GCE AS/A LEVEL – NEW

2300U20-1

S19-2300U20-1

MATHEMATICS – AS unit 2
APPLIED MATHEMATICS A

WEDNESDAY, 22 MAY 2019 – MORNING

1 hour 45 minutes

2300U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0 1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 75.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0 1 Three events A, B, C are such that $P(B) = \frac{1}{5}$, $P(C) = \frac{1}{6}$, $P(A \cup B) = \frac{3}{4}$, $P(A \cup C) = \frac{5}{6}$.

The events A and B are independent.

a) Find $P(A)$. [3]

b) Determine whether or not A and C are independent. [3]

c) What can be said about the events B and C if $B \cap C = \varnothing$? [1]

0 2 Bethan owns a clothing company. Records of past sales show that 30% of people who come into her store buy at least one item of clothing. She would like to increase the proportion of people who buy at least one item. She believes that social media advertising will achieve this. Let p be the proportion of people who buy at least one item of clothing after Bethan has carried out social media advertising.

a) State suitable hypotheses to test Bethan's belief. [1]

She considers the first 50 people who come into her store after she carries out social media advertising to be a random sample, and observes that 21 of them buy at least one item of clothing.

b) Carry out an appropriate test at the 5% level of significance, stating clearly the conclusion that Bethan should reach. [5]

c) What else would Bethan need to consider in order to decide whether or not to continue social media advertising? [1]

Ali also owns a clothing company. From past experience he knows that 29% of people who come into his store buy at least one item of clothing. He starts selling a new brand of clothing and notices that during the following week 35% of people buy at least one item of clothing. He carries out a statistical test using this data on the following hypotheses, where θ is the proportion of people who buy at least one item of clothing.

$$H_0: \theta = 0{\cdot}29 \qquad H_1: \theta > 0{\cdot}29$$

d) State the fundamental error in Ali's statistical test and explain what he should have done, including appropriate hypotheses. [3]

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0 3 The number of patients arriving at the Accident and Emergency (A&E) department of a hospital has a mean of 5·3 per hour. A Poisson distribution is used to model the number of patients arriving at A&E in a particular time interval.

a) Determine the probability that exactly 7 patients arrive in a one-hour period. [2]

b) Find the smallest integer n such that the probability of at least n patients arriving in a 90-minute interval is less than 0·09. [4]

c) Give **one** reason why a Poisson model may not be appropriate in this context. [1]

