Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

S25-1300U30-1

1300U30-1

WEDNESDAY, 4 JUNE 2025 – MORNING

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	11	
2	12	
3	6	
4	6	
5	8	
6	8	
7	5	
8	5	
9	4	
10	3	
11	10	
12	7	
13	11	
14	11	
15	7	
16	6	
Total	120	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. (a) Differentiate each of the following functions with respect to x.

(i) $\left(e^{-x} + x^2\right)^3$ [2]

$y = (e^{-x} + x^2)^3$

$\frac{dy}{dx} = 3(e^{-x} + x^2)^2 \times (-e^{-x} + 2x)$

$\frac{dy}{dx} = 3(2x - e^{-x})(e^{-x} + x^2)^2$

(ii) $e^{2x} \sin 3x$ [3]

Product rule: $y = e^{2x} \sin 3x$

$\frac{dy}{dx} = (2e^{2x}) \sin 3x + e^{2x}(3\cos 3x)$

$\frac{dy}{dx} = 2e^{2x} \sin 3x + 3e^{2x} \cos 3x$

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(iii) $\dfrac{\cos 2x}{x^2+1}$ [3]

Quotient rule:

$\frac{dy}{dx} = \frac{(x^2 + 1)(-2\sin 2x) - (2x)\cos 2x}{(x^2 + 1)^2}$

$\frac{dy}{dx} = \frac{-2(x^2 + 1)\sin 2x - 2x\cos 2x}{(x^2 + 1)^2}$

(b) A function is defined implicitly by

$$2x - 3x^2y^3 = 5.$$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

$2x - 3x^2y^3 = 5$

Differentiating implicitly with respect to x:

$2 - (6xy^3 + 3x^2 \times 3y^2\frac{dy}{dx}) = 0$

$9x^2y^2\frac{dy}{dx} = 2 - 6xy^3$

$\frac{dy}{dx} = \frac{2 - 6xy^3}{9x^2y^2}$

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2. (a) Find all values of x in the range $0 \leqslant x \leqslant 2\pi$ satisfying the equation

$$\cos 2x + \sin x = 0.$$

Give your answers in terms of π. [6]

$\cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$\sin x = -\frac{1}{2}$: $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$, $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\sin x = 1$: $x = \frac{\pi}{2}$

$\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

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(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$2\sec^2\theta - 5 = \tan\theta.$$ [6]

$2\sec^2\theta - 5 = \tan\theta$

$2(1 + \tan^2\theta) - 5 = \tan\theta$

$2\tan^2\theta - \tan\theta - 3 = 0$

$(2\tan\theta - 3)(\tan\theta + 1) = 0$

$\tan\theta = \frac{3}{2}$ or $\tan\theta = -1$

$\tan\theta = \frac{3}{2}$: $\theta = 56.309...° = 56.3°$, $\theta = 180° + 56.3° = 236.3°$

$\tan\theta = -1$: $\theta = 135°$, $\theta = 315°$

$\therefore\ \theta = 56.3°, 135°, 236.3°, 315°$

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3. Given that θ is small, find an estimate for the solution of the equation $\dfrac{\cos\theta}{1+\sin 2\theta} = 0\cdot 8$. [6]

For small θ: $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\sin 2\theta \approx 2\theta$

$\frac{1 - \theta^2/2}{1 + 2\theta} = 0.8$

$1 - \frac{\theta^2}{2} = 0.8 + 1.6\theta$

$0.5\theta^2 + 1.6\theta - 0.2 = 0$

$\theta = \frac{-1.6 \pm \sqrt{(1.6^2 - 4 \times 0.5 \times (-0.2))}}{2 \times 0.5} = -1.6 \pm \sqrt{2.96}$

$\theta = 0.120465...$ or $\theta = -3.320465...$

Since θ is small, reject $\theta = -3.32$

$\therefore\ \theta \approx 0.12$ (radians)

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4. (a) Using the trapezium rule with 5 ordinates, find an approximate value for $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$.

