Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

2305U20-1

S25-2305U20-1

FRIDAY, 16 MAY 2025 – AFTERNOON

FURTHER MATHEMATICS – AS unit 2
FURTHER STATISTICS A

1 hour 30 minutes

2305U201
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**11**	
2	**7**	
3	**9**	
4	**11**	
5	**17**	
6	**15**	
Total	**70**	

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The random variable X has the binomial distribution B(n, p).

(a) Given that E(X) = 4 and E(X^2) = 18·4, calculate the values of n and p. [6]

$E(X) = np = 4$

$Var(X) = E(X^2) - [E(X)]^2 = 18.4 - 4^2 = 2.4$

$Var(X) = np(1 - p)$

$\Rightarrow \quad 2.4 = 4(1 - p)$

$1 - p = 0.6$

$p = 0.4$

$n = 4 \div 0.4 = 10$

$n = 10, \quad p = 0.4$

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The random variable Y has the binomial distribution B(12, 0·3). Given that X and Y are independent variables,

(b) calculate E($3XY$), [2]

X and Y independent, so E(XY) = E(X) × E(Y)

E(Y) = 12 × 0.3 = 3.6

E(3XY) = 3 × E(X) × E(Y) = 3 × 4 × 3.6

= 43.2

(c) calculate Var($6X - 5Y + 1$). [3]

Var(6X − 5Y + 1) = 6^2Var(X) + $(-5)^2$Var(Y) (X, Y independent)

Var(X) = 10 × 0.4 × 0.6 = 2.4

Var(Y) = 12 × 0.3 × 0.7 = 2.52

Var(6X − 5Y + 1) = 36 × 2.4 + 25 × 2.52

= 86.4 + 63

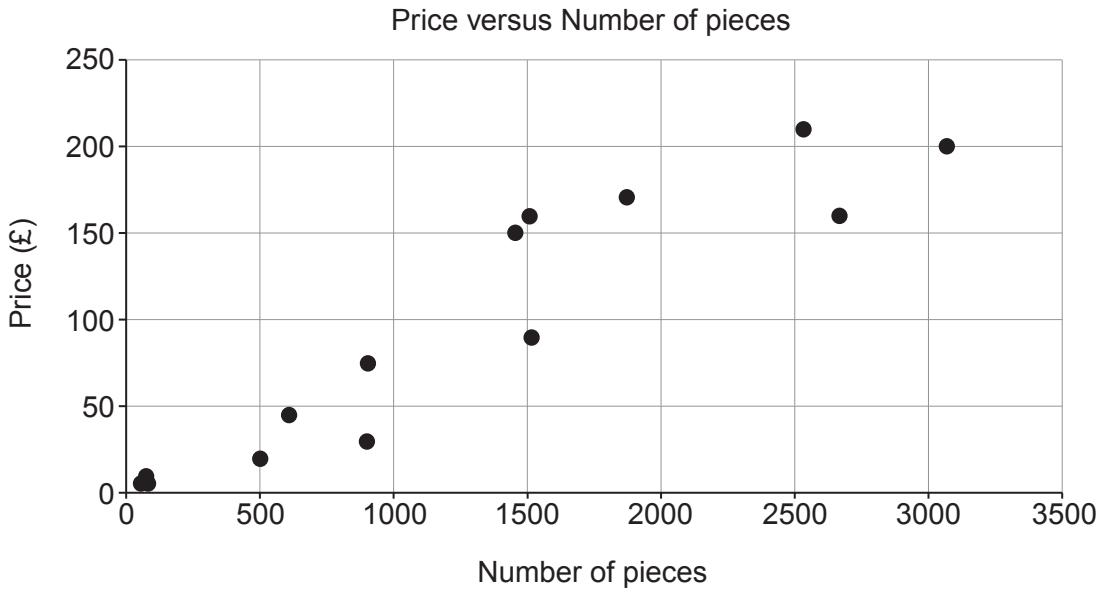
= 149.4

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2. The scatter diagram below shows the price (£y) and the number of pieces (x) for a random sample of 14 building-block sets.



