GCE AS/A LEVEL – NEW

2300U20-1

S18-2300U20-1

MATHEMATICS – AS unit 2
APPLIED MATHEMATICS A

WEDNESDAY, 23 MAY 2018 – MORNING

1 hour 45 minutes

2300U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Write only within the white area of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer

e.g. **0** **1** .

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

© WJEC CBAC Ltd.

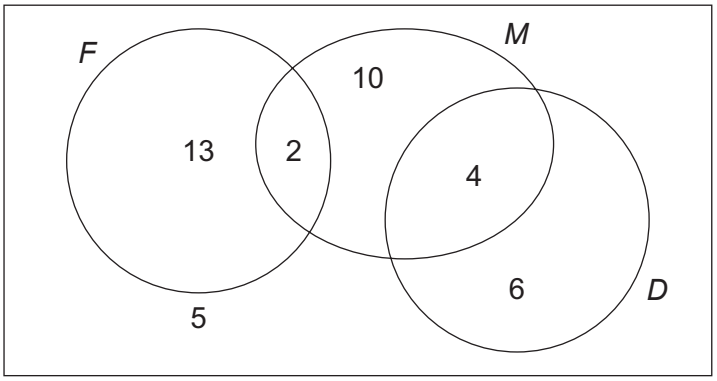
stemup.co.uk

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0 1 The random variable X has the binomial distribution B(16, 0·3).
Showing your calculation, find $P(X = 7)$. [2]

0 2 The Venn diagram shows the subjects studied by 40 sixth form students. *F* represents the set of students who study French, *M* represents the set of students who study Mathematics and *D* represents the set of students who study Drama.
The diagram shows the number of students in each set.



a) Explain what $M \cap D'$ means in this context. [1]

b) One of these students is chosen at random. Find the probability that this student studies

i) exactly two of these subjects,

ii) Mathematics or French or both. [3]

c) Determine whether studying Mathematics and studying Drama are statistically independent for these students. [3]

0 3 Naomi produces oak tabletops, each of area 4·8 m^2. Defects in the oak tabletops occur randomly at a rate of 0·25 per m^2.

a) Find the probability that a randomly chosen tabletop will contain at most 2 defects. [3]

b) Find the probability that, in a random sample of 7 tabletops, exactly 4 will contain at most 2 defects each. [3]

© WJEC CBAC Ltd.

stemup.co.uk

0 4 Edward can correctly identify 20% of types of wild flower. He studies some books to see if he can improve how often he can correctly identify types of wild flower. He collects a random sample of 10 types of wild flower in order to test whether or not he has improved.

a) **i)** Write suitable hypotheses for this test.

ii) State a suitable test statistic that he could use. [2]

b) Using a 5% level of significance, find the critical region for this test. [3]

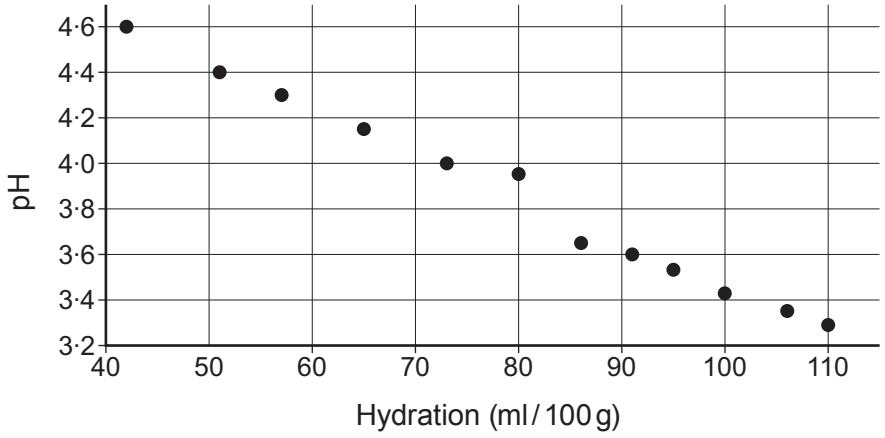
c) State the probability of a Type I error for this test and explain what it means in this context. [2]

d) Edward correctly identifies 4 of the 10 types of wild flower he collected.
What conclusion should Edward reach? [2]

0 5 A baker is aware that the pH of his sourdough, y, and the hydration, x, affect the taste and texture of the final product. The hydration is measured in ml of water per 100 g of flour (ml / 100 g). The baker researches how the pH of his sourdough changes as the hydration changes.