Show your working and give your answer correct to three decimal places. [4]

$h = \frac{\pi/3}{4} = \frac{\pi}{12}$, $y = \sec^2 x$

$x = 0$: $y_0 = 1$

$x = \frac{\pi}{12}$: $y_1 = 1.071796...$

$x = \frac{\pi}{6}$: $y_2 = \frac{4}{3} = 1.333333...$

$x = \frac{\pi}{4}$: $y_3 = 2$

$x = \frac{\pi}{3}$: $y_4 = 4$

$\int_0^{\pi/3} \sec^2 x \, dx \approx \frac{\pi/12}{2}[(1 + 4) + 2(1.071796 + 1.333333 + 2)]$

$= \frac{\pi}{24} \times 13.810260... = 1.807758...$

≈ 1.808 (3 d.p.)

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(b) Hence, deduce an approximate value for $\int_0^{\frac{\pi}{3}} \left(\tan^2 x + 2\right) \mathrm{d}x$. [2]

$\tan^2 x + 2 = (\sec^2 x - 1) + 2 = \sec^2 x + 1$

$\int_0^{\pi/3} (\tan^2 x + 2)\, dx = \int_0^{\pi/3} \sec^2 x\, dx + \int_0^{\pi/3} 1\, dx$

$\approx 1.808 + \frac{\pi}{3} = 1.808 + 1.047197\ldots = 2.855$

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5. A curve is defined by $f(x) = x^3 - 6x^2 + 12x - 5$.

(a) Find the coordinates of the stationary point and determine its nature. [6]

$f'(x) = 3x^2 - 12x + 12$

At a stationary point $f'(x) = 0$:

$3x^2 - 12x + 12 = 0 \Rightarrow x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0 \Rightarrow x = 2$

$f(2) = 2^3 - 6 \times 2^2 + 12 \times 2 - 5 = 8 - 24 + 24 - 5 = 3$

Stationary point is (2, 3)

Nature: $f'(x) = 3(x - 2)^2 \geq 0$ for all x, so the gradient is positive on both sides of $x = 2$ (no change of sign).

Also $f''(x) = 6x - 12$: $f''(x) < 0$ for $x < 2$ and $f''(x) > 0$ for $x > 2$, so the concavity changes at $x = 2$.

$\therefore$ (2, 3) is a point of inflection.

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(b) Determine the range of values of x where the curve is convex. [2]

Curve is convex when $f''(x) > 0$

$f''(x) = 6x - 12 > 0$

$x > 2$

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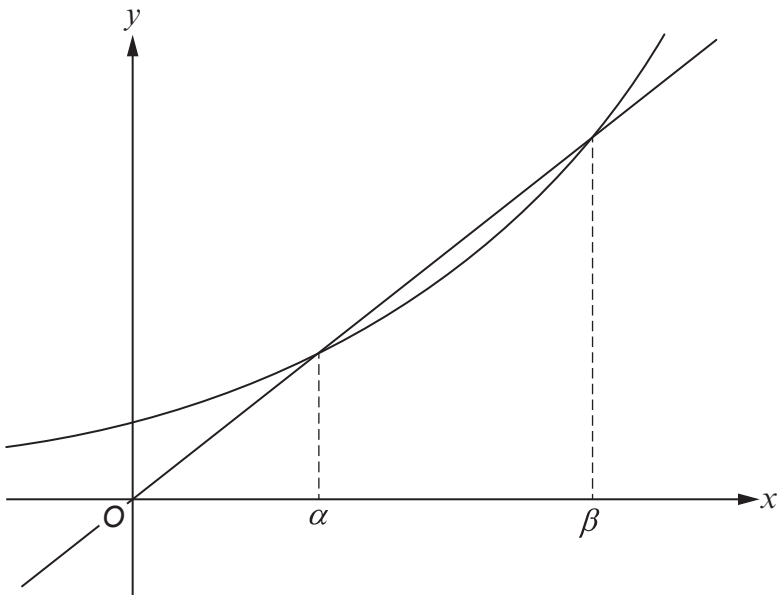
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6. The diagram below shows a sketch of the curve with equation $y = e^x$ and the straight line with equation $y = 3x$.



