GCE AS/A LEVEL

2300U10-1

Z22-2300U10-1

THURSDAY, 19 MAY 2022 – AFTERNOON

MATHEMATICS – AS unit 1
PURE MATHEMATICS A

2 hours 30 minutes

2300U101
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid. You may use a pencil for graphs and diagrams only.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, for example, **0** **1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

0 1 Write down the inverse function of $y = e^x$. On the same set of axes, sketch the graphs of $y = e^x$ and its inverse function, clearly labelling the coordinates of the points where the graphs cross the x and y axes. [3]

0 2 Showing all your working, simplify the following expression. [6]

$$5\sqrt{48} + \frac{2+5\sqrt{3}}{5+3\sqrt{3}} - \left(2\sqrt{3}\right)^3$$

0 3 The line L_1 passes through the points $A(0, 5)$ and $B(3, -1)$.

a) Find the equation of the line L_1. [3]

The line L_2 is perpendicular to L_1 and passes through the origin O.

b) Write down the equation of L_2. [1]

The lines L_1 and L_2 intersect at the point C.

c) Calculate the area of triangle OAC. [4]

d) Find the equation of the line L_3 which is parallel to L_1 and passes through the point $D(4, 2)$. [2]

e) The line L_3 intersects the y-axis at the point E. Find the area of triangle ODE. [1]

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0 4 Solve the inequality $x^2 + 3x - 6 > 4x - 4$. [4]

0 5 The curve C_1 has equation $y = -x^2 + 2x + 3$ and the curve C_2 has equation $y = x^2 - x - 6$. The two curves intersect at the points *A* and *B*.

a) Determine the coordinates of *A* and *B*. [4]

b) On the same set of axes, sketch the graphs of C_1 and C_2. Clearly label the points where the two curves intersect. [3]

c) In the diagram drawn in part (b), shade the region satisfying the following inequalities: [2]

$$x > 0,$$

$$y < -x^2 + 2x + 3,$$

$$y > x^2 - x - 6.$$

0 6 In each of the two statements below, x and y are real numbers. One of the statements is true while the other is false.

A : $x^2 + y^2 \geqslant 2xy$, for all real values of x and y.

B : $x + y \geqslant 2\sqrt{xy}$, for all real values of x and y.

a) Identify the statement which is false. Find a counter example to show that this statement is in fact false. [3]

b) Identify the statement which is true. Give a proof to show that this statement is in fact true. [2]

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0 7 A circle *C* has centre *A* and equation $x^2 + y^2 - 4x - 6y = 3$.

a) Find the coordinates of *A* and the radius of *C*. [3]

The line *L* with equation $y = x + 5$ intersects *C* at the points *P* and *Q*.

b) Determine the coordinates of *P* and *Q*. [4]

c) The point *B* is on *PQ* and is such that *AB* is perpendicular to *PQ*. Find the length of *PB*. [2]

d) Show that the area of the smaller segment enclosed by *C* and *L* is $4\pi - 8$. [2]

0 8 **a)** The graph *G* shows the relationship between the variables y and x, where $y \propto x$. Sketch the graph *G*. [1]

b) Mary and Jeff work for a company which pays its employees by hourly rates. Mary's hourly rate is twice Jeff's hourly rate. On a certain day, Jeff worked three times as long as Mary and was paid £120. Calculate Mary's earnings on that day. [3]

c) Atmospheric pressure, P units, decreases as the height, H metres, above sea level increases. The rate of decrease is 12% for every 1000 m. At sea level, the pressure P is 1013 units. Write down the model for P in terms of H and find the pressure at the top of Mount Everest, which is 8848 m above sea level. [3]