The results of his research are shown in the diagram below.

How pH changes with hydration



a) Describe the relationship between pH and hydration. [2]

b) The equation of the regression line for y on x is

$$y = 5{\cdot}4 - 0{\cdot}02x.$$

i) Interpret the gradient and intercept of the regression line in this context.

ii) Estimate the pH of the sourdough when the hydration is 20 ml / 100 g.
Comment on the reliability of this estimate. [4]

© WJEC CBAC Ltd.

stemup.co.uk

2300U201
03

0 6 Basel is a keen learner of languages. He finds a website on which a large number of language tutors offer their services. Basel records the cost, in dollars, of a one hour lesson from a random sample of tutors. He puts the data into a computer program which gives the following summary statistics.

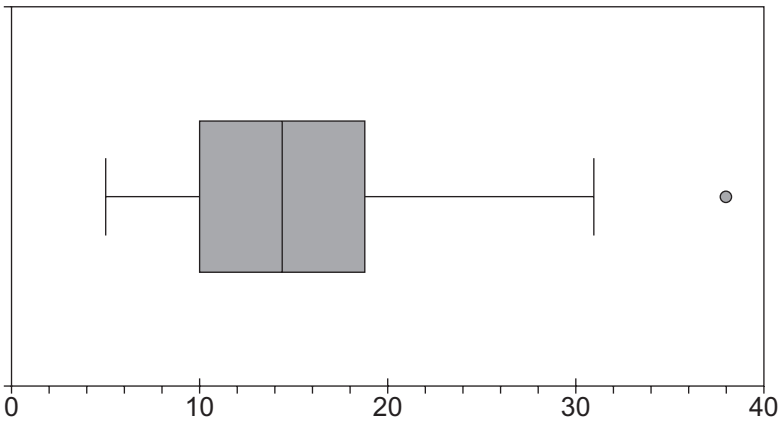
```
Cost per 1 hour lesson
Min.     :10.0
1st Qu.  :16.0
Median   :17.2
Mean     :19.8
3rd Qu.  :21.0
Max.     :40.0
```

a) Showing all calculations, comment on any outliers for the cost of a one hour lesson with a language tutor. [4]

b) Describe the skewness of the data and explain what it means in this context. [2]

Dafydd is also a keen learner of languages. He takes his own random sample of the cost, in dollars, for a one hour lesson. He produces the following box plot.

Cost in dollars for a one hour lesson



c) **i)** What will happen to the mean if the outlier is removed?

ii) What will happen to the median if the outlier is removed? [2]

d) Compare and contrast the distributions of the cost of one hour language lessons for Dafydd's sample and Basel's sample. [2]

© WJEC CBAC Ltd.

stemup.co.uk

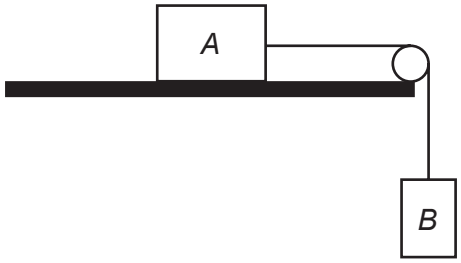
Section B: Mechanics

0 7 A particle moves along the horizontal x-axis so that its velocity $v\,\text{ms}^{-1}$ at time t seconds is given by

$$v = 6t^2 - 8t - 5.$$

At time $t = 1$, the particle's displacement from the origin is -4 m. Find an expression for the displacement of the particle at time t seconds. [3]

0 8 The diagram shows two objects *A* and *B*, of mass 3 kg and 5 kg respectively, connected by a light inextensible string passing over a light smooth pulley fixed at the end of a smooth horizontal surface. Object *A* lies on the horizontal surface and object *B* hangs freely below the pulley.