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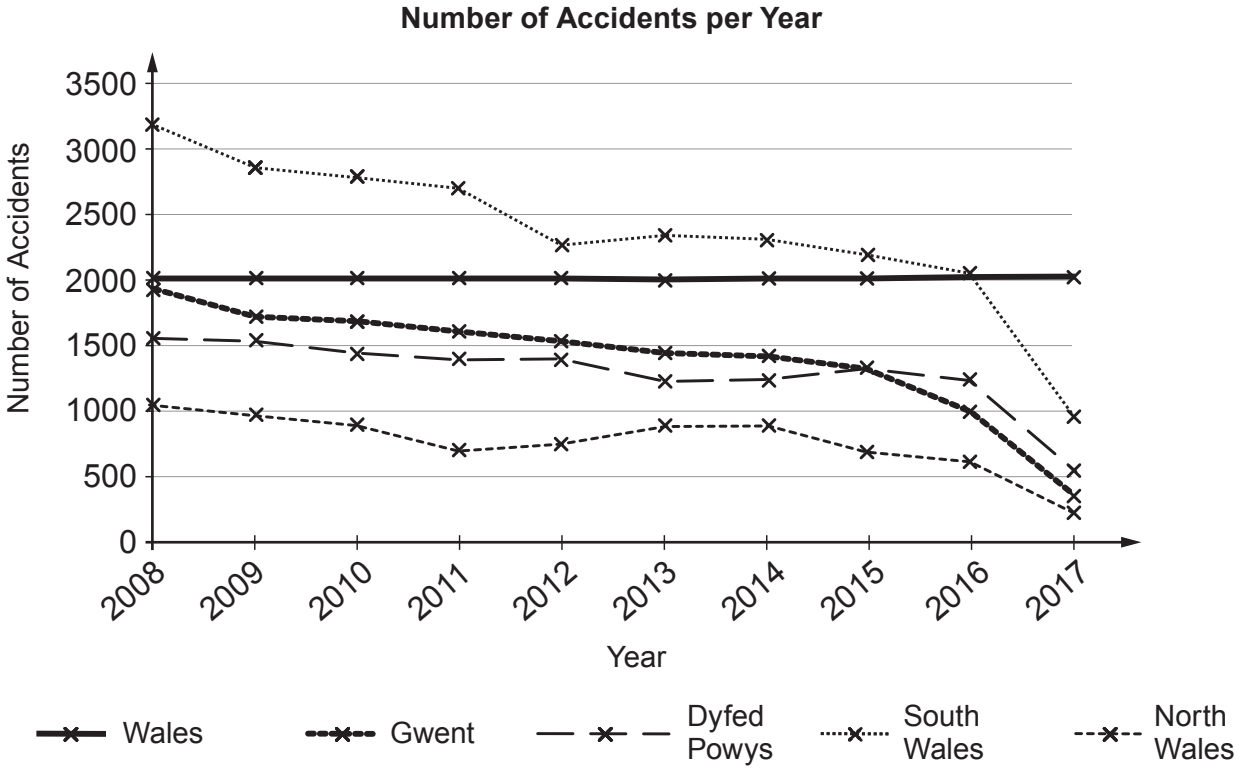
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0 4 Angharad collects the following data from the statswales.gov.wales website which shows the number of road accidents in different regions of Wales.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Wales	7784	7126	6850	6434	5971	5895	5876	5543	4921	2079
North Wales	1956	1724	1695	1600	1535	1445	1415	1333	997	341
Dyfed Powys	1560	1542	1454	1407	1412	1231	1249	1325	1238	543
South Wales	3208	2880	2805	2715	2283	2345	2328	2196	2066	958
Gwent	1060	980	896	712	741	874	884	689	620	237

a) Angharad is interested in road safety and produces the following graph.



i) State two errors that she has made.

ii) Angharad states that South Wales is the most dangerous region of Wales in which to drive. What other information should she take into consideration before making this statement? [3]

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Wales is also divided into 22 smaller regions. The numbers of accidents in 2017 in the 22 smaller regions are summarised in the table below.

