# GCE AS/A LEVEL – NEW

2300U10-1

S18-2300U10-1

# MATHEMATICS – AS unit 1
## PURE MATHEMATICS A

WEDNESDAY, 16 MAY 2018 – MORNING

2 hours 30 minutes

2300U101
01

### ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

### INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Write only within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0** **1** .

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

### INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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**Reminder**: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

**0 1** Showing all your working, simplify

**a)** $\dfrac{24\sqrt{a}}{\left(\sqrt{a}+3\right)^2-\left(\sqrt{a}-3\right)^2}$, [3]

**b)** $\dfrac{3\sqrt{7}+5\sqrt{3}}{\sqrt{7}+\sqrt{3}}$. [4]

**0 2** The points *A* and *B* have coordinates (–1, 10) and (5, 1) respectively. The straight line *L* has equation $2x - 3y + 6 = 0$.

**a)** The line *L* intersects the line *AB* at the point *C*. Find the coordinates of *C*. [5]

**b)** Determine the ratio in which the line *L* divides the line *AB*. [2]

**c)** The line *L* crosses the $x$-axis at the point *D*. Find the coordinates of *D*. [1]

**d)** **i)** Show that *L* is perpendicular to *AB*.

**ii)** Calculate the area of the triangle *ACD*. [6]

**0 3** Solve the following equation for values of $\theta$ between 0° and 360°.

$$2 - 3\cos^2\theta = 2\sin\theta.$$ [6]

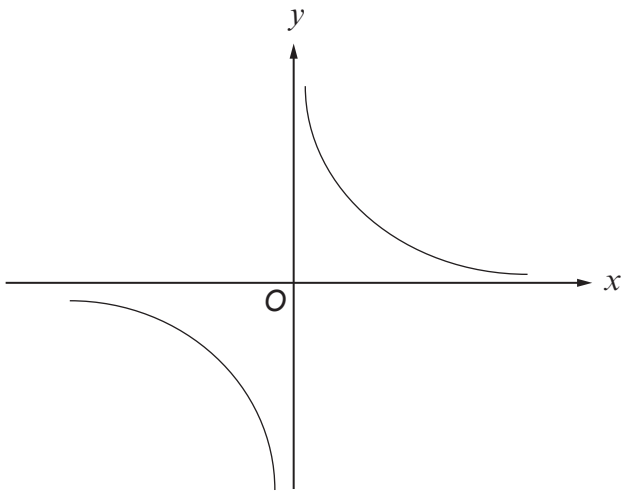
**0 4** **a)** Given that $y = \dfrac{5}{x} + 6\sqrt[3]{x}$, find $\dfrac{dy}{dx}$ when $x = 8$. [3]

**b)** Find $\int\left(5x^{\frac{3}{2}} + 12x^{-5} + 7\right)dx$. [3]

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**0 5** The diagram below shows a sketch of $y = f(x)$.

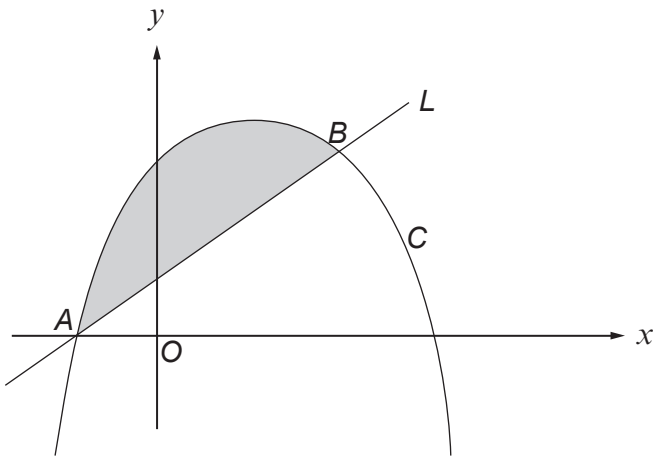



**a)** Sketch the graph of $y = 4 + f(x)$, clearly indicating any asymptotes. [2]

**b)** Sketch the graph of $y = f(x - 3)$, clearly indicating any asymptotes. [2]



**0 6**




The sketch shows the curve *C* with equation $y = 14 + 5x - x^2$ and line *L* with equation $y = x + 2$. The line intersects the curve at the points *A* and *B*.

