Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

S24-1300U40-1

1300U40-1

TUESDAY, 11 JUNE 2024 – AFTERNOON

MATHEMATICS – A2 unit 4
APPLIED MATHEMATICS B

1 hour 45 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**3**	
2	**8**	
3	**8**	
4	**21**	
5	**7**	
6	**8**	
7	**7**	
8	**7**	
9	**11**	
Total	**80**	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

Section A: Statistics

1. The table below shows the destination from school of 180 year 11 pupils. Most pupils either continued education, in school or college, or went into some form of employment.

	School	College	Employment	Other	Total
Boys	33	49	8	2	**92**
Girls	40	40	7	1	**88**
Total	**73**	**89**	**15**	**3**	**180**

A reporter selects two pupils at random to interview. Given that the first pupil is in school or college, find the probability that both pupils are girls. [3]

In school or college: 73 + 89 = 162 pupils, of whom 40 + 40 = 80 are girls

P(both girls | 1st in school or college) = $\frac{80}{162} \times \frac{87}{179}$

(once a girl is chosen, 87 girls remain among the other 179 pupils)

$= \frac{40}{81} \times \frac{87}{179} = \frac{1160}{4833}$

= 0.240016... = 0.2400 (4 s.f.)

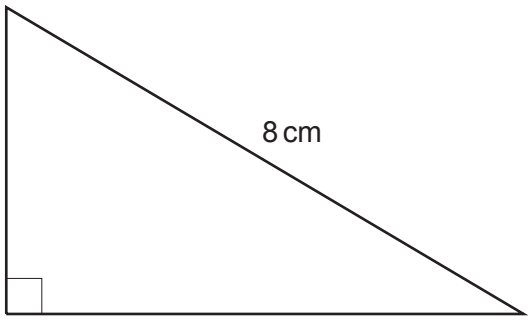
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2. The smallest angle θ, in degrees, of a right-angled triangle with hypotenuse 8 cm, is uniformly distributed across all possible values.



(a) Find the mean and standard deviation of θ. [3]

$\theta \sim U[0, 45]$ (the smallest angle is between 0° and 45°)

$\text{Mean} = \frac{0 + 45}{2} = 22.5°$

$\text{Variance} = \frac{(45 - 0)^2}{12} = 168.75$

$SD = \sqrt{168.75} = \frac{15\sqrt{3}}{2} = 12.990... = 12.99° \text{ (4 s.f.)}$

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(b) The shortest side of the triangle is of length X cm. Find the probability that X is greater than 5. [5]

$X = 8 \sin \theta$

$P(X > 5) = P(8 \sin \theta > 5)$

$= P(\sin \theta > \frac{5}{8})$

$= P(\theta > 38.682...°)$

$= \frac{45 - 38.682...}{45}$

$= 0.14039... = 0.1404$ (4 s.f.)

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3. Awena has a large data set of body measurements, and she wants to investigate relationships between body dimensions. In this particular investigation, she is testing for a correlation between forearm girth and bicep girth. The diagrams below show how to measure these.



Forearm girth



Bicep girth

(a) Why is it appropriate for Awena to use a one-tailed test? [1]

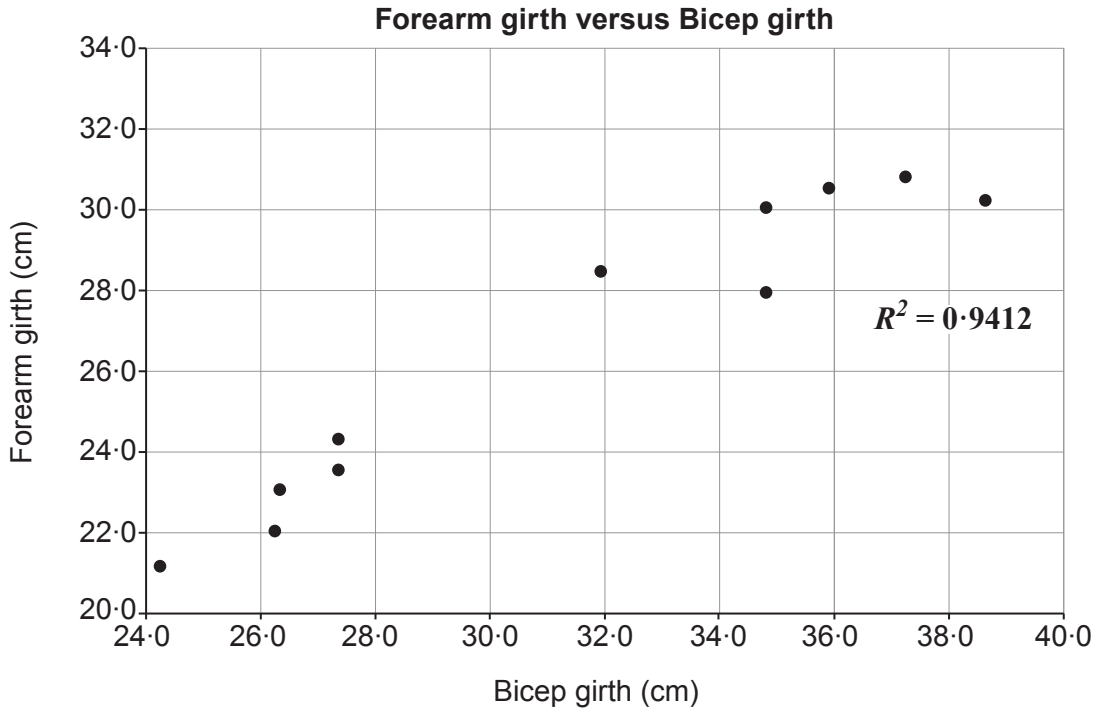
There is no need to check for negative correlation: in the general population, people with bigger biceps would be expected to have bigger forearms, so any correlation would be positive.

