GCE AS/A LEVEL NEW

2300U10-1

S19-2300U10-1

WEDNESDAY, 15 MAY 2019 – MORNING

MATHEMATICS – AS unit 1
PURE MATHEMATICS A

2 hours 30 minutes

2300U101
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0** **1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0 1 Solve the following equation for values of θ between 0° and 360°.

$$3\tan\theta + 2\cos\theta = 0$$ [6]

0 2 Find all the values of k for which the equation $x^2 + 2kx + 9k = -4x$ has two distinct real roots. [7]

0 3 Use an **algebraic method** to solve the equation $12x^3 - 29x^2 + 7x + 6 = 0$.
Show all your working. [6]

0 4 The line L_1 passes through the points $A(-1, 3)$ and $B(2, 9)$. The line L_2 has equation $2y + x = 25$ and intersects L_1 at the point C. L_2 also intersects the x-axis at the point D.

a) Show that the equation of the line L_1 is $y = 2x + 5$. [3]

b) **i)** Find the coordinates of the point D.

ii) Show that L_1 and L_2 are perpendicular.

iii) Determine the coordinates of C. [5]

c) Find the length of CD. [2]

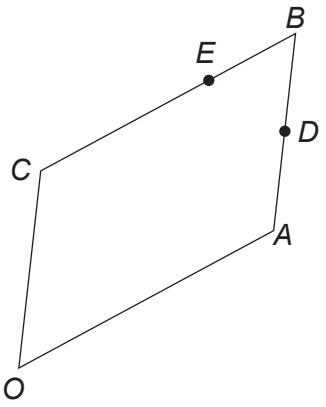
d) Calculate the angle ADB. Give your answer in degrees, correct to one decimal place. [5]

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0 5 Given that n is an integer such that $1 \leqslant n \leqslant 4$, prove that $2n^2 + 5$ is a prime number. [3]

0 6 *OABC* is a parallelogram with *O* as origin.



The position vector of *A* is $\mathbf{a}$ and the position vector of *C* is $\mathbf{c}$. The midpoint of *AB* is *D*. The point *E* divides the line *CB* such that *CE* : *EB* = 2 : 1.

a) Find, in terms of $\mathbf{a}$ and $\mathbf{c}$,

i) the vector **AC**,

ii) the position vector of *D*,

iii) the position vector of *E*. [3]

b) Determine whether or not *DE* is parallel to *AC*, clearly stating your reason. [2]

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0 7 Given that a, b are integers, simplify the following. Show all your working.

a) $\dfrac{2\sqrt{3}+a}{\sqrt{3}-1}$ [3]

b) $\dfrac{2\sqrt{6b^2}}{\sqrt{2}} - \sqrt{27} + \sqrt{192}$ [3]

0 8 **a)** Given that $y = 2x^2 - 5x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ from first principles. [5]

b) Given that $y = \dfrac{16}{5}x^{\frac{1}{4}} + \dfrac{48}{x}$, find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 16$. [3]

0 9 The points $A(-2, 4)$ and $B(6, 10)$ are such that AB is the diameter of a circle.

a) Show that the centre of the circle has coordinates (2, 7). [1]

b) The equation of the circle is $x^2 + y^2 + ax + by + c = 0$.
Determine the values of a, b, c. [3]

A straight line, with equation $y = x + 6$, passes through the point A and cuts the circle again at the point C.

c) Find the coordinates of C. [5]

d) Calculate the exact area of the triangle ABC. [3]

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1 0 **a)** Solve the following simultaneous equations.

$$3^{3x} \times 9^{y} = 27$$

$$2^{-3x} \times 8^{-y} = \frac{1}{64}$$ [6]

b) Find the value of x satisfying the equation

$$\log_a 3 + 2\log_a x - \log_a(x - 1) = \log_a(5x + 2).$$ [7]

1 1 Two quantities are related by the equation $Q = 1{\cdot}25P^3$. Explain why the graph of $\log_{10} Q$ against $\log_{10} P$ is a straight line. State the gradient of the straight line and the intercept on the $\log_{10} Q$ axis of the graph. [4]