The straight line intersects the curve at the points where $x = \alpha$ and $x = \beta$. Approximate values for α and β are $0{\cdot}62$ and $1{\cdot}5$ respectively.

(a) The iterative formula $x_{n+1} = \frac{1}{3}e^{x_n}$ is used to find a more accurate approximation for α. Taking $x_0 = 0{\cdot}62$, use the iterative formula to obtain the values of x_1, x_2 and x_3. Give your answers correct to three decimal places. [2]

$x_0 = 0.62$

$x_1 = \frac{1}{3}e^{0.62} = 0.619642\ldots = 0.620$

$x_2 = \frac{1}{3}e^{0.619642\ldots} = 0.619421\ldots = 0.619$

$x_3 = \frac{1}{3}e^{0.619421\ldots} = 0.619284\ldots = 0.619$

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(b) The Newton-Raphson method is used to find a more accurate approximation for β.

(i) Taking $f(x) = e^x - 3x$ and starting with $x_0 = 1 \cdot 5$, apply the Newton-Raphson method once to obtain a second approximation, x_1. Give your answer correct to three decimal places. [3]

(ii) Show that x_1 is the value of β correct to three decimal places. [2]

(iii) Explain why the Newton-Raphson method fails if the starting value in the approximation of β is $\ln 3$. [1]

(i) $f(x) = e^x - 3x, \quad f'(x) = e^x - 3$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_1 = 1.5 - \dfrac{e^{1.5} - 4.5}{e^{1.5} - 3} = 1.5 - \dfrac{-0.018310\ldots}{1.481689\ldots}$

$x_1 = 1.512358\ldots = 1.512 \quad (3 \text{ d.p.})$

(ii) $f(1.5115) = e^{1.5115} - 3 \times 1.5115 = -0.00097\ldots < 0$

$f(1.5125) = e^{1.5125} - 3 \times 1.5125 = 0.00056\ldots > 0$

Change of sign and f is continuous, so β lies between 1.5115 and 1.5125, i.e. β = 1.512 correct to 3 d.p.

(iii) If $x_0 = \ln 3$, then $f'(\ln 3) = e^{\ln 3} - 3 = 3 - 3 = 0$, so the Newton-Raphson formula would require division by zero (the tangent is horizontal), hence the method fails.

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7. The first and last terms of an arithmetic series are –3 and 61 respectively. The sum of all of the terms is 957. Determine

(a) the number of terms, [2]

$S_n = \frac{n}{2}(a + l)$

$\frac{n}{2}(-3 + 61) = 957$

$29n = 957$

$n = 33$

(b) the common difference, [2]

$l = a + (n - 1)d$

$61 = -3 + (33 - 1)d$

$32d = 64$

$d = 2$

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(c) the middle term. [1]

Middle term is the 17th term:

$u_{17} = -3 + (17 - 1) \times 2 = 29$

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8. By algebraic division, or otherwise, simplify the expression $\frac{x^4+5x^3-7x^2-29x+30}{3(x+5)(x+3)(x-2)}$. [5]

Let $g(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$

$g(1) = 1 + 5 - 7 - 29 + 30 = 0$, so $(x - 1)$ is a factor

$g(-5) = 625 - 625 - 175 + 145 + 30 = 0$, so $(x + 5)$ is a factor

$g(-3) = 81 - 135 - 63 + 87 + 30 = 0$, so $(x + 3)$ is a factor

$g(2) = 16 + 40 - 28 - 58 + 30 = 0$, so $(x - 2)$ is a factor

Check (dividing by $x - 1$): $g(x) = (x - 1)(x^3 + 6x^2 - x - 30)$

and $x^3 + 6x^2 - x - 30 = (x + 5)(x + 3)(x - 2)$

$\therefore\ g(x) = (x - 1)(x + 5)(x + 3)(x - 2)$

$$\frac{x^4 + 5x^3 - 7x^2 - 29x + 30}{3(x + 5)(x + 3)(x - 2)} = \frac{(x - 1)(x + 5)(x + 3)(x - 2)}{3(x + 5)(x + 3)(x - 2)}$$

$$= \frac{1}{3}(x - 1) \quad \text{or} \quad \frac{x - 1}{3}$$

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9. Use proof by contradiction to show that there are no integers x and y for which $3x+15y=2$. [4]

Assume that there are integers x and y such that 3x + 15y = 2.