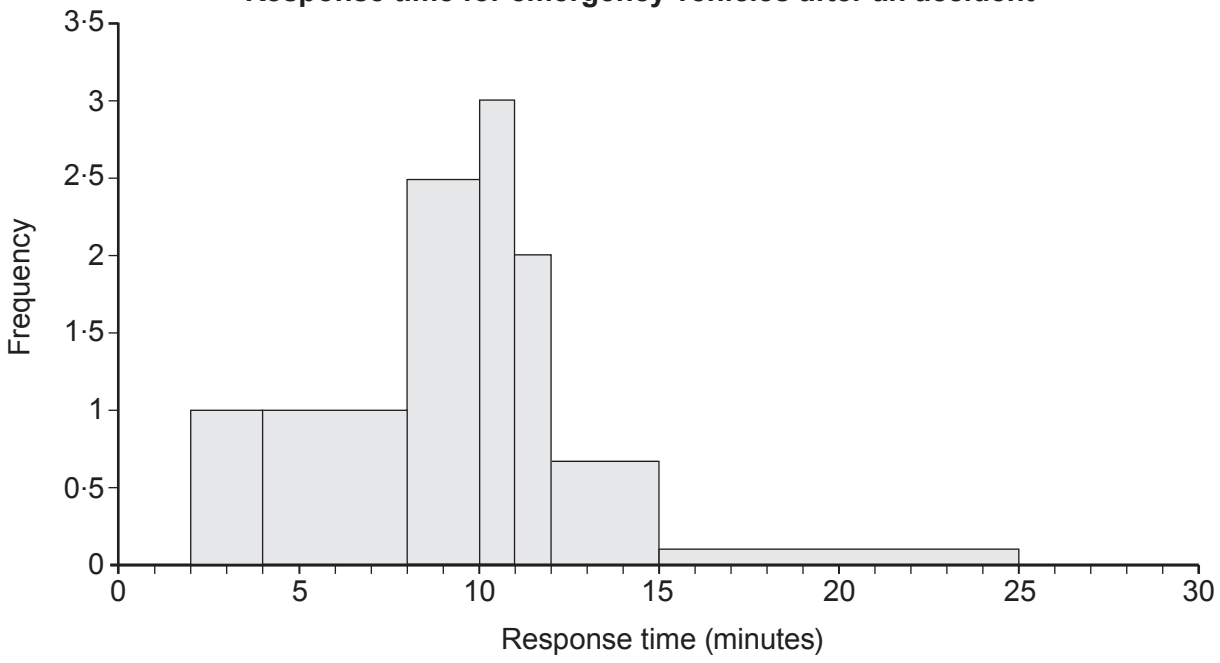
Number of Accidents	Midpoint (x)	Frequency (f)
0 - 50	25	6
51 - 100	75·5	9
101 - 150	125·5	2
151 - 200	175·5	4
201 - 250	225·5	0
251 - 300	275·5	1

b) For these 22 smaller regions, calculate estimates for the mean and standard deviation of the number of accidents per region in 2017.

You may use $\sum fx^2 = 285654$. [4]

c) Angharad also investigates the response time for emergency vehicles after 19 accidents in the region where she lives. She produces the histogram below.

Response time for emergency vehicles after an accident



i) State a correction that Angharad needs to make to this histogram.

ii) Angharad says, "More vehicles respond in a time between 10 and 11 minutes than in any other 1-minute interval." Comment on her statement.

iii) Comment on the skewness of this histogram. [3]

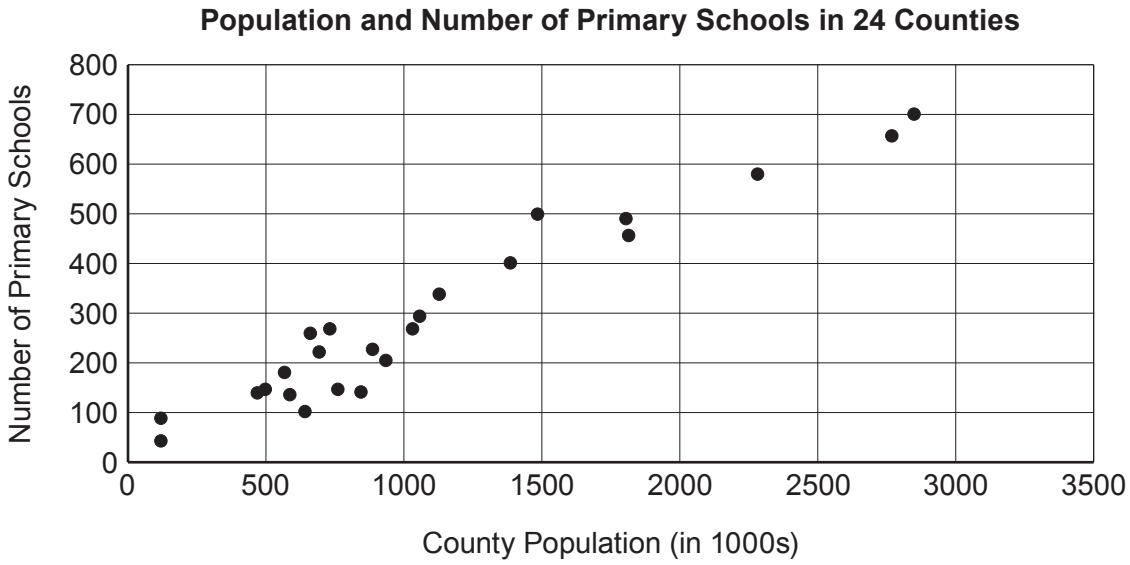
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0 5 Huw investigates the opinion of the public about their local primary schools. He visits his town centre and administers a questionnaire to 120 people over the course of several days.