1 2 In the binomial expansion of $(2 - 5x)^8$, find

a) the number of terms, [1]

b) the 4th term, when the expansion is in ascending powers of x, [2]

c) the greatest positive coefficient. [3]

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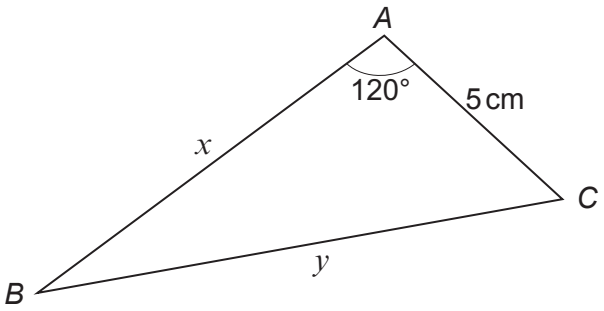
1 3 A curve *C* has equation $y = \frac{1}{9}x^3 - kx + 5$. A point *Q* lies on *C* and is such that the tangent to *C* at *Q* has gradient –9. The x-coordinate of *Q* is 3.

a) Show that $k = 12$. [3]

b) Find the coordinates of each of the stationary points of *C* and determine their nature. [6]

c) Sketch the curve *C*, clearly labelling the stationary points and the point where the curve crosses the y-axis. [2]

1 4 The diagram below shows a triangle *ABC* with *AC* = 5 cm, *AB* = x cm, *BC* = y cm and angle *BAC* = 120°. The area of the triangle *ABC* is 14 cm^2.



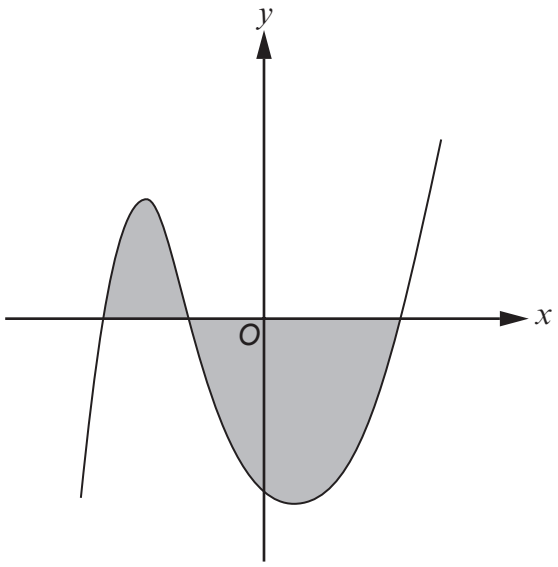
Find the value of x and the value of y. Give your answers correct to 2 decimal places. [6]

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1 5 Prove that $f(x) = x^3 - 6x^2 + 13x - 7$ is an increasing function. [4]

1 6 The diagram below shows a curve with equation $y = (x + 2)(x - 2)(x + 1)$.



Calculate the total area of the two shaded regions. [8]

END OF PAPER

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $3\tan\theta + 2\cos\theta = 0$

$3\frac{\sin\theta}{\cos\theta} + 2\cos\theta = 0$

$3\sin\theta + 2\cos^2\theta = 0$

$3\sin\theta + 2(1 - \sin^2\theta) = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 2$ (no solution, since $-1 \leq \sin\theta \leq 1$)

$\sin\theta = -\frac{1}{2}$: $\theta = 180° + 30° = 210°$, $\theta = 360° - 30° = 330°$

