Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

S25-2305U10-1

2305U10-1

MONDAY, 12 MAY 2025 – AFTERNOON

FURTHER MATHEMATICS – AS unit 1
FURTHER PURE MATHEMATICS A

1 hour 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	13	
2	14	
3	10	
4	5	
5	11	
6	6	
7	11	
Total	70	

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01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The function f is defined by $f(x) = x^3 - 8x^2 + 25x - 26$.

(a) Show that $f(3-2\mathrm{i}) = 0$. You must show all your working. [3]

$(3 - 2i)^2 = 9 - 12i + 4i^2 = 5 - 12i$

$(3 - 2i)^3 = (5 - 12i)(3 - 2i) = 15 - 10i - 36i + 24i^2$

$= -9 - 46i$

$f(3 - 2i) = (-9 - 46i) - 8(5 - 12i) + 25(3 - 2i) - 26$

$= -9 - 46i - 40 + 96i + 75 - 50i - 26$

$= (-9 - 40 + 75 - 26) + (-46 + 96 - 50)i$

$= 0 + 0i$

$\therefore f(3 - 2i) = 0$ as required

(b) Solve the equation $f(x) = 0$. [4]

Coefficients are real, so $3 + 2i$ is also a root.

$(x - 3 + 2i)(x - 3 - 2i) = (x - 3)^2 - (2i)^2$

$= x^2 - 6x + 9 + 4 = x^2 - 6x + 13$

So $x^2 - 6x + 13$ is a factor of $f(x)$.

$x^3 - 8x^2 + 25x - 26 = (x^2 - 6x + 13)(x - 2)$

Check: $-2x^2 - 6x^2 = -8x^2$, $12x + 13x = 25x$, $13 \times (-2) = -26$ ✓

$(x^2 - 6x + 13)(x - 2) = 0$

$x = 3 - 2i$, $x = 3 + 2i$, $x = 2$

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(c) The roots of the equation $f(x) = 0$ are denoted by α, β, γ. Find a new cubic equation with roots $\frac{13}{\alpha}, \frac{13}{\beta}, \frac{13}{\gamma}$. [6]

From $f(x) = 0$:

$\alpha + \beta + \gamma = 8, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 25, \quad \alpha\beta\gamma = 26$

Sum: $\frac{13}{\alpha} + \frac{13}{\beta} + \frac{13}{\gamma} = \frac{13(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma} = \frac{13 \times 25}{26} = \frac{25}{2}$

Pairs: $\frac{169}{\alpha\beta} + \frac{169}{\beta\gamma} + \frac{169}{\gamma\alpha} = \frac{169(\alpha + \beta + \gamma)}{\alpha\beta\gamma} = \frac{169 \times 8}{26} = 52$

Product: $\frac{13}{\alpha} \times \frac{13}{\beta} \times \frac{13}{\gamma} = \frac{2197}{\alpha\beta\gamma} = \frac{2197}{26} = \frac{169}{2}$

New equation: $x^3 - (\text{sum})x^2 + (\text{pairs})x - (\text{product}) = 0$

$x^3 - \frac{25}{2}x^2 + 52x - \frac{169}{2} = 0$

i.e. $2x^3 - 25x^2 + 104x - 169 = 0$

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2. Three points, A, B, C, are such that $A(3, 2, 5)$, $B(4, 0, 1)$, and $C(-3, 2, 0)$.
The line L_1 passes through the points A and B, and the line L_2 passes through the points A and C.

(a) Show that the line L_1 has the vector equation $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ and find a vector equation for the line L_2. [3]

$AB = b - a = (4i + k) - (3i + 2j + 5k) = i - 2j - 4k$

L_1 passes through A with direction AB:

L_1: $r = 3i + 2j + 5k + \lambda(i - 2j - 4k)$ as required

$AC = c - a = (-3i + 2j) - (3i + 2j + 5k) = -6i - 5k$

L_2: $r = 3i + 2j + 5k + \mu(-6i - 5k)$

(b) Find the angle between the lines L_1 and L_2. [5]

Directions: $a = i - 2j - 4k$, $b = -6i - 5k$

$a.b = (1)(-6) + (-2)(0) + (-4)(-5) = -6 + 0 + 20 = 14$

$|a| = \sqrt{1^2 + (-2)^2 + (-4)^2} = \sqrt{21}$

$|b| = \sqrt{(-6)^2 + 0^2 + (-5)^2} = \sqrt{61}$

$\cos\theta = \frac{a.b}{|a||b|} = \frac{14}{\sqrt{21} \times \sqrt{61}} = 0.39115...$

$\theta = 66.973...°$

Angle between L_1 and L_2 = 67.0° (1 d.p.)

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(c) The line L_1 intersects the plane $2x-3y+z=13$ at the point E.

