Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE AS/A LEVEL

S24-2305U30-1

2305U30-1

FRIDAY, 24 MAY 2024 – AFTERNOON

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

1 hour 30 minutes

2305U301
01

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**14**	
2	**10**	
3	**5**	
4	**7**	
5	**9**	
6	**10**	
7	**15**	
Total	**70**	

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. Two particles *A* and *B*, of masses 2 kg and 5 kg respectively, are moving in the same direction along a smooth horizontal surface when they collide directly. Before the collision, *B* is moving with speed $1{\cdot}2\,\text{ms}^{-1}$ and, immediately after the collision, its speed is $3{\cdot}8\,\text{ms}^{-1}$. The coefficient of restitution between the particles *A* and *B* is 0·3.

(a) (i) Find the impulse exerted by *A* on *B*.

(ii) Given that the particles *A* and *B* were in contact for 0·08 seconds, find the average force between *A* and *B*. [4]

(i) Impulse = change in momentum of B

$I = 5(3.8 - 1.2)$

$I = 13\ Ns$

(ii) $I = Ft$

$13 = F \times 0.08$

$F = \frac{13}{0.08} = 162.5\ N$

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(b) Calculate the speed of *A* before and after the collision. [7]

Let A have speed u before and v after (both in direction of motion).

Conservation of momentum:

$2u + 5(1.2) = 2v + 5(3.8)$

$2u + 6 = 2v + 19$

$u - v = 6.5$ (1)

Restitution:

$3.8 - v = 0.3(u - 1.2)$

$3.8 - v = 0.3u - 0.36$

$0.3u + v = 4.16$ (2)

(1) + (2): $1.3u = 10.66$

$u = 8.2$

$v = 8.2 - 6.5 = 1.7$

Speed of A before collision = $8.2\ ms^{-1}$

Speed of A after collision = $1.7\ ms^{-1}$

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(c) After the collision between A and B, particle B continues to move with speed $3{\cdot}8\,\text{ms}^{-1}$ until it collides directly with a stationary particle C of mass 4 kg. When B and C collide, they coalesce to form a single particle.

(i) Write down the coefficient of restitution between B and C.

(ii) Determine the speed of the combined particle after the collision. [3]

(i) $e = 0$ (the particles coalesce)

(ii) Conservation of momentum:

$5(3.8) + 4(0) = (5 + 4)V$

$19 = 9V$

$V = \frac{19}{9} = 2.111\ldots = 2.11\ \text{ms}^{-1}$

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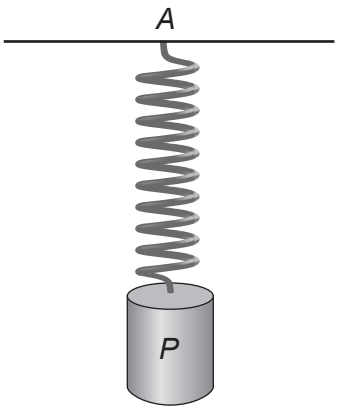
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2. The diagram below shows a light spring of natural length 1·2 m and modulus of elasticity 84 N. One end of the spring *A* is fixed and the other end is attached to an object *P* of mass 4 kg.



Initially, *P* is held at rest with the spring stretched to a total length of 2·2 m and *AP* vertical.

(a) Show that the elastic energy stored in the spring is 35 J. [2]

Extension $x = 2.2 - 1.2 = 1$ m

$EE = \frac{\lambda x^2}{2l} = \frac{84 \times 1^2}{2 \times 1.2}$

$= \frac{84}{2.4} = 35$ J as required

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(b) The object P is then released. Find the speed of P at the instant when the elastic energy in the spring is reduced to 5·6 J. [8]

When EE = 5.6 J, let the extension be x:

$5.6 = \frac{84x^2}{2 \times 1.2} = 35x^2$

$x^2 = 0.16 \Rightarrow x = 0.4$ m (x = −0.4 rejected)

P rises from extension 1 m to extension 0.4 m,

so $h = 2.2 - 1.2 - 0.4 = 0.6$ m

Gain in PE = $mgh = 4 \times 9.8 \times 0.6 = 23.52$ J

$KE = \frac{1}{2} \times 4 \times v^2 = 2v^2$

Conservation of energy (initial KE = 0, initial PE = 0):

$35 = 5.6 + 23.52 + 2v^2$

$2v^2 = 5.88$

$v^2 = 2.94$

$v = \sqrt{2.94} = 1.7146...$

Speed of P = 1.71 ms^{-1}

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3. Three forces $(4\mathbf{i}-7\mathbf{j}+9\mathbf{k})$N , $(5\mathbf{i}+3\mathbf{j}-8\mathbf{k})$N and $(-2\mathbf{i}+6\mathbf{j}-11\mathbf{k})$N act on a particle.