**a)** Find the coordinates of *A* and *B*. [4]

**b)** Calculate the area enclosed by *L* and *C*. [6]

# TURN OVER

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**0 7** Prove that

$$\frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta} \equiv \tan\theta\,.$$ [3]

**0 8** Given that $(x - 2)$ and $(x + 2)$ are factors of the polynomial $2x^3 + px^2 + qx - 12$,

**a)** find the values of $p$ and $q$, [4]

**b)** determine the other factor of the polynomial. [1]

**0 9** The triangle $ABC$ is such that $AC = 16$ cm, $AB = 25$ cm and $A\hat{B}C = 32°$. Find two possible values for the area of the triangle $ABC$. [5]

**1 0** **a)** Use the binomial theorem to expand $\left(a + \sqrt{b}\right)^4$. [2]

**b)** Hence, deduce an expression in terms of $a$ and $b$ for $\left(a + \sqrt{b}\right)^4 + \left(a - \sqrt{b}\right)^4$. [2]

**1 1** **a)** The vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = 9\mathbf{i} - 40\mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. Determine the range of values for $\mu$ such that $\mu|\mathbf{v}| > |\mathbf{u}|$. [3]

**b)** The point $A$ has position vector $11\mathbf{i} - 4\mathbf{j}$ and the point $B$ has position vector $21\mathbf{i} + \mathbf{j}$. Determine the position vector of the point $C$, which lies between $A$ and $B$, such that $AC : CB$ is $2 : 3$. [3]

**1 2** Find the values of $m$ for which the equation $4x^2 + 8x - 8 = m(4x - 3)$ has real roots. [5]

**1 3** A curve $C$ has equation $y = x^3 - 3x^2$.

**a)** Find the stationary points of $C$ and determine their nature. [7]

**b)** Draw a sketch of $C$, clearly indicating the stationary points and the points where the curve crosses the coordinate axes. [3]

**c)** **Without performing the integration**, state whether $\int_0^3 (x^3 - 3x^2)\mathrm{d}x$ is positive or negative, giving a reason for your answer. [1]

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**1 4** In each of the two statements below, $c$ and $d$ are real numbers. One of the statements is true, while the other is false.

A : $(2c - d)^2 = 4c^2 - d^2$, for all values of $c$ and $d$.

B : $8c^3 - d^3 = (2c - d)(4c^2 + 2cd + d^2)$, for all values of $c$ and $d$.

**a)** Identify the statement which is false. Show, by counter example, that this statement is in fact false. [2]

**b)** Identify the statement which is true. Give a proof to show that this statement is in fact true. [2]

**1 5** The value of a car, £$V$, may be modelled as a continuous variable. At time $t$ years, the value of the car is given by $V = Ae^{kt}$, where $A$ and $k$ are constants. When the car is new, it is worth £30 000. When the car is two years old, it is worth £20 000. Determine the value of the car when it is six years old, giving your answer correct to the nearest £100. [6]

**1 6** The curve *C* has equation $y = 7 + 13x - 2x^2$. The point *P* lies on *C* and is such that the tangent to *C* at *P* has equation $y = x + c$, where $c$ is a constant. Find the coordinates of *P* and the value of $c$. [5]

**1 7** **a)** Solve $2\log_{10}x = 1 + \log_{10}5 - \log_{10}2$. [4]

**b)** Solve $3 = 2e^{0{\cdot}5x}$. [2]

**c)** Express $4^x - 10 \times 2^x$ in terms of $y$, where $y = 2^x$.

Hence solve the equation $4^x - 10 \times 2^x = -16$. [5]

