Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

2305U30-1

S25-2305U30-1

TUESDAY, 20 MAY 2025 – AFTERNOON

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

1 hour 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**6**	
2	**13**	
3	**11**	
4	**7**	
5	**9**	
6	**10**	
7	**14**	
Total	**70**	

2305U301
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

JUN252305U30101

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A particle of mass 1·5 kg is moving under the action of a single force $\mathbf{F}$ N so that its velocity $\mathbf{v}$ ms^{-1} at time t seconds is given by

$$\mathbf{v} = 2t^3\mathbf{i} + 4t\mathbf{j} - t^2\mathbf{k}.$$

(a) Find an expression for $\mathbf{F}$ at time t s. [3]

$a = \frac{dv}{dt} = 6t^2i + 4j - 2tk$

$F = ma = 1.5(6t^2i + 4j - 2tk)$

$F = 9t^2i + 6j - 3tk \quad N$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) Determine the value of $\mathbf{F}.\mathbf{v}$ when $t = 2$, clearly stating the units of your answer. [3]

$F \cdot v = (9t^2)(2t^3) + (6)(4t) + (-3t)(-t^2)$

$= 18t^5 + 24t + 3t^3$

When $t = 2$:

$F \cdot v = 18(2)^5 + 24(2) + 3(2)^3$

$= 576 + 48 + 24$

$= 648\ W \quad (J\ s^{-1})$

2305U301
03

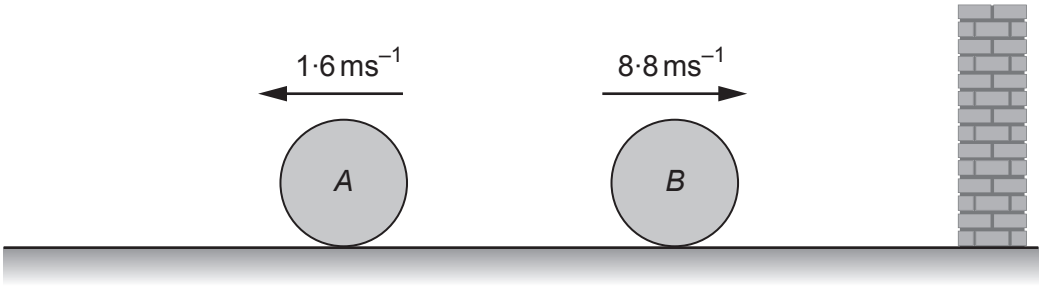
© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

2. Two smooth spheres *A* and *B*, of equal radii, are moving on a smooth horizontal surface when they collide directly. Sphere *A* has mass $2m$ kg and sphere *B* has mass m kg. The coefficient of restitution between the two spheres is 0·8.

The diagram below shows that, immediately **after the collision**, sphere *A* is moving with speed 1·6 ms^{-1} and sphere *B* is moving with speed 8·8 ms^{-1} in the opposite direction.



(a) Find the speed of *A* and the speed of *B* immediately **before** the collision. [7]

Let u_A, u_B be the velocities of A and B before the collision,

taking the direction from A to B (towards the wall) as positive.

After: $v_A = -1.6$, $v_B = 8.8$

Conservation of momentum:

$2m\,u_A + m\,u_B = 2m(-1.6) + m(8.8)$

$2u_A + u_B = 5.6$ (1)

Restitution:

$8.8 - (-1.6) = -0.8(u_B - u_A)$

$10.4 = 0.8(u_A - u_B)$

$u_A - u_B = 13$ (2)

(1) + (2): $3u_A = 18.6 \Rightarrow u_A = 6.2$

$u_B = 6.2 - 13 = -6.8$

Speed of A before collision = 6.2 ms^{-1}

Speed of B before collision = 6.8 ms^{-1}

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) Given that $m = 5$, calculate the magnitude and direction of the impulse exerted by *A* on *B*. [3]

Impulse on B = change in momentum of B (m_B = 5 kg)

$I = 5(8.8) - 5(-6.8)$

$I = 5(8.8 + 6.8) = 78$

Magnitude of impulse = 78 Ns

Direction: opposite to the original motion of B

(i.e. towards the wall, the direction from A to B)

2305U3301
05

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

After the collision, sphere B continues to move with speed $8{\cdot}8\,\text{ms}^{-1}$ until it collides with a smooth vertical wall, which is perpendicular to its direction of motion. The coefficient of restitution between the wall and sphere B is denoted by e.