a) What type of sampling is this? [1]

b) How could Huw's sampling be improved? [1]

Huw wishes to investigate the relationship between the population and the number of primary schools in a county. He searches the internet for data and selects a random sample of 24 counties from the UK. He produces the following scatter diagram.



The equation of the regression line for this dataset is given by

Number of Primary Schools = 22·7 + 0·2406 × County Population (in 1000s).

c) State a limitation of this regression line. [1]

d) Calculate an estimate for the number of primary schools in a county with a population of 889 000. [2]

e) Statisticians often say, "Correlation does not imply causation". Comment on this statement in this context. [1]

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Section B: Mechanics

0 6 Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are acting on a body of mass 2 kg such that

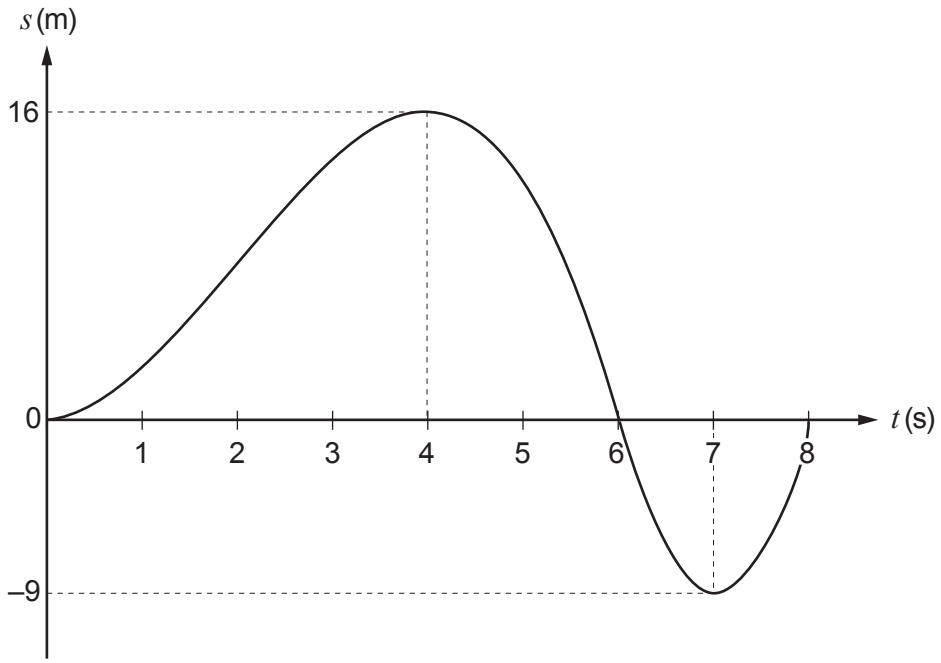
$$\mathbf{F}_1 = (6\mathbf{i} - 7\mathbf{j})\ \text{N},$$
$$\mathbf{F}_2 = (a\mathbf{i} + 2\mathbf{j})\ \text{N},$$
$$\mathbf{F}_3 = (5\mathbf{i} + b\mathbf{j})\ \text{N},$$

where a, b are constants.