Then 3(x + 5y) = 2

$\Rightarrow$ $x + 5y = \frac{2}{3}$

Since x and y are integers, x + 5y is also an integer.

But $\frac{2}{3}$ is not an integer, which is a contradiction.

Therefore the assumption is false: there are no integers x and y for which 3x + 15y = 2.

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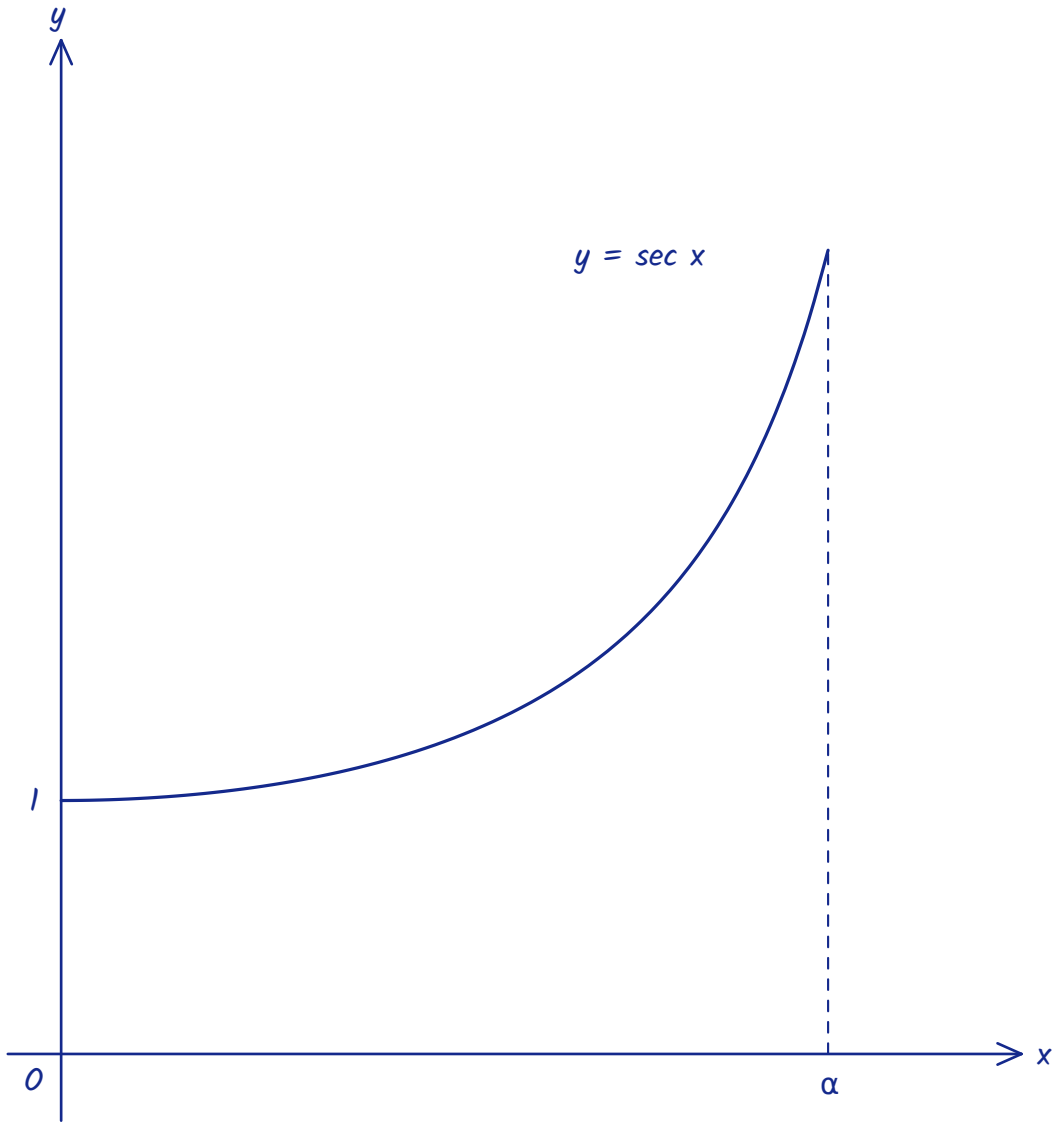
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10. (a) Sketch the graph of $y = \sec x$ for $0 \leqslant x \leqslant \alpha$, where $0 < \alpha < \frac{\pi}{2}$. [1]