(c) Determine the range of values of e to ensure that B will collide with A again. [3]

Speed of B after rebounding from the wall = $8.8e$

B moves back towards A, which moves away at $1.6\ ms^{-1}$.

To collide with A again, B must be faster than A:

$8.8e > 1.6$

$e > \frac{1.6}{8.8} = \frac{2}{11}$ $(= 0.1818...)$

Range: $\frac{2}{11} < e \leq 1$

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

3. At time $t = 0$, a drone *A* has position vector $(3\mathbf{i} - 10\mathbf{j} + 20\mathbf{k})$m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})\text{ms}^{-1}$. At time $t = 0$, another drone *B* has position vector $(-2\mathbf{i} + 30\mathbf{j} + 15\mathbf{k})$m and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\text{ms}^{-1}$.

(a) Write down the position vectors of *A* and *B* at time t seconds. [3]

$r = r_0 + vt$

$r_A = (3i - 10j + 20k) + (-i + 2j + k)t$

$= (3 - t)i + (-10 + 2t)j + (20 + t)k$

$r_B = (-2i + 30j + 15k) + (3i - 4j + 2k)t$

$= (-2 + 3t)i + (30 - 4t)j + (15 + 2t)k$

(b) Given that *AB* is the distance between *A* and *B* at time t seconds, show that

$$AB^2 = 53t^2 - 530t + 1650.$$

[4]

$AB = r_B - r_A$

$= (-5 + 4t)i + (40 - 6t)j + (-5 + t)k$

$AB^2 = (-5 + 4t)^2 + (40 - 6t)^2 + (-5 + t)^2$

$= (25 - 40t + 16t^2) + (1600 - 480t + 36t^2)$

$+ (25 - 10t + t^2)$

$= 53t^2 - 530t + 1650$ as required

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(c) Determine the value of t when A and B are closest together. Hence find the least distance between A and B. [4]

$\frac{d}{dt}(AB^2) = 106t - 530$

At minimum, $\frac{d}{dt}(AB^2) = 0$

$106t - 530 = 0 \quad \Rightarrow \quad t = 5$

When $t = 5$:

$AB^2 = 53(5)^2 - 530(5) + 1650$

$= 1325 - 2650 + 1650 = 325$

Least distance $= \sqrt{325} = 5\sqrt{13} = 18.027\ldots = 18.0$ m (3 s.f.)

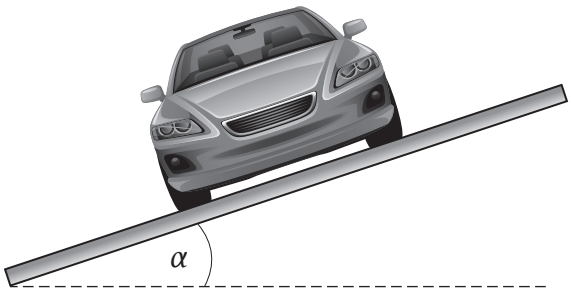
2305U301 09

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

4. The diagram below shows a car, of mass 1500 kg, on a track banked at an angle α to the horizontal.



The car is moving in a horizontal circle of radius 150 m.

When the car is moving with a constant speed of v ms^{-1}, it has no tendency to sideslip. The normal reaction of the track on the car is 17 150 N.

(a) Show that α is approximately 31°. [3]

No tendency to sideslip, so no friction.

Resolve vertically: $R \cos \alpha = mg$

$17\,150 \cos \alpha = 1500 \times 9.8 = 14\,700$

$\cos \alpha = \frac{14\,700}{17\,150} = \frac{6}{7}$

$\alpha = \cos^{-1}\left(\frac{6}{7}\right) = 31.0027...° \approx 31°$ as required

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) Calculate the value of v. [4]

$\sin\alpha = \sqrt{(1 - (\frac{6}{7})^2)} = \frac{\sqrt{13}}{7}$

N2L horizontally towards the centre:

$R\sin\alpha = \frac{mv^2}{r}$

$17\,150 \times \frac{\sqrt{13}}{7} = 1500 \times \frac{v^2}{150}$

$2450\sqrt{13} = 10v^2$

$v^2 = 245\sqrt{13} = 883.3600...$

$v = 29.721... = 29.7\ ms^{-1}$ (3 s.f.)