(i) Find the coordinates of the point E. [4]

(ii) Find the distance from the point C to the plane $2x-3y+z=13$. [2]

(i) On L_1: $x = 3 + \lambda$, $y = 2 - 2\lambda$, $z = 5 - 4\lambda$

Substitute into $2x - 3y + z = 13$:

$2(3 + \lambda) - 3(2 - 2\lambda) + (5 - 4\lambda) = 13$

$6 + 2\lambda - 6 + 6\lambda + 5 - 4\lambda = 13$

$4\lambda + 5 = 13 \Rightarrow \lambda = 2$

$x = 3 + 2 = 5$, $y = 2 - 4 = -2$, $z = 5 - 8 = -3$

$E(5, -2, -3)$

(ii) $C(-3, 2, 0)$, plane $2x - 3y + z - 13 = 0$

$$\text{Distance} = \frac{|2(-3) - 3(2) + 1(0) - 13|}{\sqrt{(2^2 + (-3)^2 + 1^2)}} = \frac{|-25|}{\sqrt{14}}$$

$$= \frac{25}{\sqrt{14}} = 6.6815\ldots = 6.68 \text{ (3 s.f.)}$$

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3. (a) Given that $A_r = \frac{5}{r-1} - \frac{4}{r} - \frac{1}{r+1}$ for $r \geqslant 2$, find an expression for $\sum_{r=2}^{n} A_r$, in the form $\frac{a}{2} + \frac{b}{n} + \frac{c}{n+1}$, where a, b, c are integers to be found. [5]

ΣA_r $(r = 2$ to $n)$:

$r = 2$: $\frac{5}{1} - \frac{4}{2} - \frac{1}{3}$

$r = 3$: $\frac{5}{2} - \frac{4}{3} - \frac{1}{4}$

$r = 4$: $\frac{5}{3} - \frac{4}{4} - \frac{1}{5}$

⋮ ⋮

$r = n - 2$: $\frac{5}{n-3} - \frac{4}{n-2} - \frac{1}{n-1}$

$r = n - 1$: $\frac{5}{n-2} - \frac{4}{n-1} - \frac{1}{n}$

$r = n$: $\frac{5}{n-1} - \frac{4}{n} - \frac{1}{n+1}$

Terms in $\frac{1}{k}$ cancel for $3 \leq k \leq n - 1$, since $5 - 4 - 1 = 0$

$\Sigma A_r = \frac{5}{1} - \frac{4}{2} + \frac{5}{2} - \frac{1}{n} - \frac{4}{n} - \frac{1}{n+1}$

$= 5 - 2 + \frac{5}{2} - \frac{5}{n} - \frac{1}{n+1}$

$= \frac{11}{2} - \frac{5}{n} - \frac{1}{n+1}$

$a = 11, \quad b = -5, \quad c = -1$

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(b) Find the smallest value of n such that $\sum_{r=2}^{n} A_r > 5$. [5]

$\frac{11}{2} - \frac{5}{n} - \frac{1}{n+1} > 5$

$\frac{5}{n} + \frac{1}{n+1} < \frac{1}{2}$

Multiply by $2n(n+1)$, which is positive:

$10(n+1) + 2n < n(n+1)$

$12n + 10 < n^2 + n$

$n^2 - 11n - 10 > 0$

Critical values: $n = \frac{11 \pm \sqrt{(121+40)}}{2} = \frac{11 \pm \sqrt{161}}{2}$

$n = 11.844...$ or $n = -0.844...$

$n^2 - 11n - 10 > 0$ for $n > 11.844...$ (n positive)

Smallest value of n is 12

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4. The complex number z is given by $z = 7 - 24\mathrm{i}$. By writing $\sqrt{z} = a + \mathrm{i}b$, where a, b are real, find the two values of $\sqrt{z}$. [5]

$(a + ib)^2 = 7 - 24i$

$a^2 + 2abi + i^2b^2 = 7 - 24i$

$a^2 - b^2 + 2abi = 7 - 24i$

Real parts: $a^2 - b^2 = 7$ Imaginary parts: $2ab = -24$

$b = \frac{-12}{a} \Rightarrow a^2 - \frac{144}{a^2} = 7$

$a^4 - 144 = 7a^2$

$a^4 - 7a^2 - 144 = 0$

$(a^2 - 16)(a^2 + 9) = 0$

$a^2 = 16$ ($a^2 = -9$ has no real solutions)

$a = 4, b = -3$ or $a = -4, b = 3$

$\sqrt{z} = 4 - 3i$ or $\sqrt{z} = -4 + 3i$

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5. (a) The matrix $\mathbf{M}$ is such that $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Given that $\mathbf{M}^2 = \mathbf{I}$, show that $a^2 = d^2$. [4]

$M^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$

$M^2 = I \Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing entries: $a^2 + bc = 1$ and $bc + d^2 = 1$

(also $ab + bd = 0$ and $ac + cd = 0$)

$a^2 = 1 - bc$ and $d^2 = 1 - bc$

$\therefore a^2 = d^2$

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(b) Prove, by mathematical induction, that

$$\begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}^n = \begin{bmatrix} \frac{1}{2}+\frac{(-1)^n}{2} & 1-(-1)^n \\ \frac{1}{4}-\frac{(-1)^n}{4} & \frac{1}{2}+\frac{(-1)^n}{2} \end{bmatrix}$$

is true for all positive integers n. [7]

When $n = 1$: LHS $= \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}$

RHS $= \begin{bmatrix} \frac{1}{2}+\frac{(-1)}{2} & 1-(-1) \\ \frac{1}{4}-\frac{(-1)}{4} & \frac{1}{2}+\frac{(-1)}{2} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix} =$ LHS

So the result is true for $n = 1$.