**1 8** The coordinates of three points *A*, *B*, *C* are (4, 6), (–3, 5) and (5, –1) respectively.

**a)** Show that $B\hat{A}C$ is a right angle. [3]

**b)** A circle passes through all three points *A*, *B*, *C*. Determine the equation of the circle. [5]

**END OF PAPER**

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| Surname | StemUp Model Solution |
|---|---|
| Answer booklet | Write the question number in the left-hand margin at the start of each answer. |

Question number

1 (a)

$$\frac{24\sqrt{a}}{(\sqrt{a}+3)^2-(\sqrt{a}-3)^2} = \frac{24\sqrt{a}}{[(\sqrt{a}+3)+(\sqrt{a}-3)][(\sqrt{a}+3)-(\sqrt{a}-3)]}$$

$$= \frac{24\sqrt{a}}{(2\sqrt{a})(6)} = \frac{24\sqrt{a}}{12\sqrt{a}}$$

$$= 2$$

1 (b)

$$\frac{3\sqrt{7}+5\sqrt{3}}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{3\times 7 - 3\sqrt{21} + 5\sqrt{21} - 5\times 3}{7 - \sqrt{21} + \sqrt{21} - 3}$$

$$= \frac{21 - 15 + 2\sqrt{21}}{4} = \frac{6+2\sqrt{21}}{4}$$

$$= \frac{3+\sqrt{21}}{2}$$

2 (a)

Gradient of AB $= \frac{1-10}{5-(-1)} = \frac{-9}{6} = -\frac{3}{2}$

Equation of AB: $y - 1 = -\frac{3}{2}(x-5)$

$2y - 2 = -3x + 15 \Rightarrow 3x + 2y = 17$

L: $2x - 3y = -6$

$(\times 3)\quad 9x + 6y = 51$

$(\times 2)\quad 4x - 6y = -12$

Adding: $13x = 39 \Rightarrow x = 3$

$2y = 17 - 9 = 8 \Rightarrow y = 4$

C is the point $(3, 4)$

| Question number | |
|---|---|
| 2 (b) | $AC : CB = (3 - (-1)) : (5 - 3)$ |
| | $= 4 : 2 = 2 : 1$ |
| | $L$ divides $AB$ in the ratio $2 : 1$ |
| 2 (c) | At $D$, $y = 0$: $2x + 6 = 0 \Rightarrow x = -3$ |
| | $D$ is the point $(-3, 0)$ |
| 2 (d)(i) | $L$: $3y = 2x + 6 \Rightarrow y = \frac{2}{3}x + 2$, so gradient of $L = \frac{2}{3}$ |
| | grad $AB \times$ grad $L = -\frac{3}{2} \times \frac{2}{3} = -1$ |
| | The product of the gradients is $-1$, hence $L$ is perpendicular to $AB$. |
| 2 (d)(ii) | $L \perp AB$, so angle $ACD = 90°$ and $CA$, $CD$ are base and height. |
| | $CA = \sqrt{((3 - (-1))^2 + (4 - 10)^2)} = \sqrt{(16 + 36)} = \sqrt{52}$ |
| | $CD = \sqrt{((3 - (-3))^2 + (4 - 0)^2)} = \sqrt{(36 + 16)} = \sqrt{52}$ |
| | Area of triangle $ACD = \frac{1}{2} \times CA \times CD = \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$ |
| | Area of triangle $ACD = 26$ |
| 3 | $2 - 3(1 - \sin^2\theta) = 2\sin\theta$ |
| | $2 - 3 + 3\sin^2\theta = 2\sin\theta$ |
| | $3\sin^2\theta - 2\sin\theta - 1 = 0$ |
| | $(3\sin\theta + 1)(\sin\theta - 1) = 0$ |
| | $\sin\theta = 1$ or $\sin\theta = -\frac{1}{3}$ |
| | $\sin\theta = 1 \Rightarrow \theta = 90°$ |
| | $\sin\theta = -\frac{1}{3} \Rightarrow$ principal value $\theta = -19.47°$ |