a) Given that the acceleration of the body is $(7\mathbf{i} - 3\mathbf{j})\,\text{ms}^{-2}$, find the values of the constants a, b. [4]

b) A fourth force, $\mathbf{F}_4$, is then applied to the body causing it to move with constant velocity. Find $\mathbf{F}_4$, giving your answer in vector form. [2]

0 7 An athlete is running along a straight horizontal track during the interval $0 \leqslant t \leqslant 8$, where t is the time in seconds. The displacement, s metres, of the athlete relative to the origin O is modelled by the displacement-time graph shown below.



a) Determine the total distance travelled and hence find the average speed of the athlete. [3]

b) Write down the times when the athlete is instantaneously at rest. [1]

c) Write down the time interval where

i) the velocity of the athlete is negative,

ii) the velocity is increasing **and** the speed is decreasing. [2]

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0 8 Lowri kicks a football vertically upwards with a speed of u ms^{-1} from a point which is 0·9 m above horizontal ground. The ball reaches a greatest height of 10·9 m **above the ground**. You may assume that there is no air resistance acting on the ball during its motion.

a) Show that $u = 14$. [3]

b) Determine the time between the ball being kicked and the ball hitting the ground. Give your answer correct to one decimal place. [4]

c) In addition to the assumption given in the question, write down one further assumption that you have made in your solution. [1]

d) Lowri now goes into a sports hall where the height of the ceiling is 11·5 m. She believes that if she kicks a lighter ball vertically upwards from the same height of 0·9 m above the floor, with the same initial speed of 14 ms^{-1}, then the ball would hit the ceiling.

Explain why this may not be the case according to this model. [1]

0 9 A particle moves in a straight line so that its acceleration a ms^{-2} at time t seconds is given by

$$a = \begin{cases} 2t - 8 & \text{for } 0 \leqslant t \leqslant 5, \\ 2 & \text{for } t > 5. \end{cases}$$

At time $t = 0$, the particle has velocity 12 ms^{-1}.

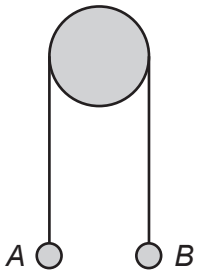
a) For $0 \leqslant t \leqslant 5$, find an expression for the velocity of the particle at t s. [3]

b) Hence, determine the velocity of the particle when $t = 14$. [3]

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1 0 Two particles *A* and *B*, of mass 3 kg and M kg respectively, are connected by a light inextensible string passing over a smooth fixed pulley. Initially, the particles are held at rest with the string just taut either side of the pulley.



The system is then released and the particles move with an acceleration of magnitude $4{\cdot}2\,\text{ms}^{-2}$.

a) Calculate the possible values of the tension in the string and the corresponding values of M. [7]

b) Now suppose that the pulley is **not** smooth. Write down one assumption which you made in your solution to part *(a)* that is no longer valid. [1]

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a) A, B independent, so $P(A \cap B) = P(A) \times P(B) = (1/5)P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$3/4 = P(A) + 1/5 - (1/5)P(A)$

$(4/5)P(A) = 3/4 - 1/5 = 11/20$

$P(A) = 11/16 \quad (= 0.6875)$

1 (b) $P(A \cap C) = P(A) + P(C) - P(A \cup C) = 11/16 + 1/6 - 5/6 = 1/48$

$P(A) \times P(C) = 11/16 \times 1/6 = 11/96$

Since $1/48 \neq 11/96$, $P(A \cap C) \neq P(A) \times P(C)$,

so A and C are not independent.

1 (c) B and C are mutually exclusive (they cannot both occur).

2 (a) H_0: $p = 0.3$ $\quad$ H_1: $p > 0.3$

2 (b) Let X = number of people out of 50 who buy at least one item.

Under H_0, $X \sim B(50, 0.3)$

p-value $= P(X \geqslant 21) = 1 - P(X \leqslant 20) = 1 - 0.9522$

$P(X \geqslant 21) = 0.0478$

Since $0.0478 < 0.05$, reject H_0.