Assume the result is true for $n = k$, i.e.

$$\begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}^k = \begin{bmatrix} \frac{1}{2}+\frac{(-1)^k}{2} & 1-(-1)^k \\ \frac{1}{4}-\frac{(-1)^k}{4} & \frac{1}{2}+\frac{(-1)^k}{2} \end{bmatrix}$$

Consider $n = k + 1$:

$$\begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}^{k+1} = \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}^k \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{1}{2} + \frac{(-1)^k}{2} & 1 - (-1)^k \\ \frac{1}{4} - \frac{(-1)^k}{4} & \frac{1}{2} + \frac{(-1)^k}{2} \end{bmatrix} \begin{bmatrix} 0 & 2 \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(1 - (-1)^k) & 2(\frac{1}{2} + \frac{(-1)^k}{2}) \\ \frac{1}{2}(\frac{1}{2} + \frac{(-1)^k}{2}) & 2(\frac{1}{4} - \frac{(-1)^k}{4}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} - \frac{(-1)^k}{2} & 1 + (-1)^k \\ \frac{1}{4} + \frac{(-1)^k}{4} & \frac{1}{2} - \frac{(-1)^k}{2} \end{bmatrix}$$

Since $-(-1)^k = (-1)(-1)^k = (-1)^{k+1}$:

$$= \begin{bmatrix} \frac{1}{2} + \frac{(-1)^{k+1}}{2} & 1 - (-1)^{k+1} \\ \frac{1}{4} - \frac{(-1)^{k+1}}{4} & \frac{1}{2} + \frac{(-1)^{k+1}}{2} \end{bmatrix}$$

which is the result with $n = k + 1$.

If the result is true for $n = k$, it is also true for $n = k + 1$.

Since it is true for $n = 1$, by mathematical induction it is true for all positive integers n.

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6. The complex number z is represented by the point $P(x, y)$ in an Argand diagram and

$$|z-2+5\mathrm{i}| = 3|z+q-\mathrm{i}|.$$

Given that the locus of P is a circle, centre C, where the x-coordinate of C is $-\frac{5}{2}$, find the value of q. [6]

Let $z = x + iy$:

$|(x - 2) + i(y + 5)| = 3|(x + q) + i(y - 1)|$

$\sqrt{(x - 2)^2 + (y + 5)^2} = 3\sqrt{(x + q)^2 + (y - 1)^2}$

$(x - 2)^2 + (y + 5)^2 = 9[(x + q)^2 + (y - 1)^2]$

$x^2 - 4x + 4 + y^2 + 10y + 25$

$= 9x^2 + 18qx + 9q^2 + 9y^2 - 18y + 9$

$0 = 8x^2 + 8y^2 + (18q + 4)x - 28y + 9q^2 - 20$

$x^2 + y^2 + \frac{18q + 4}{8}x - \frac{7}{2}y + \frac{9q^2 - 20}{8} = 0$

x-coordinate of centre $C = -\frac{18q + 4}{2 \times 8} = -\frac{18q + 4}{16}$

$-\frac{18q + 4}{16} = -\frac{5}{2}$

$18q + 4 = 40$

$q = 2$

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7. The transformation T in the plane consists of a reflection in the line $y = \sqrt{3}x$, followed by a rotation through 180° about the origin, followed by a translation by the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(a) Find the 3 × 3 matrix which represents T. [6]

$y = \sqrt{3}x$ makes angle θ with the x-axis where $\tan\theta = \sqrt{3}$, $\theta = 60°$

Reflection: $\begin{bmatrix} \cos 120° & \sin 120° & 0 \\ \sin 120° & -\cos 120° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation 180°: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Translation: $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

T = (Translation)(Rotation)(Reflection)

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$T = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 2 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

TURN OVER FOR QUESTION 7(b)

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(b) Show that there are no fixed points under the transformation T. [5]

For a fixed point: $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 2 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y + 2 = x \quad \Rightarrow \quad x + \sqrt{3}y = 4$

$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y + 1 = y \quad \Rightarrow \quad \sqrt{3}x + 3y = 2$

Multiplying the first equation by $\sqrt{3}$: $\sqrt{3}x + 3y = 4\sqrt{3}$

But $\sqrt{3}x + 3y = 2$, and $4\sqrt{3} \neq 2$

The equations are inconsistent (contradiction),

so there are no fixed points under T.

END OF PAPER

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