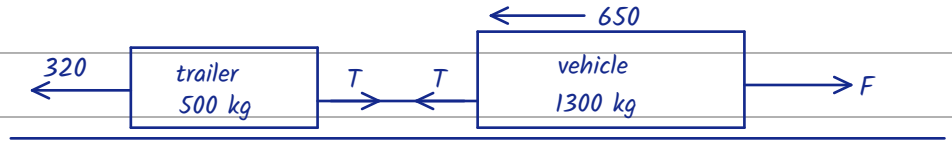
Question number	
5 (b)(ii)	Let X = number in the sample who know the name. Under H_0, $X \sim B(60, 0.7)$ $P(X \leqslant 35) = 0.0362 \quad (\leqslant 0.05)$ $P(X \leqslant 36) = 0.0632 \quad (> 0.05)$ Critical region: $X \leqslant 35$
5 (b)(iii)	$x = 40$ is not in the critical region ($P(X \leqslant 40) = 0.3308 > 0.05$), so there is insufficient evidence to reject H_0. There is insufficient evidence to suggest that fewer than 70% of participants know the name of the sponsoring company.
5 (c)	Continue sponsoring the event next year: the test implies a reasonable proportion (no evidence of fewer than 70%) know the name of the sponsor, so sponsorship is an effective way of getting brand recognition.
6 (a)	The 1914 distribution is negatively skewed (skewed to the left). The 2014 distribution is positively skewed (skewed to the right). Fertility rates were generally larger in 1914 than in 2014.
6 (b)	The decrease is negative, so the fertility rate increased in at least one country: women had 0.71 more babies on average in 2014 than in 1914.
6 (c)(i)	France 1914: between 2 and 3, so use the midpoint 2.5 Decrease $\approx 2.5 - 1.98 = 0.52$
6 (c)(ii)	Ethiopia 1914: between 6 and 7, so use the midpoint 6.5 Decrease $\approx 6.5 - 4.4 = 2.1$
6 (c)(iii)	Ethiopia had a much higher fertility rate in 1914, so it had more opportunity to decrease (Ethiopia is a developing country whose rate

Question number

is likely to have fallen more rapidly than in developed France).

6 (c)(iv) We have used the midpoint of each group as the 1914 value: we have no way of knowing the exact fertility rates of France and Ethiopia in 1914.

7



N2L for vehicle and trailer combined (a = 0.85):

$F - (650 + 320) = (1300 + 500)a$

$F - 970 = 1800 \times 0.85 = 1530$

$F = 1530 + 970 = 2500$ (shown)

N2L for the trailer:

$T - 320 = 500a$

$T - 320 = 500 \times 0.85 = 425$

Tension in the tow bar $T = 745$ N

8 (a) $u = 4$, $a = 1.5$, $t = 8$: $v = u + at$

$v = 4 + 1.5 \times 8 = 16$ ms^{-1}

8 (b) $u = 4$, $v = 78$, $a = 1.5$: $v^2 = u^2 + 2as$

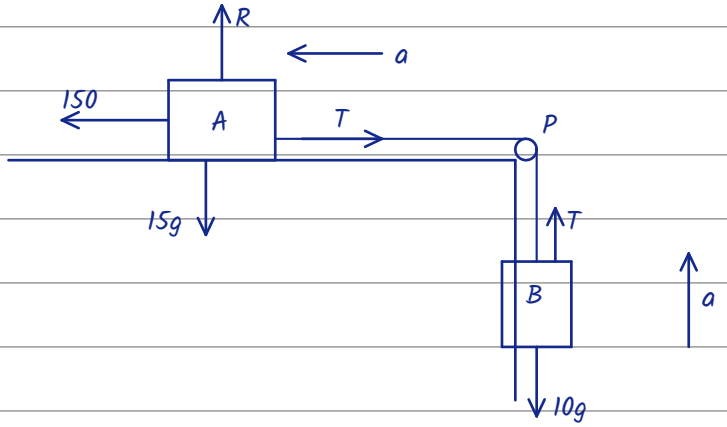
$78^2 = 4^2 + 2 \times 1.5 \times s$

$6084 = 16 + 3s \Rightarrow s = \frac{6068}{3}$

Minimum AB = 2022.66... = 2020 m (3 s.f.)

Question number

9 (a)



N2L for A: $150 - T = 15a$

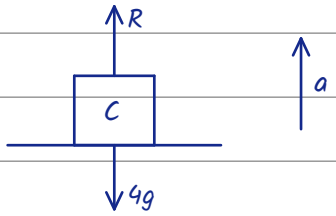
N2L for B (and C): $T - 10g = 10a$

Adding to eliminate T: $150 - 10g = 25a$

$150 - 98 = 25a \Rightarrow a = 52 \div 25 = 2.08\ ms^{-2}$

$T = 150 - 15 \times 2.08 = 150 - 31.2 = 118.8\ N$

9 (b)



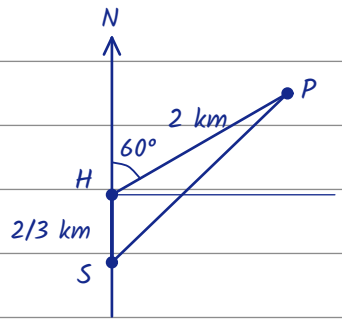
N2L for C (mass 4 kg, acceleration 2.08 upwards):

$R - 4g = 4a$

$R = 4 \times 2.08 + 4 \times 9.8 = 8.32 + 39.2$

$R = 47.52\ N$

10 (a)



Park: 2 km on a bearing of 060°

$r_{park} = (2\sin 60°)i + (2\cos 60°)j$

Question number

$= 2 \times \frac{\sqrt{3}}{2}\, i + 2 \times \frac{1}{2}\, j$

$= \sqrt{3}\, i + j$ km (shown)

10 (b) Shop to park: $SP = (\sqrt{3}\, i + j) - \left(-\frac{2}{3}\, j\right) = \sqrt{3}\, i + \frac{5}{3}\, j$

$|SP| = \sqrt{\left(\sqrt{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{\frac{52}{9}} = \frac{2\sqrt{13}}{3} = 2.4037\ldots$ km

House to shop $= \frac{2}{3}$ km

Total distance $= \frac{2}{3} + \frac{2\sqrt{13}}{3} = 3.0703\ldots = 3.07$ km

10 (c) We assumed Sarah walks in straight lines. In reality she is unlikely to walk in straight lines (she follows roads and paths, which may not be straight or flat), so the actual distance walked is likely to be greater.

11 (a) At rest when $v = 0$: $3t^2 - 24t + 36 = 0$

$3(t^2 - 8t + 12) = 0 \Rightarrow 3(t - 2)(t - 6) = 0$

$t = 2$ s and $t = 6$ s

11 (b) $a = \frac{dv}{dt} = 6t - 24$

Velocity decreasing when $a < 0$: $6t - 24 < 0 \Rightarrow 0 \leq t < 4$

$v = 0$ at $t = 2$, so P changes direction at $t = 2$.

For $0 \leq t \leq 2$: distance $= \int_0^2 (3t^2 - 24t + 36)\, dt = \left[t^3 - 12t^2 + 36t\right]_0^2$

$= 8 - 48 + 72 - 0 = 32$ m

For $2 \leq t \leq 4$: $\int_2^4 (3t^2 - 24t + 36)\, dt = \left[t^3 - 12t^2 + 36t\right]_2^4$

$= (64 - 192 + 144) - 32 = 16 - 32 = -16$

so distance = 16 m

Total distance = 32 + 16 = 48 m