$\theta = 210°,\ 330°$

2 $x^2 + 2kx + 9k + 4x = 0$

$x^2 + (2k + 4)x + 9k = 0$

Two distinct real roots $\Rightarrow b^2 - 4ac > 0$

$(2k + 4)^2 - 4 \times 1 \times 9k > 0$

$4k^2 + 16k + 16 - 36k > 0$

$4k^2 - 20k + 16 > 0$

$k^2 - 5k + 4 > 0$

$(k - 1)(k - 4) > 0$ critical values $k = 1$, $k = 4$

$k < 1$ or $k > 4$

3 Let $f(x) = 12x^3 - 29x^2 + 7x + 6$

$f(1) = 12 - 29 + 7 + 6 = -4 \neq 0$

Question number

$f(2) = 96 - 116 + 14 + 6 = 0$, so $(x - 2)$ is a factor

$f(x) = (x - 2)(12x^2 + ax + b)$

x^2: $-29 = a - 24 \Rightarrow a = -5$; constant: $6 = -2b \Rightarrow b = -3$

$f(x) = (x - 2)(12x^2 - 5x - 3)$

$f(x) = (x - 2)(3x + 1)(4x - 3)$

$f(x) = 0$: $x = 2$, $x = -\frac{1}{3}$, $x = \frac{3}{4}$

4 (a) Gradient of L_1 = gradient of $AB = \frac{9 - 3}{2 - (-1)} = \frac{6}{3} = 2$

$y - 3 = 2(x + 1)$

$y - 3 = 2x + 2$

$y = 2x + 5$ as required

4(b)(i) $2y + x = 25$: when $y = 0$, $x = 25$

$D(25, 0)$

4(b)(ii) $2y = -x + 25 \Rightarrow y = -\frac{1}{2}x + \frac{25}{2}$, so gradient of $L_2 = -\frac{1}{2}$

grad L_1 × grad $L_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

Therefore L_1 and L_2 are perpendicular.

4(b)(iii) Substitute $y = 2x + 5$ into $2y + x = 25$:

$2(2x + 5) + x = 25$

$5x + 10 = 25 \Rightarrow x = 3$, $y = 2(3) + 5 = 11$

$C(3, 11)$

4 (c) $CD = \sqrt{(25 - 3)^2 + (0 - 11)^2} = \sqrt{484 + 121}$

Question number	
	$CD = \sqrt{605} = 11\sqrt{5}$ $(= 24.6)$
4 (d)	$L_1 \perp L_2$, so angle ACD = angle BCD = 90°
	$AC = \sqrt{(3-(-1))^2 + (11-3)^2} = \sqrt{4^2 + 8^2} = 4\sqrt{5}$
	$BC = \sqrt{(3-2)^2 + (11-9)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$
	$\tan \angle ADC = \frac{AC}{DC} = \frac{4\sqrt{5}}{11\sqrt{5}} = \frac{4}{11} \Rightarrow \angle ADC = 19.983...°$
	$\tan \angle BDC = \frac{BC}{DC} = \frac{\sqrt{5}}{11\sqrt{5}} = \frac{1}{11} \Rightarrow \angle BDC = 5.194...°$
	$\angle ADB = \angle ADC - \angle BDC = 19.983...° - 5.194...° = 14.789...°$
	$\angle ADB = 14.8°$ (1 d.p.)
5	Proof by exhaustion: test every integer n with $1 \leq n \leq 4$.
	$n = 1$: $2(1)^2 + 5 = 7$
	$n = 2$: $2(2)^2 + 5 = 13$
	$n = 3$: $2(3)^2 + 5 = 23$
	$n = 4$: $2(4)^2 + 5 = 37$
	7, 13, 23 and 37 are all prime numbers.
	Therefore $2n^2 + 5$ is prime for all integers n with $1 \leq n \leq 4$.
6(a)(i)	$\underline{AC} = \underline{AO} + \underline{OC} = -\underline{a} + \underline{c} = \underline{c} - \underline{a}$
6(a)(ii)	$\underline{AB} = \underline{OC} = \underline{c}$, so $\underline{OD} = \underline{a} + \frac{1}{2}\underline{c}$
6(a)(iii)	$\underline{CB} = \underline{OA} = \underline{a}$, $CE = \frac{2}{3}CB$, so $\underline{OE} = \underline{c} + \frac{2}{3}\underline{a}$

Question number

6 (b) $\underline{DE} = \underline{OE} - \underline{OD} = \underline{c} + \frac{2}{3}\underline{a} - \underline{a} - \frac{1}{2}\underline{c} = \frac{1}{2}\underline{c} - \frac{1}{3}\underline{a}$

$\underline{AC} = \underline{c} - \underline{a}$: a multiple $k(\underline{c} - \underline{a})$ has coefficients of $\underline{c}$ and $\underline{a}$

equal in size, but $\frac{1}{2} \neq \frac{1}{3}$, so $\underline{DE} \neq k\underline{AC}$ for any scalar k.