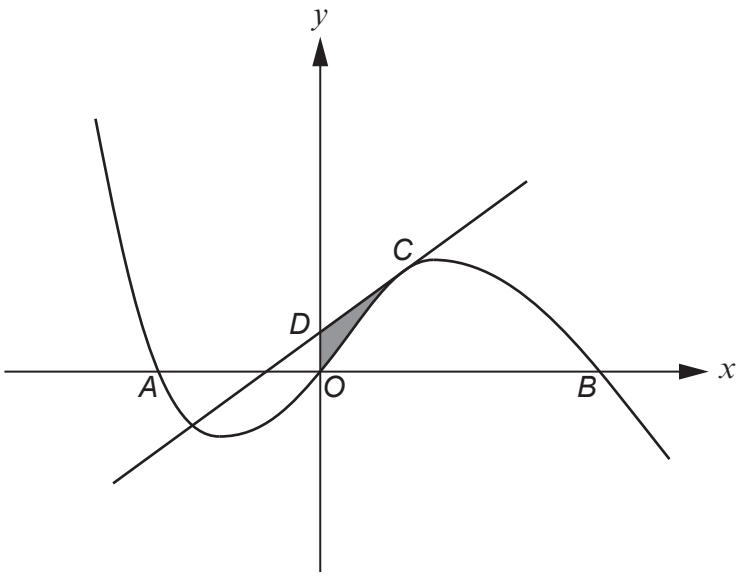
0 9 Find the range of values of k for which the quadratic equation $x^2 + 2kx + 8k = 0$ has no real roots. [4]

1 0 Showing all your working, solve the equation $2^x = 53$. Give your answer correct to two decimal places. [3]

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1 1 The diagram below shows a sketch of the curve $y = f(x)$, where $f(x) = 10x + 3x^2 - x^3$. The curve intersects the x-axis at the origin O and at the points $A(-2, 0)$, $B(5, 0)$. The tangent to the curve at the point $C(2, 24)$ intersects the y-axis at the point D.



a) Find the coordinates of D. [5]

b) Find the area of the shaded region. [6]

c) Determine the range of values of x for which $f(x)$ is an increasing function. [4]

1 2 **a)** Solve the equation $2x^3 - x^2 - 5x - 2 = 0$. [6]

b) Find all values of θ in the range $0° < \theta < 180°$ satisfying

$$\cos(2\theta - 51°) = 0{\cdot}891.$$ [3]

1 3 Find the term which is independent of x in the expansion of $\dfrac{(2-3x)^5}{x^3}$. [4]

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1 4 A curve *C* has equation $f(x) = 3x^3 - 5x^2 + x - 6$.

a) Find the coordinates of the stationary points of *C* and determine their nature. [8]

b) **Without solving the equations**, determine the number of distinct real roots for each of the following:

i) $3x^3 - 5x^2 + x + 1 = 0$,

ii) $6x^3 - 10x^2 + 2x + 1 = 0$. [4]

1 5 Solve the simultaneous equations

$$3\log_a\left(x^2y\right) - \log_a\left(x^2y^2\right) + \log_a\left(\frac{9}{x^2y^2}\right) = \log_a 36,$$

$$\log_a y - \log_a (x+3) = 0.$$ [8]

1 6 The vectors $\mathbf{a}$ and $\mathbf{b}$ are defined by $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$.

a) Find a unit vector in the direction of $\mathbf{a}$. [2]

b) Determine the angle $\mathbf{b}$ makes with the x-axis. [2]

c) The vector $\mu\mathbf{a} + \mathbf{b}$ is parallel to $4\mathbf{i} - 5\mathbf{j}$.

i) Find the vector $\mu\mathbf{a} + \mathbf{b}$ in terms of μ, $\mathbf{i}$ and $\mathbf{j}$. [1]

ii) Determine the value of μ. [4]