Initially, *B* is supported so that the objects are at rest with the string just taut. Object *B* is then released.

a) Find the magnitude of the acceleration of *A* and the tension in the string. [6]

b) State briefly what effect a rough pulley would have on the tension in the string. [1]

0 9 Three forces $\mathbf{L}$, $\mathbf{M}$ and $\mathbf{N}$ are given by

$$\mathbf{L} = 2\mathbf{i} + 5\mathbf{j},$$

$$\mathbf{M} = 3\mathbf{i} - 22\mathbf{j},$$

$$\mathbf{N} = 4\mathbf{i} - 23\mathbf{j}.$$

Find the magnitude and direction of the resultant of the three forces. [6]

TURN OVER

© WJEC CBAC Ltd.

stemup.co.uk

1 0 A person, of mass 68 kg, stands in a lift which is moving upwards with constant acceleration. The lift is of mass 770 kg and the tension in the lift cable is 8000 N.

a) Determine the acceleration of the lift, giving your answer correct to two decimal places. [3]

b) State whether the lift is getting faster, staying at the same speed or slowing down. [1]

c) Calculate the magnitude of the reaction of the floor of the lift on the person. [3]

1 1 A vehicle moves along a straight horizontal road. Points *A* and *B* lie on the road. As the vehicle passes point *A*, it is moving with constant speed 15 ms^{-1}. It travels with this constant speed for 2 minutes before a constant deceleration is applied for 12 seconds so that it comes to rest at point *B*.

a) Find the distance *AB*. [3]

The vehicle then reverses with a constant acceleration of 2 ms^{-2} for 8 seconds, followed by a constant deceleration of 1·6 ms^{-2}, coming to rest at the point *C*, which is between *A* and *B*.

b) Calculate the time it takes for the vehicle to reverse from *B* to *C*. [4]

c) Sketch a velocity-time graph for the motion of the vehicle. [3]

d) Determine the distance *AC*. [2]

END OF PAPER

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

© WJEC CBAC Ltd.

stemup.co.uk

Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 $X \sim B(16, 0.3)$

$P(X = 7) = {}^{16}C_7 \times 0.3^7 \times 0.7^9$

$= 11440 \times 0.3^7 \times 0.7^9$

$= 0.100961... = 0.101$ (3 s.f.)

2 (a) $M \cap D'$ is the set of students who study Mathematics but do not study Drama.

2 (b)(i) $P(\text{exactly two subjects}) = \frac{2 + 4}{40} = \frac{6}{40} = \frac{3}{20}$ $(= 0.15)$

2 (b)(ii) $P(M \cup F) = \frac{13 + 2 + 10 + 4}{40} = \frac{29}{40}$ $(= 0.725)$

2 (c) $P(M) = \frac{16}{40} = 0.4$, $P(D) = \frac{10}{40} = 0.25$, $P(M \cap D) = \frac{4}{40} = 0.1$

$P(M) \times P(D) = 0.4 \times 0.25 = 0.1 = P(M \cap D)$

Since $P(M) \times P(D) = P(M \cap D)$, studying Mathematics and studying Drama are statistically independent for these students.

3 (a) Let X = number of defects on a tabletop.

Mean $= 4.8 \times 0.25 = 1.2$, so $X \sim Po(1.2)$

$P(X \leq 2) = e^{-1.2}(1 + 1.2 + 1.2^2/2!)$

$= 0.879487... = 0.8795$ (4 d.p.)

Question number	
3 (b)	Let Y = number of the 7 tabletops with at most 2 defects. $Y \sim B(7, 0.8795)$ $P(Y = 4) = {}^{7}C_{4} \times 0.8795^{4} \times 0.1205^{3}$ $= 35 \times 0.8795^{4} \times 0.1205^{3} = 0.036641... = 0.0366$ (3 s.f.)
4 (a)(i)	Let p = probability that Edward correctly identifies a type of wild flower. H_0: $p = 0.2$ H_1: $p > 0.2$
4 (a)(ii)	X = the number of the 10 types of wild flower that he correctly identifies.
4 (b)	Under H_0, $X \sim B(10, 0.2)$ $P(X \geqslant 4) = 1 - P(X \leqslant 3) = 1 - 0.8791 = 0.1209 > 0.05$ $P(X \geqslant 5) = 1 - P(X \leqslant 4) = 1 - 0.9672 = 0.0328 < 0.05$ Critical region: $X \geqslant 5$
4 (c)	$P(\text{Type I error}) = P(X \geqslant 5 \mid p = 0.2) = 0.0328$ It is the probability of concluding that Edward has improved his ability to correctly identify wild flowers when in fact he has not improved.
4 (d)	4 is not in the critical region ($P(X \geqslant 4) = 0.1209 > 0.05$), so do not reject H_0. There is insufficient evidence at the 5% level to conclude that Edward has improved his ability to correctly identify wild flowers.
5 (a)	There is a strong negative linear relationship (correlation): the higher the hydration, the lower the pH.