Hence DE is not parallel to AC.

7 (a) $\frac{2\sqrt{3} + a}{\sqrt{3} - 1} = \frac{(2\sqrt{3} + a)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$= \frac{2 \times 3 + 2\sqrt{3} + a\sqrt{3} + a}{3 - 1}$

$= \frac{1}{2}(6 + 2\sqrt{3} + a\sqrt{3} + a)$

$= \frac{1}{2}(6 + a) + \frac{1}{2}(2 + a)\sqrt{3}$

7 (b) $\frac{2\sqrt{(6b^2)}}{\sqrt{2}} = \frac{2b\sqrt{6}}{\sqrt{2}} = 2b\sqrt{3}$

$\sqrt{27} = \sqrt{(9 \times 3)} = 3\sqrt{3}, \qquad \sqrt{192} = \sqrt{(64 \times 3)} = 8\sqrt{3}$

$\frac{2\sqrt{(6b^2)}}{\sqrt{2}} - \sqrt{27} + \sqrt{192} = 2b\sqrt{3} - 3\sqrt{3} + 8\sqrt{3}$

$= 2b\sqrt{3} + 5\sqrt{3}$

$= (2b + 5)\sqrt{3}$

8 (a) $y + \delta y = 2(x + \delta x)^2 - 5(x + \delta x)$

$y + \delta y = 2x^2 + 4x\delta x + 2(\delta x)^2 - 5x - 5\delta x$

Subtract $y = 2x^2 - 5x$:

$\delta y = 4x\delta x + 2(\delta x)^2 - 5\delta x$

Question number

$\frac{\delta y}{\delta x} = 4x + 2\delta x - 5$

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} (4x + 2\delta x - 5)$

$\frac{dy}{dx} = 4x - 5$

8 (b) $y = \frac{16}{5}x^{1/4} + 48x^{-1}$

$\frac{dy}{dx} = \frac{16}{5} \times \frac{1}{4}x^{-3/4} - 48x^{-2} = \frac{4}{5}x^{-3/4} - \frac{48}{x^2}$

$x = 16$: $\frac{dy}{dx} = \frac{4}{5} \times 16^{-3/4} - \frac{48}{16^2} = \frac{4}{5} \times \frac{1}{8} - \frac{48}{256}$

$= \frac{1}{10} - \frac{3}{16} = \frac{8}{80} - \frac{15}{80}$

$\frac{dy}{dx} = -\frac{7}{80}$ $(= -0.0875)$

9 (a) Centre = midpoint of AB $= \left(\frac{-2+6}{2}, \frac{4+10}{2}\right) = (2, 7)$

9 (b) Radius $= \sqrt{(6-2)^2 + (10-7)^2} = \sqrt{3^2 + 4^2} = 5$

$(x-2)^2 + (y-7)^2 = 5^2$

$x^2 - 4x + 4 + y^2 - 14y + 49 - 25 = 0$

$x^2 + y^2 - 4x - 14y + 28 = 0$

$a = -4, \quad b = -14, \quad c = 28$

9 (c) Substitute $y = x + 6$ into $(x-2)^2 + (y-7)^2 = 25$:

$(x-2)^2 + (x-1)^2 = 25$

Question number

$x^2 - 4x + 4 + x^2 - 2x + 1 = 25$

$2x^2 - 6x - 20 = 0 \Rightarrow x^2 - 3x - 10 = 0$

$(x + 2)(x - 5) = 0 \Rightarrow x = -2$ (point A) or $x = 5$

$x = 5$: $y = 5 + 6 = 11$

$C(5, 11)$

9 (d) AB is a diameter, so angle ACB = 90° (angle in a semicircle)