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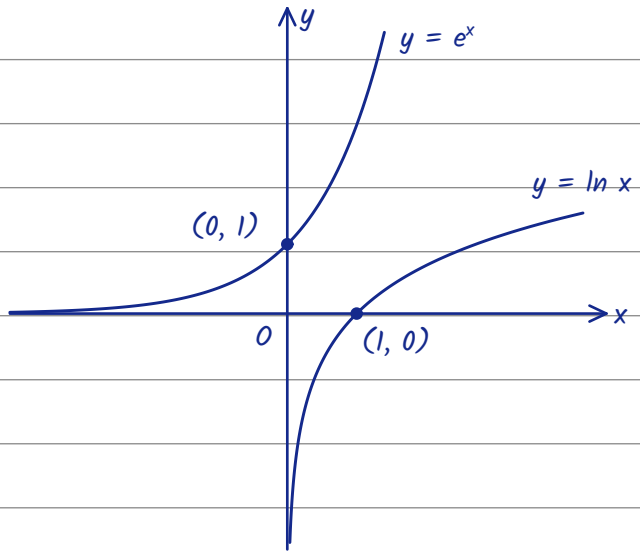
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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1

Inverse function: $y = \ln x$



2

$5\sqrt{48} = 5 \times 4\sqrt{3} = 20\sqrt{3}$

$(2\sqrt{3})^3 = 8 \times 3\sqrt{3} = 24\sqrt{3}$

$$\frac{2+5\sqrt{3}}{5+3\sqrt{3}} = \frac{(2+5\sqrt{3})(5-3\sqrt{3})}{(5+3\sqrt{3})(5-3\sqrt{3})} = \frac{10-6\sqrt{3}+25\sqrt{3}-45}{25-27} = \frac{19\sqrt{3}-35}{-2} = \frac{35-19\sqrt{3}}{2}$$

$$\text{Expression} = 20\sqrt{3} + \frac{35-19\sqrt{3}}{2} - 24\sqrt{3} = \frac{40\sqrt{3}+35-19\sqrt{3}-48\sqrt{3}}{2}$$

$$= \frac{35-27\sqrt{3}}{2}$$

3 (a)

Gradient of $L_1 = \frac{-1-5}{3-0} = -2$

$y - 5 = -2(x - 0)$

L_1: $y = -2x + 5$ $\quad (y + 2x = 5)$

3 (b)

Gradient of $L_2 = \frac{1}{2}$, $\quad L_2$: $y = \frac{1}{2}x$

Question number

3 (c)

At C: $\frac{1}{2}x = -2x + 5 \Rightarrow \frac{5}{2}x = 5 \Rightarrow x = 2,\ y = 1$

C is the point (2, 1)

Area OAC $= \frac{1}{2} \times OA \times$ (x-coordinate of C) $= \frac{1}{2} \times 5 \times 2 = 5$ (square units)

3 (d)

Gradient of $L_3 = -2$ (parallel to L_1)

$y - 2 = -2(x - 4) \Rightarrow L_3: y = -2x + 10$

3 (e)

E is (0, 10)

Area ODE $= \frac{1}{2} \times OE \times$ (x-coordinate of D) $= \frac{1}{2} \times 10 \times 4 = 20$ (square units)

4

$x^2 + 3x - 6 > 4x - 4$

$x^2 - x - 2 > 0$

$(x + 1)(x - 2) > 0$

Critical values: $x = -1$, $x = 2$

$x < -1$ or $x > 2$

5 (a)