There is sufficient evidence at the 5% level of significance to suggest that social media advertising has increased the proportion of people who buy at least one item of clothing.

Question number	
2 (c)	She would need to consider the cost of the social media advertising compared with the extra profit from the increased sales.
2 (d)	Ali has formed his alternative hypothesis ($\theta > 0.29$) using his observations (35% in the week), i.e. after looking at the data. Hypotheses should be formed independently of (before seeing) the data. With no prior reason to expect an increase he should have used a two-tailed test: H_0: $\theta = 0.29$ $\quad$ H_1: $\theta \neq 0.29$
3 (a)	Let Y = number of patients arriving in one hour, $Y \sim Po(5.3)$ $P(Y = 7) = e^{-5.3} \times 5.3^7 \div 7!$ $P(Y = 7) = 0.116342... = 0.1163$
3 (b)	Let X = number of patients arriving in 90 minutes, $X \sim Po(1.5 \times 5.3) = Po(7.95)$ From calculator: $P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.8916 = 0.1084$ (> 0.09) $P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9386 = 0.0614$ (< 0.09) $\therefore$ smallest $n = 13$
3 (c)	Patients may not arrive independently of each other, e.g. several people injured in the same car accident arrive together. (Also some times of day are busier than others, so the rate is not constant.)
4 (a)(i)	1. She has plotted the year as a data value: the "Wales" line is a horizontal line at about 2000 instead of the numbers of accidents for Wales. 2. She has mixed up Gwent and North Wales (the Gwent line shows the North Wales data and vice versa).

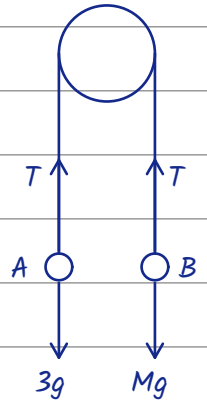
Question number	
4 (a)(ii)	She should take into account the population of each region (or the number of vehicles / length of road in each region), e.g. by comparing accidents per 100 000 people.
4 (b)	$\Sigma fx = 25 \times 6 + 75.5 \times 9 + 125.5 \times 2 + 175.5 \times 4$ $+ 225.5 \times 0 + 275.5 \times 1 = 2058$ Mean $\bar{x} = \Sigma fx \div \Sigma f = 2058 \div 22 = 93.5454... = 93.5$ accidents $\sigma = \sqrt{(\Sigma fx^2 \div \Sigma f - \bar{x}^2)} = \sqrt{(285654 \div 22 - (2058 \div 22)^2)}$ $\sigma = \sqrt{(12984.27... - 8750.75...)} = \sqrt{4233.52...}$ Standard deviation = 65.0655... = 65.1 accidents
4 (c)(i)	The vertical axis should be labelled frequency density, not frequency.
4 (c)(ii)	Her statement may not be true. The 10 to 11 interval has frequency 3, but the 8 to 10 class has frequency $2.5 \times 2 = 5$, and it is not possible to tell from the histogram how those 5 times are spread: all 5 could have been between 9 and 10 minutes.
4 (c)(iii)	The histogram has positive skew (there is a long right-hand tail).
5 (a)	Opportunity sampling.
5 (b)	He should go to different areas (not only his own town centre), as many of his respondents will be from the same community, so the sample is biased.
5 (c)	It is only valid for counties with populations within the range of the data (about 100 000 to 2 800 000); outside this range it is extrapolation (e.g. it predicts about 23 schools for a population of 0).