Question number	
5 (b)(i)	Gradient: each additional 1 ml of water per 100 g of flour decreases the pH by 0.02 on average. Intercept: at zero hydration (0 ml / 100 g) the pH would be 5.4.
5 (b)(ii)	$y = 5.4 - 0.02 \times 20 = 5.4 - 0.4 = 5$, so pH ≈ 5 Unreliable: 20 ml / 100 g is outside the range of the data (about 40 to 110 ml / 100 g), so this is extrapolation.
6 (a)	$IQR = 21 - 16 = 5$, $1.5 \times IQR = 1.5 \times 5 = 7.5$ Lower limit: $16 - 7.5 = 8.5$. Min = 10 > 8.5, so no outliers below. Upper limit: $21 + 7.5 = 28.5$. Max = 40 > 28.5, so 40 is an outlier. There may be other outliers above 28.5 as well, but the summary statistics do not show the individual values, so we cannot tell.
6 (b)	Positive skew ($Q_3 - Q_2 = 3.8 > Q_2 - Q_1 = 1.2$, mean 19.8 > median 17.2). Most tutors are relatively cheap, but a few tutors are very expensive.
6 (c)(i)	The mean will decrease.
6 (c)(ii)	The median may stay the same or it may decrease: we cannot tell without knowing the individual values.
6 (d)	Dafydd's lessons are cheaper on average than Basel's (median about \$14 compared with \$17.20). Dafydd's lessons are more variable in cost than Basel's (IQR about \$9 compared with \$5). Both distributions are positively skewed.

Question number

7

$x = \int (6t^2 - 8t - 5)\, dt$

$x = \frac{6t^3}{3} - \frac{8t^2}{2} - 5t + C = 2t^3 - 4t^2 - 5t + C$

When $t = 1$, $x = -4$: $\quad -4 = 2 - 4 - 5 + C$

$C = 3$

$x = 2t^3 - 4t^2 - 5t + 3$

8 (a)

N2L for A (horizontally): $\quad T = 3a$

N2L for B (vertically downwards): $\quad 5g - T = 5a$

Adding: $\quad 5g = 8a$

$a = \frac{5 \times 9.8}{8} = 6.125 \text{ m s}^{-2}$

$T = 3 \times 6.125 = 18.375 \text{ N}$

Acceleration of A = 6.125 m s^{-2}, $\quad$ tension = 18.375 N

8 (b)

With a rough pulley the tension in the string would not be the same on both sides of the pulley.

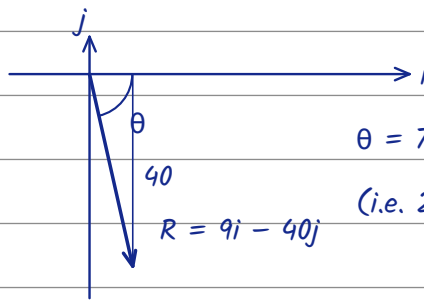
9

$R = L + M + N = (2 + 3 + 4)i + (5 - 22 - 23)j$

$R = 9i - 40j$

$|R| = \sqrt{(9^2 + 40^2)} = \sqrt{1681} = 41 \text{ N}$

$\theta = \tan^{-1}\left(\frac{40}{9}\right) = 77.32°$



θ = 77.32° below the positive i-direction

(i.e. 282.68° anticlockwise from i)

Magnitude 41 N, direction 77.32° clockwise from (below) the i-direction.