Question number

$\theta = 180° + 19.47° = 199.47°$ or $\theta = 360° - 19.47° = 340.53°$

$\therefore \theta = 90°, 199.47°, 340.53°$

4 (a) $y = 5x^{-1} + 6x^{1/3}$

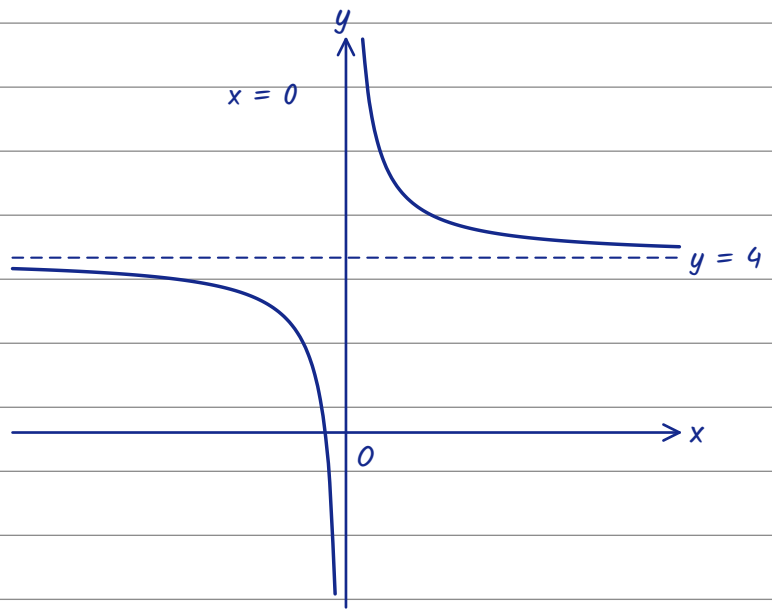
$\frac{dy}{dx} = -5x^{-2} + 6 \times \frac{1}{3}x^{-2/3} = -\frac{5}{x^2} + 2x^{-2/3}$

When $x = 8$: $\frac{dy}{dx} = -\frac{5}{64} + 2 \times \frac{1}{4} = \frac{27}{64}$ $(= 0.421875)$

4 (b) $\int (5x^{3/2} + 12x^{-5} + 7)\,dx = 5 \times \frac{x^{5/2}}{5/2} + 12 \times \frac{x^{-4}}{-4} + 7x + C$

$= 2x^{5/2} - 3x^{-4} + 7x + C$

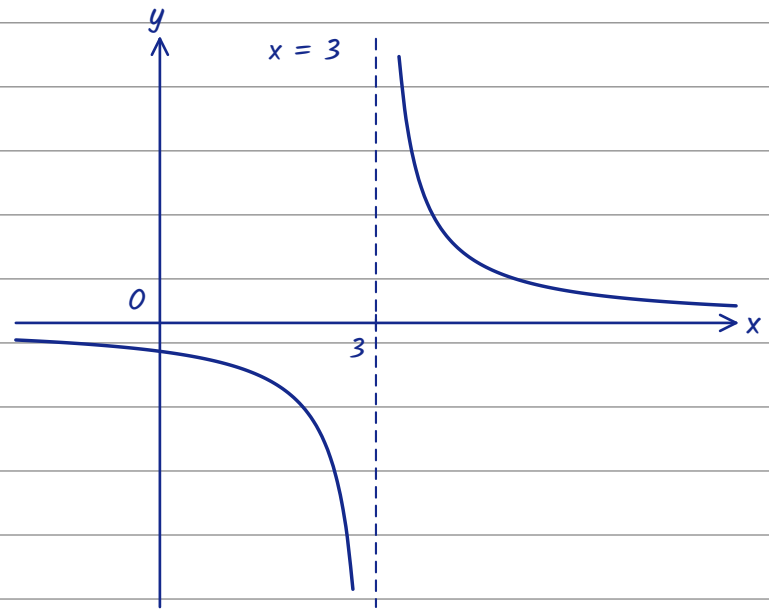
5 (a)