(a) Find the resultant $\mathbf{R}$ of the three forces. [1]

$R = (4 + 5 - 2)i + (-7 + 3 + 6)j + (9 - 8 - 11)k$

$R = 7i + 2j - 10k$ N

(b) The points *A* and *B* have position vectors $(3\mathbf{i}+4\mathbf{j}-12\mathbf{k})$m and $(a\mathbf{i}+7\mathbf{j}-10\mathbf{k})$m respectively, where a is a constant. The work done by $\mathbf{R}$ in moving the particle from *A* to *B* is 21 J. Calculate the value of a. [4]

Displacement $AB = (ai + 7j - 10k) - (3i + 4j - 12k)$

$= (a - 3)i + 3j + 2k$

Work done = $R \cdot AB$

$21 = (7i + 2j - 10k) \cdot ((a - 3)i + 3j + 2k)$

$21 = 7(a - 3) + 2(3) + (-10)(2)$

$21 = 7a - 21 + 6 - 20$

$7a = 56$

$a = 8$

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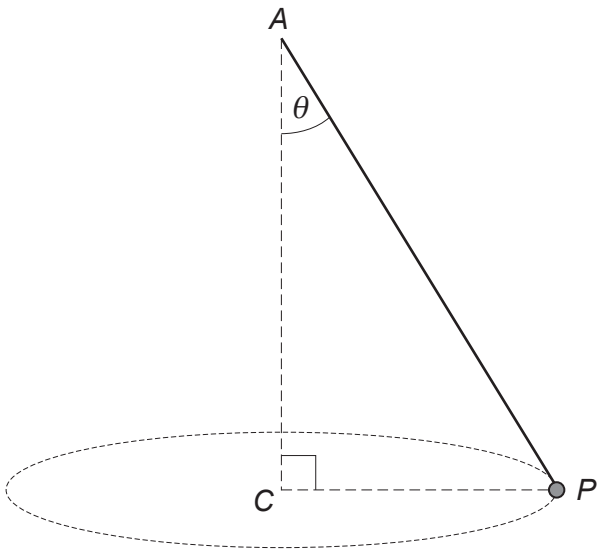
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4. The diagram below shows a particle *P*, of mass 5 kg, attached to one end of a light inextensible string of length 3 m. The other end is fixed at a point *A*. The particle *P* is moving in a horizonal circle with centre *C*, where the point *C* is vertically below *A*. The string is inclined at an angle θ to the downward vertical, where $\tan\theta = \frac{20}{21}$.



Find the speed of the particle. [7]

$\tan\theta = \frac{20}{21} \Rightarrow \sin\theta = \frac{20}{29}, \quad \cos\theta = \frac{21}{29}$

Resolve vertically (T = tension): $T\cos\theta = 5g$

$T = 5 \times 9.8 \times \frac{29}{21} = \frac{203}{3} = 67.666\ldots$ N

Radius $r = 3\sin\theta = 3 \times \frac{20}{29} = \frac{60}{29}$ m

N2L towards C: $T\sin\theta = 5 \times \frac{v^2}{r}$

$\frac{203}{3} \times \frac{20}{29} = 5 \times \frac{v^2}{60/29}$

$\frac{140}{3} = \frac{29v^2}{12}$

$v^2 = \frac{560}{29} = 19.3103\ldots$

$v = 4.394\ldots$

Speed of particle = 4.39 ms^{-1}

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5. A particle of mass 2 kg is moving under the action of a force $\mathbf{F}$ N which, at time t seconds, is given by

$$\mathbf{F} = 4t\mathbf{i} - \sqrt{t}\,\mathbf{j} + 6\mathbf{k}.$$

When $t = 1$, the velocity of the particle is $\left(3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k}\right)\text{ms}^{-1}$.

(a) Find an expression for the velocity vector of the particle at time t s. [5]

$a = \frac{F}{m} = \frac{1}{2}(4ti - \sqrt{t}\,j + 6k) = 2ti - \frac{1}{2}t^{1/2}j + 3k$

$v = \int a\,dt = t^2i - \frac{1}{2} \times \frac{2}{3}t^{3/2}j + 3tk + c$

$= t^2i - \frac{1}{3}t^{3/2}j + 3tk + c$

When $t = 1$: $i - \frac{1}{3}j + 3k + c = 3i - \frac{1}{3}j - k$

$c = 2i - 4k$

$v = (t^2 + 2)i - \frac{1}{3}t^{3/2}j + (3t - 4)k \ \ ms^{-1}$

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(b) Determine the values of t when the particle is moving in a direction perpendicular to the vector $(-\mathbf{i}+3\mathbf{k})$. [4]

Perpendicular $\Rightarrow$ $v \cdot (-i + 3k) = 0$

$((t^2 + 2)i - \frac{1}{3}t^{3/2}j + (3t - 4)k) \cdot (-i + 3k) = 0$

$-(t^2 + 2) + 3(3t - 4) = 0$

$-t^2 - 2 + 9t - 12 = 0$

$t^2 - 9t + 14 = 0$

$(t - 2)(t - 7) = 0$

$t = 2$ or $t = 7$

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6. A slope is inclined at an angle of 5° to the horizontal. A car, of mass 1500 kg, has an engine that is working at a constant rate of P W. The resistance to motion of the car is constant at 4500 N. When the car is moving up the slope, its acceleration is a ms^{-2} at the instant when its speed is 10 ms^{-1}. When the car is moving down the slope, its **deceleration** is a ms^{-2} at the instant when its speed is 20 ms^{-1}.