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

5. A light elastic string, of natural length 1·4 m and modulus of elasticity 156·8 N, has one end attached to a fixed point *O* and the other end attached to a particle *P* of mass m kg.

Initially, *P* is held at rest at the point *O*. It is then released and allowed to fall under gravity.

(a) Determine the extension in the string at the instant when the tension in the string is 36 N. [2]

Hooke's Law: $T = \frac{\lambda x}{l}$

$36 = \frac{156.8x}{1.4} = 112x$

$x = \frac{36}{112} = \frac{9}{28} = 0.321 \text{ m}$ (3 s.f.)

(b) Given that the kinetic energy of *P* is 15·12 J when it is 2 m below *O*, show that $m = 1 \cdot 8$. [7]

When P is 2 m below O, extension $x = 2 - 1.4 = 0.6$ m

$EE = \frac{\lambda x^2}{2l} = \frac{156.8 \times 0.6^2}{2 \times 1.4} = 20.16$ J

Take PE = 0 at the level 2 m below O.

Initially: $PE = mg(2) = 19.6m$, $KE = 0$, $EE = 0$

Finally: $PE = 0$, $KE = 15.12$ J, $EE = 20.16$ J

Conservation of energy:

$mg(2) = 15.12 + \frac{156.8 \times 0.6^2}{2 \times 1.4}$

$19.6m = 15.12 + 20.16$

$19.6m = 35.28$

$m = \frac{35.28}{19.6} = 1.8$ as required

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

6. A vehicle, of mass 2500 kg, is travelling on a straight road which has a horizontal section followed by an incline. When in motion, the vehicle experiences a variable resistive force of $(1000 + 50v)$ N, where v ms^{-1} is the speed of the vehicle.

(a) Find the power developed by the vehicle's engine when the vehicle is travelling on the horizontal section with a constant speed of 20 ms^{-1}. [4]

Tractive force $F = \frac{P}{v} = \frac{P}{20}$

Constant speed, so $a = 0$. N2L: $F - (1000 + 50v) = 0$

$\frac{P}{20} = 1000 + 50(20) = 2000$

$P = 40\,000$ W $(= 40$ kW$)$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) The vehicle travels up the slope which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{2}{49}$. The vehicle's engine is working at a constant rate of 100 kW. Calculate the speed of the vehicle at the instant when its acceleration is $0{\cdot}3\,\text{ms}^{-2}$. [6]

Tractive force $F = \frac{100 \times 1000}{v} = \frac{100\,000}{v}$

Weight component down slope $= 2500g \sin\alpha = 2500 \times 9.8 \times \frac{2}{49} = 1000$ N

N2L up the slope: $F - R - mg\sin\alpha = ma$

$\frac{100\,000}{v} - (1000 + 50v) - 2500g\left(\frac{2}{49}\right) = 2500(0.3)$

$\frac{100\,000}{v} - (1000 + 50v) - 1000 = 750$

$\frac{100\,000}{v} - 50v - 2750 = 0$

$100\,000 - 50v^2 - 2750v = 0$

$v^2 + 55v - 2000 = 0$

$(v - 25)(v + 80) = 0$

$v = 25$ or $v = -80$ (rejected, since $v > 0$)

Speed of vehicle $= 25\ \text{ms}^{-1}$

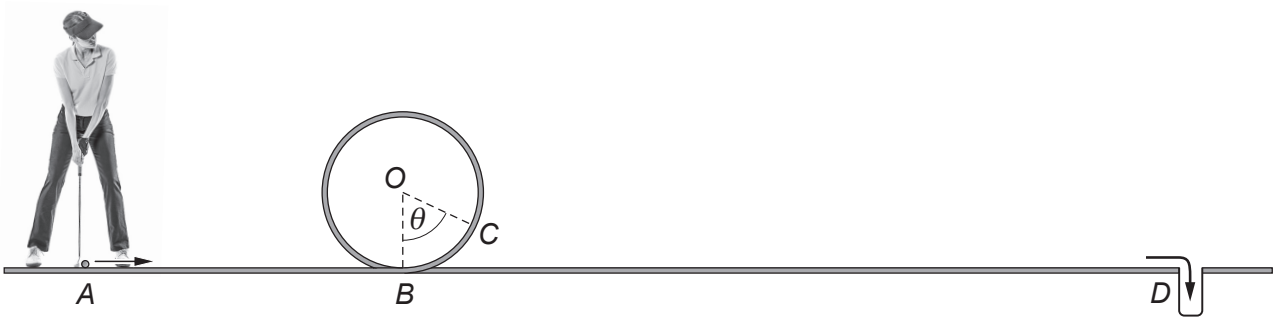
© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