Asymptotes: $y = 4$ and $x = 0$

Question number

5 (b)




Asymptotes: $x = 3$ and $y = 0$

6 (a) $x + 2 = 14 + 5x - x^2$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2 \Rightarrow y = 0$; $x = 6 \Rightarrow y = 8$

A is $(-2, 0)$ and B is $(6, 8)$

6 (b) Area under $C = \int_{-2}^{6} (14 + 5x - x^2)\,dx = \left[14x + \frac{5}{2}x^2 - \frac{x^3}{3}\right]_{-2}^{6}$

$= (84 + 90 - 72) - (-28 + 10 + \frac{8}{3})$

$= 102 - (-\frac{46}{3}) = \frac{352}{3}$

Area of triangle under $L = \frac{1}{2} \times 8 \times 8 = 32$

Required area $= \frac{352}{3} - 32 = \frac{256}{3} = 85\frac{1}{3}$

Question number

7

$$LHS = \frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta} \equiv \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$$

$$\equiv \frac{\sin\theta \times 1}{\cos\theta} \quad \text{since } \sin^2\theta + \cos^2\theta \equiv 1$$

$$\equiv \frac{\sin\theta}{\cos\theta} \equiv \tan\theta = RHS$$

8 (a)

$f(x) = 2x^3 + px^2 + qx - 12$

$f(2) = 0$: $16 + 4p + 2q - 12 = 0 \Rightarrow 2p + q = -2$

$f(-2) = 0$: $-16 + 4p - 2q - 12 = 0 \Rightarrow 2p - q = 14$

Adding: $4p = 12 \Rightarrow p = 3$

$q = -2 - 2(3) = -8$

$p = 3, \quad q = -8$

8 (b)

$2x^3 + 3x^2 - 8x - 12 = (x^2 - 4)(ax + b)$

Comparing coefficients: $a = 2, \quad -4b = -12 \Rightarrow b = 3$

The other factor is $(2x + 3)$

| Question number | |
|---|---|
| 9 | Sine rule: $\frac{\sin C}{25} = \frac{\sin 32°}{16}$ |
| | $\sin C = \frac{25 \sin 32°}{16} = 0.82799...$ |
| | $C = 55.8937°$ or $C = 180° - 55.8937° = 124.1063°$ |
| | $A = 180° - 32° - 55.8937° = 92.1063°$ |
| | or $A = 180° - 32° - 124.1063° = 23.8937°$ |
| | Area $= \frac{1}{2} \times 25 \times 16 \times \sin 92.1063° = 199.86... = 199.9\ cm^2$ |
| | or Area $= \frac{1}{2} \times 25 \times 16 \times \sin 23.8937° = 81.008... = 81.0\ cm^2$ |
| | Possible areas: $199.9\ cm^2$ and $81.0\ cm^2$ |
| 10 (a) | $(a + \sqrt{b})^4 = a^4 + 4a^3(\sqrt{b}) + 6a^2(\sqrt{b})^2 + 4a(\sqrt{b})^3 + (\sqrt{b})^4$ |
| | $= a^4 + 4a^3\sqrt{b} + 6a^2b + 4ab\sqrt{b} + b^2$ |
| 10 (b) | $(a - \sqrt{b})^4 = a^4 - 4a^3\sqrt{b} + 6a^2b - 4ab\sqrt{b} + b^2$ |
| | $(a + \sqrt{b})^4 + (a - \sqrt{b})^4 = 2a^4 + 12a^2b + 2b^2$ |
| 11 (a) | $\lvert u \rvert = \sqrt{(9^2 + (-40)^2)} = \sqrt{1681} = 41$ |
| | $\lvert v \rvert = \sqrt{(3^2 + (-4)^2)} = \sqrt{25} = 5$ |
| | $\mu\lvert v \rvert > \lvert u \rvert \Rightarrow 5\mu > 41$ |
| | $\mu > 8.2$ |
| 11 (b) | $AB = b - a = (21i + j) - (11i - 4j) = 10i + 5j$ |
| | $AC : CB = 2 : 3$, so $AC = \frac{2}{5} AB$ |
| | $c = a + \frac{2}{5} AB = (11i - 4j) + \frac{2}{5}(10i + 5j)$ |