The summary statistics for these data are shown below.

$$\sum x = 17738 \qquad \sum y = 1330 \qquad S_{xx} = 12916820$$

$$S_{yy} = 77550 \qquad S_{xy} = 934655 \qquad n = 14$$

(a) Calculate the equation of the least squares regression line for the price on the number of pieces. [5]

Regression line of y on x: $y = a + bx$

$b = \frac{S_{xy}}{S_{xx}} = \frac{934655}{12916820} = 0.072359...$

$\bar{x} = 17738 \div 14 = 1267, \qquad \bar{y} = 1330 \div 14 = 95$

$a = \bar{y} - b\bar{x} = 95 - 0.072359... \times 1267$

$a = 3.3204...$

$y = 3.32 + 0.0724x$

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(b) Calculate an estimate for the price of a building-block set that has 1200 pieces. Comment on the reliability of your answer. [2]

$y = 3.32 + 0.0724 \times 1200$

$y = 90.20$ Estimated price = £90.20

This estimate is reliable, since 1200 pieces lies within the range of the data (interpolation, not extrapolation) and the scatter diagram shows a reasonably strong linear relationship.

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3. A game at a charity fundraising event involves two independent, identical spinners numbered 1, 2, 3 and 4. The score on each spinner is uniformly distributed. Both spinners are spun and the scores are added.

If the total score is 8, the prize is £10.

If the total score is 6, the prize is £8.

If at least one of the spinners lands on the number 1, there is no prize.

In all other cases, the prize is £3.

It costs £3 to play the game.

Let the random variable X be the player's profit in pounds for a single game.

(a) (i) Find the expected value of X. [5]

(ii) Interpret the answer to (i) in relation to the charity fundraising event. [1]

(iii) Find Var(X). [2]

(i) Total 8: (4,4) $P = \frac{1}{4} \times \frac{1}{4} = 1/16$ $X = 10 - 3 = 7$

Total 6: (2,4), (4,2), (3,3) $P = 3 \times 1/16 = 3/16$ $X = 8 - 3 = 5$

At least one 1: $P = 1 - (\frac{3}{4})^2 = 7/16$ $X = -3$

Otherwise: $P = 1 - 1/16 - 3/16 - 7/16 = 5/16$ $X = 3 - 3 = 0$

x	-3	0	5	7
$P(X = x)$	$7/16$	$5/16$	$3/16$	$1/16$

$$E(X) = -3 \times \frac{7}{16} + 0 \times \frac{5}{16} + 5 \times \frac{3}{16} + 7 \times \frac{1}{16}$$

$$= \frac{-21 + 15 + 7}{16}$$

$$E(X) = \frac{1}{16} = £0.0625 \quad (6.25p)$$

(ii) On average the player makes a profit of about 6p per game, so the charity loses about 6.25p each time the game is played: the game is not good for the fundraising event.

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(iii) $E(X^2) = 9 \times \frac{7}{16} + 0 \times \frac{5}{16} + 25 \times \frac{3}{16} + 49 \times \frac{1}{16} = \frac{187}{16}$

$Var(X) = \frac{187}{16} - \left(\frac{1}{16}\right)^2$

$Var(X) = \frac{2991}{256} = 11.68359... = 11.7 \text{ (3 s.f.)}$

(b) Find the amount to which the £10 prize would have to be changed in order to make the player's expected profit £0. [1]

Let the new prize be £P, profit X = P − 3:

$-3 \times 7/16 + 5 \times 3/16 + (P - 3) \times 1/16 = 0$

$-21 + 15 + P - 3 = 0 \Rightarrow P = 9.$ The prize must be £9

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4. A judge at a triple jump competition wants to test whether there is an association between the height of a male athlete and the greatest distance he jumps.

(a) State a pair of appropriate hypotheses for such a test. [1]

H_0: There is no association between the height of an athlete and the greatest distance he jumps.

H_1: There is an association between the height of an athlete and the greatest distance he jumps.

The height and the greatest distance jumped for 7 randomly selected male athletes in the competition are shown in the table below.