7. The diagram below shows part of a 'Crazy Golf' course. This part of the course consists of a straight horizontal section *AB*, leading into a circular loop, followed by another straight horizontal section *BD*.

The objective of the game is to hit a golf ball, from rest at the point *A*, so that it remains in contact with the surface for its entire journey until it reaches the point *D*, where it is required to drop into a hole.



The loop section may be modelled as a vertical circle of radius 0·5 m and the ball as a particle of mass 0·04 kg. You may assume that the entire surface of the course is smooth and is set in the same vertical plane.

The ball is hit horizontally from *A*. At the point *B*, which is the lowest point of the circle, the speed of the ball is $u\,\text{ms}^{-1}$.

(a) When the ball is at a point *C* on the inside of the circular loop, the line *OC* makes an angle θ with the downward vertical.

(i) Find, in terms of θ and u^2, an expression for v^2, where $v\,\text{ms}^{-1}$ is the speed of the ball at *C*. [4]

(ii) Show that the reaction of the surface on the ball at *C*, R N, is given by

$$R = 0\cdot08u^2 - 0\cdot784 + 1\cdot176\cos\theta.$$ [4]

(iii) Given that the ball does not complete a full circle, but instead, it leaves the loop at a point where $\theta = 120°$, calculate the value of u. [2]

(i) Height of C above B = $r - r\cos\theta = 0.5(1 - \cos\theta)$

Conservation of energy from B to C:

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(0.5)(1 - \cos\theta)$

$v^2 = u^2 - 2g(0.5)(1 - \cos\theta)$

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(a)(i) continued

$v^2 = u^2 - 9.8(1 - \cos\theta)$

$v^2 = u^2 - 9.8 + 9.8\cos\theta$

(ii) N2L towards O:

$R - mg\cos\theta = \frac{mv^2}{r}$

$R = \frac{0.04}{0.5}(u^2 - 9.8 + 9.8\cos\theta) + 0.04(9.8)\cos\theta$

$R = 0.08u^2 - 0.784 + 0.784\cos\theta + 0.392\cos\theta$

$R = 0.08u^2 - 0.784 + 1.176\cos\theta$ as required

(iii) Ball leaves the loop when $R = 0$, with $\theta = 120°$:

$0 = 0.08u^2 - 0.784 + 1.176\cos 120°$

$0 = 0.08u^2 - 0.784 - 0.588$

$0.08u^2 = 1.372 \Rightarrow u^2 = 17.15$

$u = \sqrt{17.15} = 4.1412... = 4.14\ ms^{-1}$ (3 s.f.)

TURN OVER FOR QUESTION 7(b)

© WJEC CBAC Ltd.

stemup.co.uk

Examiner only

(b) The model is refined to include a constant force of magnitude 0·03 N that opposes the motion of the ball. When the ball is hit so that it moves with speed 5 ms^{-1} from *A*, the ball successfully completes the loop. The total length of the course from *A* to *D*, which includes the circular loop, is 10 m. The ball will drop into the hole at *D* if its speed is less than 4 ms^{-1}.

By using the work-energy principle, determine whether the ball drops into the hole. [4]

Work done against resistance = $Fd = 0.03 \times 10 = 0.3$ J

A and D are at the same level, so no change in PE.

Work-energy principle:

Initial KE − work done against resistance = final KE

$\frac{1}{2}(0.04)(5)^2 - 0.3 = \frac{1}{2}(0.04)v^2$

$0.5 - 0.3 = 0.02v^2$

$v^2 = 10$

$v = \sqrt{10} = 3.162\ldots\ ms^{-1}$

Since 3.16 < 4, the ball does drop into the hole at D.

END OF PAPER

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

Question number	Additional page, if required. Write the question number(s) in the left-hand margin.

Examiner only

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

© WJEC CBAC Ltd.

stemup.co.uk