Question number

$= 11i - 4j + 4i + 2j$

Position vector of C is $15i - 2j$

12

$4x^2 + 8x - 8 = 4mx - 3m$

$4x^2 + (8 - 4m)x + (3m - 8) = 0$

Real roots $\Rightarrow$ $b^2 - 4ac \geq 0$

$(8 - 4m)^2 - 4 \times 4 \times (3m - 8) \geq 0$

$64 - 64m + 16m^2 - 48m + 128 \geq 0$

$16m^2 - 112m + 192 \geq 0 \Rightarrow m^2 - 7m + 12 \geq 0$

$(m - 3)(m - 4) \geq 0$

Critical values $m = 3$, $m = 4$; positive quadratic is $\geq 0$ outside them

$m \leq 3$ or $m \geq 4$

13 (a)

$\frac{dy}{dx} = 3x^2 - 6x$

At stationary points $\frac{dy}{dx} = 0$: $3x^2 - 6x = 0$

$3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$

$x = 0$: $y = 0$; $x = 2$: $y = 8 - 12 = -4$

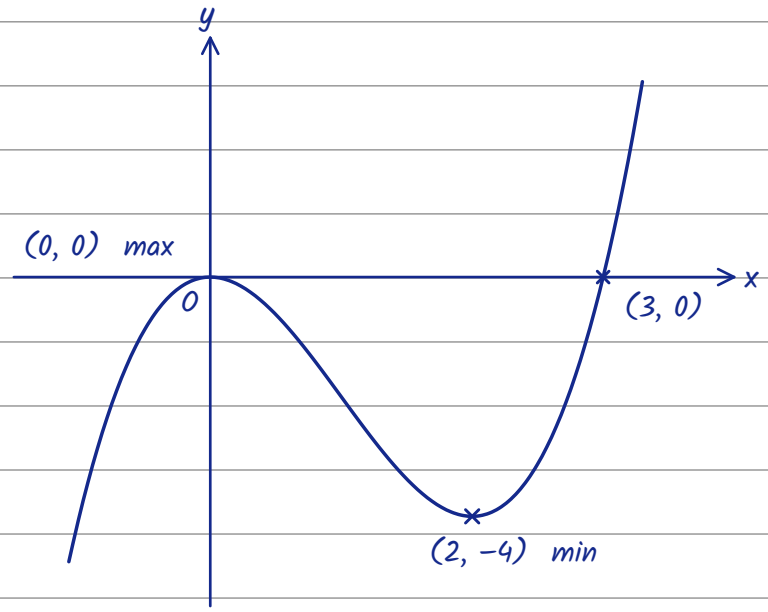
$\frac{d^2y}{dx^2} = 6x - 6$

At $x = 0$: $\frac{d^2y}{dx^2} = -6 < 0$, so $(0, 0)$ is a maximum point

At $x = 2$: $\frac{d^2y}{dx^2} = 6 > 0$, so $(2, -4)$ is a minimum point

Question number

13 (b) Crosses x-axis when $x^2(x - 3) = 0$: $x = 0, x = 3$




13 (c) The integral is negative, since the curve lies below the x-axis ($y \leq 0$) for all x in the interval $0 \leq x \leq 3$.