Athlete	A	B	C	D	E	F	G
Height (m)	1·91	1·92	1·80	1·81	1·85	1·90	1·93
Distance jumped (m)	17·18	17·57	17·47	17·44	17·98	17·25	17·43

(b) Calculate Spearman's rank correlation coefficient for this data set. [5]

Athlete	A	B	C	D	E	F	G
Height rank	3	2	7	6	5	4	1
Dist. rank	7	2	3	4	1	6	5
d	−4	0	4	2	4	−2	−4
d^2	16	0	16	4	16	4	16

$\Sigma d^2 = 16 + 0 + 16 + 4 + 16 + 4 + 16 = 72$

$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 72}{7 \times 48} = 1 - \frac{432}{336}$

$r_s = -0.2857...$ $(= -2/7)$

$r_s = -0.286$ (3 s.f.)

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(c) (i) Investigate whether or not there is an association between height and the greatest distance jumped for male athletes in this competition. [3]

(ii) The judge states that height does not affect your ability in the triple jump, but then notices that all 7 athletes in the sample are above average male height. Comment on the judge's statement. [1]

(i) $n = 7$, 5% two-tailed test: critical value = ± 0.7857

Since $-0.2857 > -0.7857$ ($|r_s| < 0.7857$), there is insufficient evidence to reject H_0.

There is insufficient evidence to suggest an association between height and the greatest distance jumped for male athletes in this competition.

(ii) The judge's statement may not be valid: the sample contains only tall athletes, so it says nothing about shorter men. Being tall may still be an advantage in the triple jump (tall people are more likely to be triple jumpers), and there might be an association over a bigger range of heights.

(d) Identify an athlete who could have improved his distance jumped in the competition by 20 cm without affecting your answer to part (b). [1]

Athlete B (or E): his distance rank would not change.

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5. Mark is a fan of 'Gaming Extreme', which is a multidisciplinary sport in which there are four ways of winning. Athletes compete in different age categories.

The table below shows the age category and manner of victory for 3829 randomly selected contests from the global population of Gaming Extreme contests.

Age Category	Outlasting	Timing out	Zoning	Goaling
Men Senior	391	211	7	3
Men U18	335	368	7	3
Men U16	391	578	10	5
Men U14	343	598	12	1
Women Senior	71	159	1	0
Women U18	9	17	0	0
Women U16	47	111	1	0
Women U14	34	116	0	0

Mark wants to investigate whether age category is independent of manner of victory. He produces the following computer output for a chi-squared test for independence.

```
> chisq.test(Gaming)

    Chi-squared test

data: Gaming
X-squared = 203.93, df = 21, p-value < 2.2e-16
```

(a) (i) Comment on the *p*-value. [1]

(ii) Comment on the validity of the test in light of the last two columns. [2]

(i) The p-value is extremely small (less than 2.2×10^{-16}), so there is very strong evidence that age category and manner of victory are not independent.

(ii) The observed frequencies in the last two columns (Zoning and Goaling) are comparatively very small, so they are likely to give rise to small expected frequencies (less than 5). This calls into question the validity of the test and of any conclusions drawn from it.

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TURN OVER FOR QUESTION 5(b)

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Mark then decides to discard the last two columns and look at whether the remaining manners of victory are independent of gender for the global population. He starts the test with the following hypotheses.

H_0: Manner of victory is independent of gender.

H_1: Manner of victory is not independent of gender.

The following table shows manner of victory and gender.

Gender	Outlasting	Timing Out	Total
Men	1460	1755	**3215**
Women	161	403	**564**
Total	**1621**	**2158**	**3779**

(b) Carry out a chi-squared test for independence at the 5% level of significance. [12]

Expected frequency = row total × column total ÷ grand total

e.g. Men, Outlasting: $E = 3215 \times 1621 \div 3779 = 1379.073$

Expected frequencies:

Gender	Outlasting	Timing Out
Men	1379.073	1835.927
Women	241.927	322.073

$\chi^2 = \Sigma (O - E)^2 \div E$

Men, Outlasting: $(1460 - 1379.073)^2 \div 1379.073 = 4.749$

Men, Timing Out: $(1755 - 1835.927)^2 \div 1835.927 = 3.567$

Women, Outlasting: $(161 - 241.927)^2 \div 241.927 = 27.071$

Women, Timing Out: $(403 - 322.073)^2 \div 322.073 = 20.335$

Test statistic $\chi^2 = 4.749 + 3.567 + 27.071 + 20.335$

$= 55.722...$

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Degrees of freedom $\nu = (2 - 1)(2 - 1) = 1$

5% critical value = 3.841

Since 55.722 > 3.841, there is sufficient evidence to reject H_0.