Question number	
5 (d)	County population = 889 000 = 889 (in 1000s) Number of primary schools = $22.7 + 0.2406 \times 889$ $= 236.5934 \approx 237$ schools
5 (e)	There is strong positive correlation, and here it is likely that a greater population does cause more primary schools (local authorities must provide education); however, increasing the number of schools would not increase the county population.
6 (a)	Resultant $R = F_1 + F_2 + F_3 = (6i - 7j) + (ai + 2j) + (5i + bj)$ $R = (11 + a)i + (b - 5)j$ N2L, $R = ma$: $(11 + a)i + (b - 5)j = 2(7i - 3j) = 14i - 6j$ i: $11 + a = 14 \Rightarrow a = 3$ j: $b - 5 = -6 \Rightarrow b = -1$
6 (b)	Constant velocity $\Rightarrow$ resultant force = 0 $F_4 = -(F_1 + F_2 + F_3) = -(14i - 6j)$ $F_4 = (-14i + 6j)$ N
7 (a)	Total distance = 16 + 16 + 9 + 9 = 50 m Average speed = total distance ÷ total time = 50 ÷ 8 Average speed = 6.25 ms^{-1}
7 (b)	$t = 4$ s and $t = 7$ s (also $t = 0$)
7 (c)(i)	$4 < t < 7$
7 (c)(ii)	$6 < t < 7$

Question number	
8 (a)	Taking upwards as positive, from the kick to the greatest height:
	$s = 10.9 - 0.9 = 10$ m, $v = 0$, $a = -9.8$ ms^{-2}
	$v^2 = u^2 + 2as$: $0 = u^2 + 2(-9.8)(10)$
	$u^2 = 196 \Rightarrow u = 14$ ms^{-1} (as required)
8 (b)	Upwards positive: $s = -0.9$, $u = 14$, $a = -9.8$
	$s = ut + \frac{1}{2}at^2$: $-0.9 = 14t + \frac{1}{2}(-9.8)t^2$
	$4.9t^2 - 14t - 0.9 = 0$
	$t = (14 \pm \sqrt{(14^2 - 4(4.9)(-0.9))}) \div (2 \times 4.9) = (14 \pm \sqrt{213.64}) \div 9.8$
	$t = 2.92004...$ or $t = -0.0629...$ (reject, $t > 0$)
	Time = 2.9 s (1 d.p.)
8 (c)	The ball is modelled as a particle (or: the acceleration due to gravity is constant).
8 (d)	The calculations are independent of the mass of the ball, so a lighter ball with the same initial speed still rises only 10 m, to 10.9 m above the floor, which is below the 11.5 m ceiling.
9 (a)	$v = \int a\,dt = \int (2t - 8)\,dt$
	$v = t^2 - 8t + C$
	When $t = 0$, $v = 12 \Rightarrow C = 12$
	$v = t^2 - 8t + 12$
9 (b)	At $t = 5$: $v = 5^2 - 8(5) + 12 = -3$ ms^{-1}
	For $t > 5$, $a = 2$ (constant), so use $v = u + at$
	$u = -3$, $a = 2$, $t = 14 - 5 = 9$
	$v = -3 + 2(9)$
	$v = 15$ ms^{-1}

Question number

10 (a)



Case 1: $M > 3$, A moves up

Case 2: $M < 3$, A moves down

Case 1: $M > 3$ (A accelerates upwards, B downwards)

N2L for A: $T - 3g = 3(4.2) \Rightarrow T = 12.6 + 29.4 = 42$ N

N2L for B: $Mg - T = M(4.2) \Rightarrow 9.8M - 42 = 4.2M$

$5.6M = 42 \Rightarrow M = 7.5$ kg

Case 2: $M < 3$ (A accelerates downwards, B upwards)

N2L for A: $3g - T = 3(4.2) \Rightarrow T = 29.4 - 12.6 = 16.8$ N

N2L for B: $T - Mg = M(4.2) \Rightarrow 16.8 - 9.8M = 4.2M$

$14M = 16.8 \Rightarrow M = 1.2$ kg

So $T = 42$ N with $M = 7.5$ kg, or $T = 16.8$ N with $M = 1.2$ kg.

10 (b)

We can no longer assume that the tension is the same on both sides of the pulley (throughout the string).