14 (a) Statement A is false.

Counter example: let $c = 2$, $d = 1$

$LHS = (2 \times 2 - 1)^2 = 3^2 = 9$

$RHS = 4 \times 2^2 - 1^2 = 16 - 1 = 15$

$9 \neq 15$, so A is not true for all c and d, hence A is false.

14 (b) Statement B is true.

$RHS = (2c - d)(4c^2 + 2cd + d^2)$

$= 8c^3 + 4c^2d + 2cd^2 - 4c^2d - 2cd^2 - d^3$

$= 8c^3 - d^3 = LHS$

Hence $8c^3 - d^3 = (2c - d)(4c^2 + 2cd + d^2)$ for all c and d.

15 $V = Ae^{kt}$

$t = 0$, $V = 30000$: $30000 = Ae^0 \Rightarrow A = 30000$

Question number

$t = 2$, $V = 20000$: $20000 = 30000e^{2k}$

$e^{2k} = \frac{2}{3}$ $(2k = \ln\frac{2}{3},\ k = -0.20273...)$

$t = 6$: $V = 30000e^{6k} = 30000(e^{2k})^3$

$V = 30000 \times (\frac{2}{3})^3 = 8888.88...$

Value of car after six years = £8900 (to the nearest £100)

16

$\frac{dy}{dx} = 13 - 4x$

Tangent $y = x + c$ has gradient 1, so at P: $13 - 4x = 1$

$x = 3$

$y = 7 + 13 \times 3 - 2 \times 3^2 = 7 + 39 - 18 = 28$

P is the point (3, 28)

Tangent passes through P: $28 = 3 + c$

$c = 25$

17 (a)

$2\log_{10}x = 1 + \log_{10}5 - \log_{10}2$

$\log_{10}x^2 - \log_{10}5 + \log_{10}2 = 1$

$\log_{10}(\frac{2x^2}{5}) = 1$

$\frac{2x^2}{5} = 10^1 = 10$

$x^2 = 25$

$x = 5$ $(x = -5$ rejected, since $\log_{10}x$ needs $x > 0)$

17 (b)

$e^{0.5x} = \frac{3}{2} = 1.5$

$0.5x = \ln 1.5$

$x = 2 \ln 1.5 = 0.81093... = 0.811$ (3 s.f.)

Question number

17 (c)

$4^x = (2^2)^x = (2^x)^2 = y^2$

$4^x - 10 \times 2^x = y^2 - 10y$

$y^2 - 10y = -16 \Rightarrow y^2 - 10y + 16 = 0$

$(y - 2)(y - 8) = 0 \Rightarrow y = 2$ or $y = 8$

$2^x = 2 \Rightarrow x = 1$; $2^x = 8 \Rightarrow x = 3$

$x = 1$ or $x = 3$

18 (a)

Gradient of AB $= \frac{6 - 5}{4 - (-3)} = \frac{1}{7}$

Gradient of AC $= \frac{6 - (-1)}{4 - 5} = \frac{7}{-1} = -7$

Gradient of AB $\times$ gradient of AC $= \frac{1}{7} \times (-7) = -1$

So AB is perpendicular to AC, hence angle BAC is a right angle.

18 (b)

Angle BAC = 90°, so BC is a diameter (angle in a semicircle).

Centre = midpoint of BC $= \left(\frac{-3 + 5}{2}, \frac{5 + (-1)}{2}\right) = (1, 2)$

$BC = \sqrt{((5 - (-3))^2 + (-1 - 5)^2)} = \sqrt{(64 + 36)} = 10$

Radius $= \frac{1}{2} \times 10 = 5$

Equation of circle: $(x - 1)^2 + (y - 2)^2 = 25$

(i.e. $x^2 + y^2 - 2x - 4y - 20 = 0$)