$AC = \sqrt{(5 - (-2))^2 + (11 - 4)^2} = \sqrt{49 + 49} = 7\sqrt{2}$

$BC = \sqrt{(6 - 5)^2 + (10 - 11)^2} = \sqrt{1 + 1} = \sqrt{2}$

Area $ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times 7\sqrt{2} \times \sqrt{2} = \frac{1}{2} \times 14$

Area $ABC = 7$

10 (a) $3^{3x} \times (3^2)^y = 3^3 \Rightarrow 3^{3x + 2y} = 3^3$

$3x + 2y = 3$ (1)

$2^{-3x} \times (2^3)^{-y} = 2^{-6} \Rightarrow 2^{-3x - 3y} = 2^{-6}$

$3x + 3y = 6$ (2)

(2) − (1): $y = 3$

$3x + 6 = 3 \Rightarrow x = -1$

$x = -1, \quad y = 3$

10 (b) $2\log_a x = \log_a x^2$

$\log_a 3 + \log_a x^2 - \log_a (x - 1) = \log_a (5x + 2)$

$\log_a\left(\frac{3x^2}{x - 1}\right) = \log_a (5x + 2)$

$\frac{3x^2}{x - 1} = 5x + 2 \Rightarrow 3x^2 = (5x + 2)(x - 1)$

$3x^2 = 5x^2 - 3x - 2$

Question number

$2x^2 - 3x - 2 = 0$

$(2x + 1)(x - 2) = 0$

$x = -\frac{1}{2}$ or $x = 2$

$x = -\frac{1}{2}$ is rejected, since $\log_a x$ is undefined for $x < 0$

$\therefore\ x = 2$

11 $Q = 1.25P^3$

Take logs: $\log_{10}Q = \log_{10}(1.25P^3)$

$\log_{10}Q = 3\log_{10}P + \log_{10}1.25$

This is of the form $Y = mX + c$ with $Y = \log_{10}Q$, $X = \log_{10}P$,

so the graph of $\log_{10}Q$ against $\log_{10}P$ is a straight line.

Gradient = 3

Intercept on the $\log_{10}Q$ axis = $\log_{10}1.25 = 0.09691... = 0.0969$

12 (a) $8 + 1 = 9$ terms

12 (b) 4th term = ${}^8C_3(2)^5(-5x)^3$

$= 56 \times 32 \times (-125x^3)$

$= -224000x^3$

12 (c) General term: ${}^8C_r(2)^{8-r}(-5x)^r$; positive coefficients when r is even

$r = 0$: 256 $r = 2$: $28 \times 64 \times 25 = 44800$

$r = 4$: $70 \times 16 \times 625 = 700000$ $r = 8$: $(-5)^8 = 390625$

$r = 6$ (7th term): ${}^8C_6(2)^2(-5)^6 = 28 \times 4 \times 15625 = 1750000$

Greatest positive coefficient = 1750000

13 (a) $\frac{dy}{dx} = \frac{1}{3}x^2 - k$

Question number

At Q, $x = 3$ and $\frac{dy}{dx} = -9$: $\frac{1}{3} \times 3^2 - k = -9$

$3 - k = -9 \Rightarrow k = 12$ as required

13 (b)

$y = \frac{1}{9}x^3 - 12x + 5, \quad \frac{dy}{dx} = \frac{1}{3}x^2 - 12$

Stationary points: $\frac{dy}{dx} = 0 \Rightarrow \frac{1}{3}x^2 - 12 = 0 \Rightarrow x^2 = 36$