There is evidence to suggest that manner of victory is not independent of gender (gender affects manner of victory).

(c) State, with a reason, whether men or women most affect the outcome of the test. Comment on your answer. [2]

Women most affect the outcome, because their χ^2 contributions (27.071 and 20.335) are much larger than the men's (4.749 and 3.567). Women have fewer Outlasting victories than expected (161 compared with 241.9) and more Timing Out victories than expected (403 compared with 322.1).

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6. (a) The random variable X has a probability density function given by

$$f(x) = \begin{cases} k\mathrm{e}^{-kx} & x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) What is the name of the distribution that X follows? [1]

(ii) Show that $\mathrm{P}(X \leqslant x) = 1 - \mathrm{e}^{-kx}$, for $x \geqslant 0$. [2]

(iii) State $F(x)$, the cumulative distribution function for X, for all values of x. [1]

(iv) Find an expression, in terms of k, for the exact value of the interquartile range of X. Simplify your answer. [5]

(i) Exponential distribution (with parameter k)

(ii) $P(X \leq x) = \int_0^x k\, e^{-kt}\, dt$

$= [\, -e^{-kt}\,]_0^x$

$= -e^{-kx} - (-e^0)$

$= 1 - e^{-kx}$ as required

(iii) $F(x) = \begin{cases} 1 - e^{-kx} & x \geq 0 \\ 0 & x < 0 \end{cases}$

(iv) Upper quartile: $F(Q_3) = 0.75$

$1 - e^{-kQ_3} = 0.75 \Rightarrow e^{-kQ_3} = 0.25$

$-kQ_3 = \ln(1/4)$

$Q_3 = \frac{\ln(1/4)}{-k} = \frac{1}{k} \ln 4$

Lower quartile: $F(Q_1) = 0.25$

$1 - e^{-kQ_1} = 0.25 \Rightarrow e^{-kQ_1} = 0.75$

$-kQ_1 = \ln(3/4)$

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$Q_1 = \frac{\ln(3/4)}{-k} = \frac{1}{k}\ln\left(\frac{4}{3}\right)$

$IQR = Q_3 - Q_1 = \frac{1}{k}\ln 4 - \frac{1}{k}\ln\left(\frac{4}{3}\right)$

$= \frac{1}{k}\ln\left(4 \div \frac{4}{3}\right)$

$IQR = \frac{1}{k}\ln 3$

TURN OVER FOR QUESTION 6(b)

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(b) The distance, Y, in miles, that a cyclist can cycle between punctures is modelled by an exponential distribution with mean 800.

(i) Calculate the probability that the distance between punctures is greater than 600, but less than 1200 miles. [2]

(ii) Three cyclists, James, Pete and Gemma, cycle the 200 miles from Holyhead to Cardiff. Calculate the probability that they will manage to complete the trip without any punctures between them. You may assume that punctures for James, Pete and Gemma occur independently of each other. [4]

(i) Mean 800, so $P(Y \leq y) = F(y) = 1 - e^{-y/800}$

$P(600 < Y < 1200) = F(1200) - F(600)$

$= (1 - e^{-1200/800}) - (1 - e^{-600/800})$

$= e^{-0.75} - e^{-1.5}$

$= 0.24923... = 0.249$ (3 s.f.)

(ii) For one cyclist, no puncture in 200 miles:

$P(Y > 200) = e^{-200/800} = e^{-0.25} = 0.7788...$

Punctures independent, so

$P(\text{no punctures between them}) = 0.7788...^3$

$= e^{-0.75} = 0.47236...$

$= 0.472$ (3 s.f.)

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