$-x^2 + 2x + 3 = x^2 - x - 6$

$2x^2 - 3x - 9 = 0$

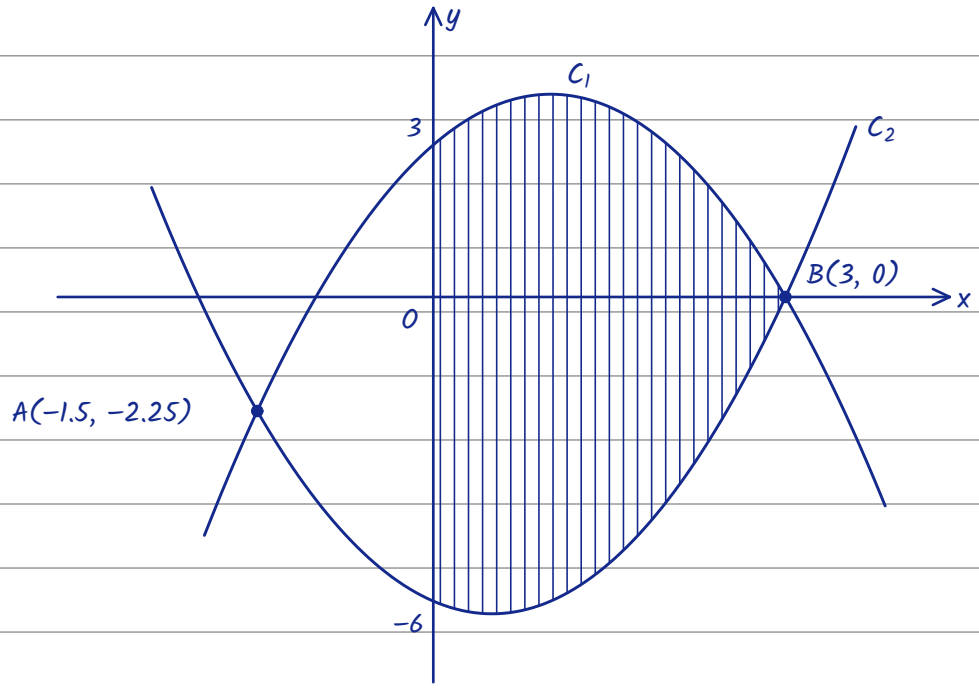
$(2x + 3)(x - 3) = 0$

$x = -\frac{3}{2}$: $y = -\frac{9}{4} - 3 + 3 = -\frac{9}{4}$ $\quad$ $x = 3$: $y = -9 + 6 + 3 = 0$

$A\left(-\frac{3}{2}, -\frac{9}{4}\right)$, $B(3, 0)$

Question number

5 (b)



5 (c) Shaded region on the diagram in (b): between C_1 (above) and C_2 (below), to the right of the y-axis ($0 < x < 3$).

6 (a) Statement B is false.

Counter example: $x = -25$, $y = -4$

$x + y = -25 + (-4) = -29$

$2\sqrt{(xy)} = 2 \times \sqrt{((-25) \times (-4))} = 2 \times \sqrt{100} = 20$

Since $-29 < 20$, $x + y \geq 2\sqrt{(xy)}$ is not true, so statement B is false.

6 (b) Statement A is true.

Proof: $(x - y)^2 \geq 0$ for all real x and y

$x^2 - 2xy + y^2 \geq 0$

$x^2 + y^2 \geq 2xy$ for all real x and y, so statement A is true.

7 (a) $x^2 - 4x + y^2 - 6y = 3$

$(x - 2)^2 - 4 + (y - 3)^2 - 9 = 3$

Question number

$(x - 2)^2 + (y - 3)^2 = 16 = 4^2$

$A(2, 3)$, radius $= 4$

7 (b)

$x^2 + (x + 5)^2 - 4x - 6(x + 5) - 3 = 0$

$x^2 + x^2 + 10x + 25 - 4x - 6x - 30 - 3 = 0$

$2x^2 - 8 = 0 \Rightarrow x^2 = 4 \Rightarrow x = -2$ or $x = 2$

$x = -2$: $y = 3$; $x = 2$: $y = 7$

$P(-2, 3)$, $Q(2, 7)$

7 (c)

$AB \perp PQ$, so B is the midpoint of PQ: $B = (0, 5)$

$PB = \sqrt{((-2 - 0)^2 + (3 - 5)^2)} = \sqrt{8} = 2\sqrt{2}$

7 (d)

AP: from $(2, 3)$ to $(-2, 3)$ is horizontal; AQ: from $(2, 3)$ to $(2, 7)$ is vertical,