$x = -6$ or $x = 6$

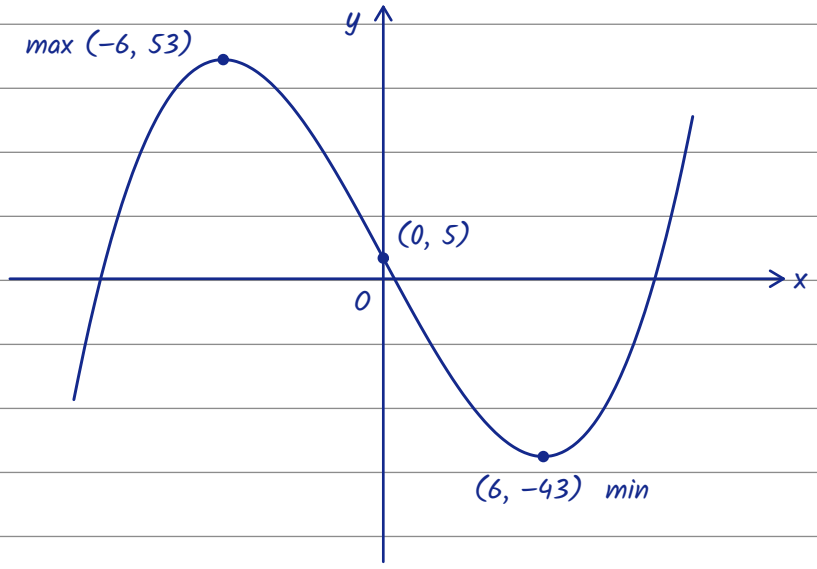
$x = -6$: $y = \frac{1}{9}(-216) + 72 + 5 = 53$; $x = 6$: $y = \frac{1}{9}(216) - 72 + 5 = -43$

$\frac{d^2y}{dx^2} = \frac{2}{3}x$

$x = -6$: $\frac{d^2y}{dx^2} = -4 < 0$, so $(-6, 53)$ is a maximum point

$x = 6$: $\frac{d^2y}{dx^2} = 4 > 0$, so $(6, -43)$ is a minimum point

13 (c)



Question number

14

Area $= \frac{1}{2}bc\sin A$: $\quad 14 = \frac{1}{2} \times 5 \times x \times \sin 120°$

$14 = \frac{5\sqrt{3}}{4}x \quad \Rightarrow \quad x = \frac{56}{5\sqrt{3}} = \frac{56\sqrt{3}}{15} = 6.4663...$

$x = 6.47$ cm (2 d.p.)

Cosine rule: $y^2 = 5^2 + x^2 - 2 \times 5 \times x \times \cos 120°$

$y^2 = 25 + 6.4663...^2 - 2 \times 5 \times 6.4663... \times (-0.5)$

$y^2 = 25 + 41.813... + 32.331... = 99.144...$

$y = 9.9571...$

$y = 9.96$ cm (2 d.p.)

15

$f'(x) = 3x^2 - 12x + 13$

$f'(x) = 3(x^2 - 4x) + 13$

$f'(x) = 3(x - 2)^2 - 12 + 13$

$f'(x) = 3(x - 2)^2 + 1$

$(x - 2)^2 \geqslant 0$ for all x, so $f'(x) \geqslant 1 > 0$ for all values of x.

Hence $f(x)$ is an increasing function.

16

Curve cuts the x-axis when $x = -2$, $x = -1$ and $x = 2$

$y = (x^2 - 4)(x + 1) = x^3 + x^2 - 4x - 4$

$$\int_{-2}^{-1} (x^3 + x^2 - 4x - 4)\,dx = \left[\frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x\right]_{-2}^{-1}$$

$$= \left(\frac{1}{4} - \frac{1}{3} - 2 + 4\right) - \left(4 - \frac{8}{3} - 8 + 8\right)$$

$$= \frac{23}{12} - \frac{4}{3} = \frac{7}{12}$$

$$\int_{-1}^{2} (x^3 + x^2 - 4x - 4)\,dx = \left[\frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x\right]_{-1}^{2}$$

Question number

$$= \left(4 + \frac{8}{3} - 8 - 8\right) - \frac{23}{12}$$

$$= -\frac{28}{3} - \frac{23}{12} = -\frac{135}{12} = -\frac{45}{4}$$

Total area $= \frac{7}{12} + \frac{45}{4} = \frac{7}{12} + \frac{135}{12} = \frac{142}{12}$

Total area $= \frac{71}{6}$ $(= 11.83)$