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(b) Explain why $\int_0^{\alpha} \sec x \, dx > \alpha$. [2]

For $0 \leq x \leq \alpha$, $\sec x \geq 1$, so the graph of $y = \sec x$ lies above the line $y = 1$ (equal only at $x = 0$).

The rectangle with sides 1 and α lies below the graph and has area α.

Since $\int_0^{\alpha} \sec x \, dx$ is the area under the graph from 0 to α, which is greater than the area of the rectangle,

$\therefore \int_0^{\alpha} \sec x \, dx > \alpha$

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11. Find $\int_1^2 \frac{5x+2}{(x+1)^2(2x-1)}\,dx$, giving your answer in the form $\ln a + b$, where a, b are constants. [10]

$\frac{5x+2}{(x+1)^2(2x-1)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}$

$5x + 2 \equiv A(x+1)(2x-1) + B(2x-1) + C(x+1)^2$

$x = -1$: $-3 = -3B \Rightarrow B = 1$

$x = \frac{1}{2}$: $\frac{9}{2} = \frac{9}{4}C \Rightarrow C = 2$

Coefficient of x^2: $0 = 2A + C \Rightarrow A = -1$

$\int_1^2 \frac{5x+2}{(x+1)^2(2x-1)}\,dx = \int_1^2 \left(\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{2x-1} \right) dx$

$= \left[-\ln|x+1| - \frac{1}{x+1} + \ln|2x-1| \right]_1^2$

$= \left(-\ln 3 - \frac{1}{3} + \ln 3\right) - \left(-\ln 2 - \frac{1}{2} + \ln 1\right)$

$= -\frac{1}{3} + \ln 2 + \frac{1}{2}$

$= \ln 2 + \frac{1}{6} \qquad (a = 2,\ b = \frac{1}{6})$

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12. Expand $\frac{2-x}{\sqrt[3]{1-2x}}$ in ascending powers of x up to and including the term in x^3.

State the range of values of x for which the expansion is valid. [7]

$\frac{2-x}{\sqrt[3]{(1-2x)}} = (2-x)(1-2x)^{-1/3}$

$(1-2x)^{-1/3} = 1 + (-\frac{1}{3})(-2x) + \frac{(-1/3)(-4/3)}{2!}(-2x)^2$

$+ \frac{(-1/3)(-4/3)(-7/3)}{3!}(-2x)^3 + \ldots$

$= 1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3 + \ldots$

$(2-x)(1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3 + \ldots)$

$= 2 + \frac{4}{3}x + \frac{16}{9}x^2 + \frac{224}{81}x^3 - x - \frac{2}{3}x^2 - \frac{8}{9}x^3 + \ldots$

$= 2 + \frac{1}{3}x + \frac{10}{9}x^2 + \frac{152}{81}x^3 + \ldots$

Valid for $|2x| < 1$, i.e. $|x| < \frac{1}{2}$ $\quad(-\frac{1}{2} < x < \frac{1}{2})$

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13. A curve is defined parametrically by

$$x = 2t^3 + 1, \qquad y = 3t^2.$$

(a) Show that the gradient of the curve at the point with parameter p is $\frac{1}{p}$. [4]

$\frac{dx}{dt} = 6t^2, \quad \frac{dy}{dt} = 6t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{6t^2} = \frac{1}{t}$

At the point with parameter p, the gradient is $\frac{1}{p}$ (as required)

(b) Find the equation of the normal to the curve at the point with parameter t, where $t = 1$. [4]

$t = 1$: $x = 2 + 1 = 3$, $y = 3$, point $(3, 3)$

Gradient of tangent $= \frac{1}{1} = 1$, so gradient of normal $= -1$

$y - 3 = -1(x - 3)$

$x + y = 6$

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(c) Determine the equation of the tangent to the curve at the point with parameter t, where $t = 0$. [3]

$t = 0$: $x = 1$, $y = 0$, point $(1, 0)$

Gradient $= \frac{1}{0}$, which is undefined (infinite),

so the tangent is vertical.