Determine the value of P and the value of a. [10]

Tractive force $F = \frac{P}{v}$

Component of weight along slope $= 1500g \sin 5° = 1281.189...$ N

Up the slope ($v = 10$), N2L:

$\frac{P}{10} - 4500 - 1500g \sin 5° = 1500a$

$\frac{P}{10} - 4500 - 1281.189... = 1500a$

$\frac{P}{10} - 5781.189... = 1500a$ (1)

Down the slope ($v = 20$), deceleration a, N2L:

$\frac{P}{20} - 4500 + 1500g \sin 5° = -1500a$

$\frac{P}{20} - 4500 + 1281.189... = -1500a$

$\frac{P}{20} - 3218.810... = -1500a$ (2)

(1) + (2) to eliminate a:

$\frac{3P}{20} - 9000 = 0$

$P = 60\,000$

Sub into (1): $6000 - 5781.189... = 1500a$

$a = \frac{218.810...}{1500} = 0.145873...$

$P = 60\,000$, $a = 0.146$ (3 s.f.)

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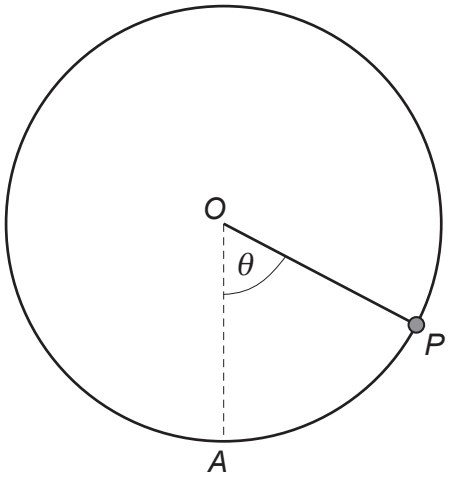
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7. One end of a light rod of length $\frac{5}{7}$m is attached to a fixed point *O* and the other end is attached to a particle *P*, of mass $m$kg. The particle *P* is projected from the point *A*, which is vertically below *O*, with a horizontal speed of $u$ms^{-1} so that it moves in a vertical circle with centre *O*. When the rod *OP* is inclined at an angle θ to the downward vertical, the speed of *P* is $v$ms^{-1} and the tension in the rod is TN.



(a) Show that

$$v^2 = u^2 - 14 + 14\cos\theta\,.$$ [4]

Height of P above A = $\frac{5}{7} - \frac{5}{7}\cos\theta = \frac{5}{7}(1 - \cos\theta)$

Conservation of energy from A to P:

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg \times \frac{5}{7}(1 - \cos\theta)$

$u^2 = v^2 + 2 \times 9.8 \times \frac{5}{7}(1 - \cos\theta)$

$u^2 = v^2 + 14(1 - \cos\theta)$

$v^2 = u^2 - 14(1 - \cos\theta)$

$v^2 = u^2 - 14 + 14\cos\theta$ as required

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(b) Hence determine the least possible value of u^2 for the particle to reach the highest point of the circle. [2]

A rod can push, so P just reaches the top if $v \geq 0$ at $\theta = 180°$.

$\cos\theta = -1$: $v^2 = u^2 - 14 - 14 = u^2 - 28 \geq 0$

Least value of $u^2 = 28$

(c) Given that $u^2 = 32 \cdot 2$,

(i) find, in terms of m and θ, an expression for T, [4]

(ii) calculate the range of values of θ such that the rod is exerting a thrust. [4]

(i) N2L towards O: $T - mg\cos\theta = \frac{mv^2}{r}$

$r = \frac{5}{7}$, $v^2 = 32.2 - 14 + 14\cos\theta = 18.2 + 14\cos\theta$

$T = \frac{7m}{5}(18.2 + 14\cos\theta) + 9.8m\cos\theta$

$T = 25.48m + 19.6m\cos\theta + 9.8m\cos\theta$

$T = m(25.48 + 29.4\cos\theta)$

(ii) Rod exerts a thrust when $T < 0$:

$m(25.48 + 29.4\cos\theta) < 0$

$\cos\theta < -\frac{25.48}{29.4} = -\frac{13}{15} = -0.8666...$

$\cos^{-1}(-\frac{13}{15}) = 150.07...°$ and $360° - 150.07...° = 209.93...°$

Thrust is exerted for $150.07° < \theta < 209.93°$

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(d) State whether your answer to (c)(ii) would be different if the mass of the particle was reduced. Give a reason for your answer. [1]

No. The answer would remain the same because the condition

$\cos\theta < -\frac{13}{15}$ does not depend on m (m cancels).

END OF PAPER

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