Question number

10 (a) N2L for lift and person together (upwards positive):

$8000 - (770 + 68)g = (770 + 68)a$

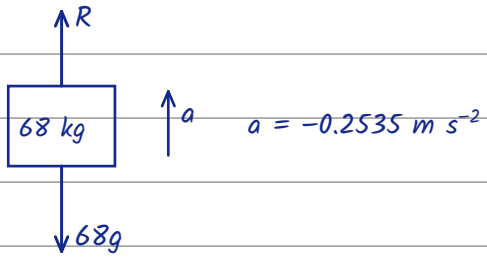
$8000 - 838 \times 9.8 = 838a$

$-212.4 = 838a$

$a = -0.253460... = -0.25 \text{ m s}^{-2}$ (2 d.p.)

10 (b) The acceleration is negative (opposite to the upward motion), so the lift is slowing down.

10 (c)



N2L for the person (upwards positive): $R - 68g = 68a$

$R = 68 \times 9.8 + 68 \times (-0.253460...)$

$R = 666.4 - 17.235... = 649.16... \approx 649 \text{ N}$

11 (a) Constant speed: distance $= 15 \times 120 = 1800 \text{ m}$

Deceleration: $s = \frac{1}{2}(u + v)t = \frac{1}{2}(15 + 0) \times 12 = 90 \text{ m}$

$AB = 1800 + 90 = 1890 \text{ m}$

11 (b) Accelerating: $v = u + at = 0 + 2 \times 8 = 16 \text{ m s}^{-1}$

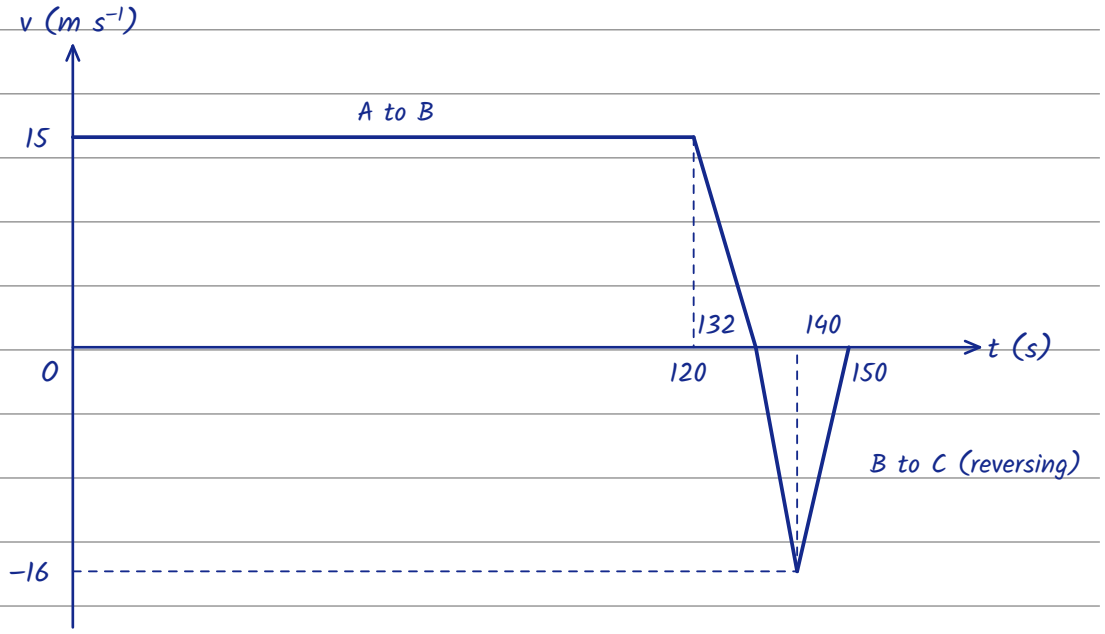
Decelerating: $v = u + at$, $u = 16$, $v = 0$, $a = -1.6$

$0 = 16 - 1.6t \Rightarrow t = 10 \text{ s}$

Time from B to C $= 8 + 10 = 18 \text{ s}$

Question number

11 (c)



11 (d)

Distance = area under graph

$BC = \frac{1}{2} \times 18 \times 16 = 144$ m

$AC = AB - BC = 1890 - 144 = 1746$ m