Equation of tangent: $x = 1$

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14. (a) Use integration by parts to find the **exact** value of $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$. [6]

$u = x^2, \quad \frac{dv}{dx} = \cos x \quad \Rightarrow \quad \frac{du}{dx} = 2x, \quad v = \sin x$

$\int_0^{\pi/2} x^2 \cos x \, dx = [\, x^2 \sin x \,]_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x \, dx$

$u = 2x, \quad \frac{dv}{dx} = \sin x \quad \Rightarrow \quad \frac{du}{dx} = 2, \quad v = -\cos x$

$\int_0^{\pi/2} 2x \sin x \, dx = [\, -2x \cos x \,]_0^{\pi/2} + \int_0^{\pi/2} 2\cos x \, dx$

$\therefore \int_0^{\pi/2} x^2 \cos x \, dx = [\, x^2 \sin x + 2x \cos x - 2\sin x \,]_0^{\pi/2}$

$= \left(\frac{\pi^2}{4} \times 1 + 0 - 2 \right) - (0 + 0 - 0)$

$= \frac{\pi^2}{4} - 2$

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(b) Use the substitution $u = 1 + \ln x$ to find $\int \frac{\ln x}{x(1+\ln x)^2}\,dx$. [5]

$u = 1 + \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$, so $du = \frac{dx}{x}$, $\ln x = u - 1$

$\int \frac{\ln x}{x(1+\ln x)^2}\,dx = \int \frac{u-1}{u^2}\,du$

$= \int \left(\frac{1}{u} - \frac{1}{u^2}\right) du$

$= \ln|u| + \frac{1}{u} + C$

$= \ln|1 + \ln x| + \frac{1}{1+\ln x} + C$

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15. A function f is defined by $f(x) = \dfrac{9}{(x-3)^2}$, where $x > 3$.

(a) Determine the range of f. [2]

For $x > 3$, $(x - 3)^2$ takes all values in $(0, \infty)$,

so $f(x) = \dfrac{9}{(x-3)^2}$ takes all positive values.

Range of f: $f(x) > 0$, i.e. $(0, \infty)$

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(b) Find an expression for $f^{-1}(x)$. [5]

Let $y = \frac{9}{(x-3)^2}$

$(x-3)^2 = \frac{9}{y}$

$x - 3 = \pm \frac{3}{\sqrt{y}}$

$x = 3 \pm \frac{3}{\sqrt{y}}$

Since $x > 3$, take the positive root: $x = 3 + \frac{3}{\sqrt{y}}$

$f^{-1}(x) = 3 + \frac{3}{\sqrt{x}}$

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16. A person borrows £50 000 which is to be repaid over 20 years in 20 equal annual amounts. The lender charges an annual compound interest rate of 5% which remains constant throughout the period of the loan. The interest is added at the end of each year.

Each instalment is paid **after** the interest has been added to the account, with the first instalment paid one year after the loan was taken out.

Calculate the amount of each annual instalment. [6]

Let each annual instalment be £R.

After 20 years the loan has grown to $50000(1.05)^{20} = 132664.885...$

The instalments, with interest, grow to

$R[(1.05)^{19} + (1.05)^{18} + ... + (1.05) + 1]$ (GP, $a = R$, $r = 1.05$, 20 terms)

$= R \times \frac{1.05^{20} - 1}{1.05 - 1} = 33.065954...R$

Loan repaid after 20 years:

$33.065954...R = 50000(1.05)^{20}$

$R = \frac{132664.885...}{33.065954...} = 4012.129...$

Annual instalment = £4012.13

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