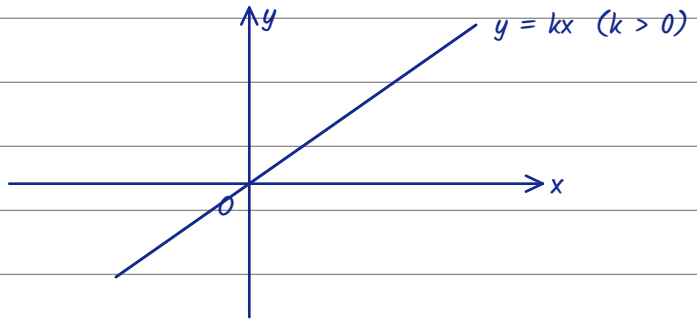
so angle $PAQ = 90°$ and $AP = AQ = 4$.

Area of segment = area of quarter circle − area of triangle APQ

$$= \frac{1}{4} \times \pi \times 4^2 - \frac{1}{2} \times 4 \times 4$$

$= 4\pi - 8$ as required

8 (a)



8 (b)

Let Jeff's hourly rate be £r and Mary work t hours.

Jeff: $r \times 3t = 120 \Rightarrow rt = 40$

Mary's earnings $= 2r \times t = 2 \times 40 =$ £80

Question number

8 (c)

$P = 1013 \times 0.88^{H/1000}$

When $H = 8848$: $P = 1013 \times 0.88^{8.848}$

$= 326.88...$

$= 327$ units (3 s.f.)

9

Discriminant $= b^2 - 4ac = (2k)^2 - 4 \times 1 \times 8k = 4k^2 - 32k$

No real roots: $4k^2 - 32k < 0$

$4k(k - 8) < 0$

Critical values: $k = 0$, $k = 8$

$0 < k < 8$

10

$\ln 2^x = \ln 53$

$x \ln 2 = \ln 53$

$x = \frac{\ln 53}{\ln 2} = 5.727920...$

$x = 5.73$ (2 d.p.)

11 (a)

$f(x) = 10x + 3x^2 - x^3 \Rightarrow \frac{dy}{dx} = 10 + 6x - 3x^2$

At $x = 2$: $\frac{dy}{dx} = 10 + 12 - 12 = 10$

Tangent at C: $y - 24 = 10(x - 2) \Rightarrow y = 10x + 4$

At $x = 0$, $y = 4$, so D is the point $(0, 4)$

11 (b)

Area of trapezium (under DC) $= \frac{1}{2}(4 + 24) \times 2 = 28$

Area under curve $= \int_0^2 (10x + 3x^2 - x^3)\,dx = \left[5x^2 + x^3 - \frac{x^4}{4}\right]_0^2$

$= (20 + 8 - 4) - (0) = 24$

Shaded area $= 28 - 24 = 4$ (square units)

11 (c)

f is increasing when $f'(x) > 0$: $10 + 6x - 3x^2 > 0$

$3x^2 - 6x - 10 < 0$

Question number

Critical values: $x = \frac{6 \pm \sqrt{((-6)^2 - 4 \times 3 \times (-10))}}{6} = \frac{3 \pm \sqrt{39}}{3}$

$x = -1.0816...$ or $x = 3.0816...$

$-1.08 < x < 3.08$ $\left(\frac{3 - \sqrt{39}}{3} < x < \frac{3 + \sqrt{39}}{3}\right)$

12 (a) $f(x) = 2x^3 - x^2 - 5x - 2$

$f(-1) = -2 - 1 + 5 - 2 = 0$, so $(x + 1)$ is a factor

$f(x) = (x + 1)(2x^2 - 3x - 2)$

$= (x + 1)(2x + 1)(x - 2)$

$x = -1$, $x = -\frac{1}{2}$, $x = 2$

12 (b) $0° < \theta < 180° \Rightarrow -51° < 2\theta - 51° < 309°$

$\cos(2\theta - 51°) = 0.891$: $2\theta - 51° = 26.99...° \approx 27°$ or $2\theta - 51° = -27°$

$2\theta = 78°$ or $2\theta = 24°$

$\theta = 39°$, $\theta = 12°$

13 Term in $(2 - 3x)^5$: $\binom{5}{r} 2^{5-r}(-3x)^r$. Dividing by x^3, need the x^3 term, so $r = 3$

Required term $= \binom{5}{3}(2)^2(-3)^3 = 10 \times 4 \times (-27)$

Required term $= -1080$

14 (a) $f'(x) = 9x^2 - 10x + 1$

Stationary points: $9x^2 - 10x + 1 = 0 \Rightarrow (9x - 1)(x - 1) = 0$

$x = \frac{1}{9}$: $y = 3\left(\frac{1}{9}\right)^3 - 5\left(\frac{1}{9}\right)^2 + \frac{1}{9} - 6 = -\frac{1445}{243} = -5.9465...$

$x = 1$: $y = 3 - 5 + 1 - 6 = -7$

$f''(x) = 18x - 10$

Question number

$x = \frac{1}{9}$: $f'' = 2 - 10 = -8 < 0$, so $\left(\frac{1}{9}, -\frac{1445}{243}\right)$ is a maximum point

$x = 1$: $f'' = 18 - 10 = 8 > 0$, so $(1, -7)$ is a minimum point

14 (b)(i) $3x^3 - 5x^2 + x + 1 = 0 \Leftrightarrow 3x^3 - 5x^2 + x - 6 = -7$, i.e. $f(x) = -7$

The line $y = -7$ touches C at the minimum $(1, -7)$ and crosses C once more to the left (since the maximum value $-5.95 > -7$).

Number of distinct real roots = 2

14 (b)(ii) $6x^3 - 10x^2 + 2x + 1 = 0 \Leftrightarrow 3x^3 - 5x^2 + x + \frac{1}{2} = 0 \Leftrightarrow f(x) = -6.5$

Since $-7 < -6.5 < -5.9465$ (between the minimum and maximum values), the line $y = -6.5$ cuts C three times.

Number of distinct real roots = 3

15 $\log_a\left(\frac{(x^2y)^3 \times 9}{x^2y^2 \times x^2y^2}\right) = \log_a 36$

$\frac{9x^6y^3}{x^4y^4} = 36 \Rightarrow \frac{9x^2}{y} = 36 \Rightarrow x^2 = 4y$

$\log_a\left(\frac{y}{x+3}\right) = 0 = \log_a 1 \Rightarrow y = x + 3$

$x^2 = 4(x + 3) \Rightarrow x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0 \Rightarrow x = -2$ or $x = 6$

$x = -2, y = 1$ or $x = 6, y = 9$

16 (a) $|a| = \sqrt{(2^2 + (-1)^2)} = \sqrt{5}$

Unit vector $= \frac{1}{\sqrt{5}}(2i - j)$

16 (b) $\tan\theta = \frac{3}{1} \Rightarrow \theta = \tan^{-1}3 = 71.565...°$

b makes an angle of 71.6° with the x-axis (below the x-axis, i.e. −71.6°)

Question number

16 (c)(i) $\mu a + b = \mu(2i - j) + (i - 3j) = (2\mu + 1)i - (\mu + 3)j$

16 (c)(ii) $(2\mu + 1)i - (\mu + 3)j = k(4i - 5j)$

$2\mu + 1 = 4k$ and $\mu + 3 = 5k$

$k = \frac{2\mu + 1}{4}$; $\mu + 3 = \frac{5(2\mu + 1)}{4} \Rightarrow 4\mu + 12 = 10\mu + 5$

$6\mu = 7 \Rightarrow \mu = \frac{7}{6}$