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Awena takes a random sample of size 11 from her data set and plots the following scatter diagram.



(b) Using the computer output above, carry out a one-tailed significance test on the sample product moment correlation coefficient at the 0·5% level. [5]

Let ρ be the population correlation coefficient between forearm girth and bicep girth.

H_0: ρ = 0 H_1: ρ > 0

Test statistic: $r = \sqrt{0.9412} = 0.97015...$

n = 11, one-tailed, 0.5% level: critical value = 0.7348

Since 0.97015... > 0.7348, there is sufficient evidence to reject H_0.

There is sufficient evidence at the 0.5% level to suggest that there is a positive correlation between bicep girth and forearm girth.

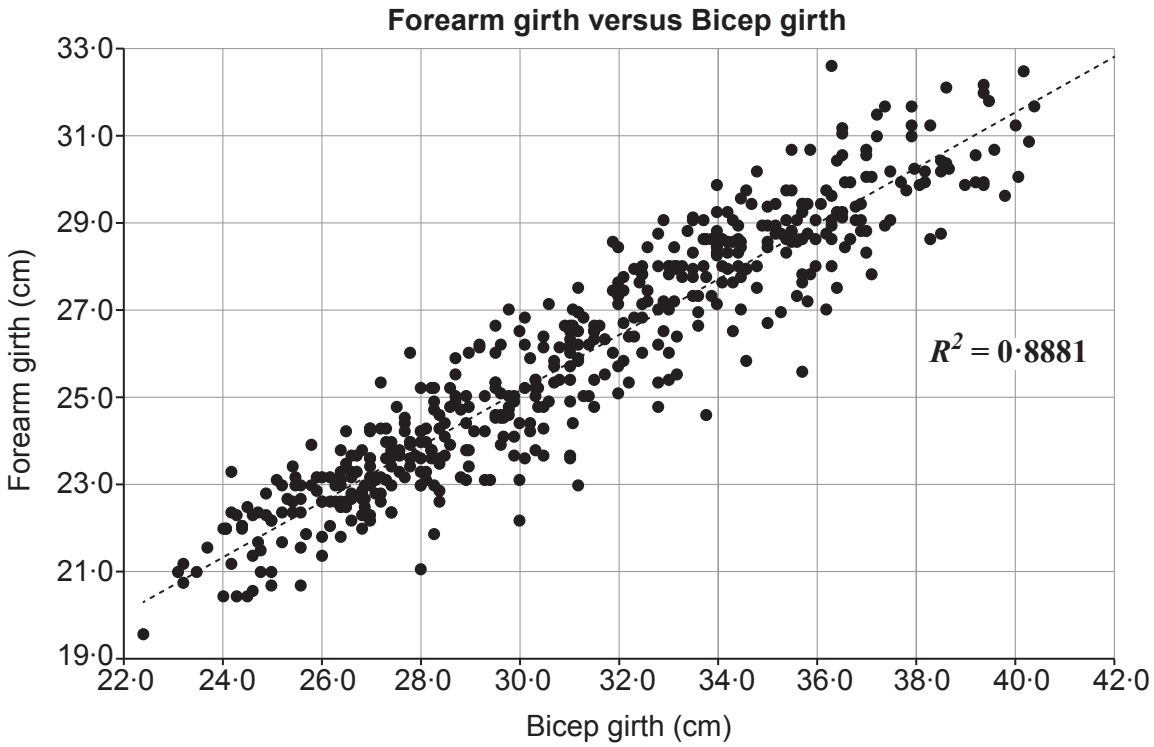
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(c) Blodwen also has access to the same large data set. She decides to do the same test using all of the 507 available data points. Her results are shown below.



(i) State the problem Blodwen will encounter when attempting to use statistical tables for her test.

(ii) How should Blodwen deal with this problem? [2]

(i) Her n = 507 is not in the statistical tables (they only go up to n = 100).

(ii) Use a software package (or an online calculator) to carry out the test, finding the critical value or the p-value for n = 507.

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4. Jake works for a parcel delivery company. The masses, in kilograms, of parcels he delivers are normally distributed with mean 2·2 and standard deviation 0·3.

(a) Calculate the probability that a randomly selected parcel will have a mass less than 1·8 kg. [2]

Let X be the mass, in kg, of a parcel. $X \sim N(2.2, 0.3^2)$

$P(X < 1.8) = P\left(Z < \frac{1.8 - 2.2}{0.3}\right) = P(Z < -1.3333...)$

$= 0.091211... = 0.0912$ (3 s.f.)

Jake delivers the lightest 80% of parcels on his bike. The rest he puts in his car and delivers by car.

(b) Find the mass of the heaviest parcel he would deliver by bike. [2]

$P(X < m) = 0.8$

$\frac{m - 2.2}{0.3} = 0.84162...$

$m = 2.2 + 0.3 \times 0.84162... = 2.45248...$

Heaviest parcel delivered by bike = 2.452 kg

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(c) He randomly selects a parcel from his car. Find the probability that it has a mass less than 3 kg. [4]

Car parcels are those with $X > m = 2.452...$, so $P(car) = 0.2$

$$P(X < 3 \mid car) = \frac{P(m < X < 3)}{P(car)}$$

$P(m < X < 3) = P(X < 3) - 0.8 = 0.996169... - 0.8 = 0.196169...$

$$P(X < 3 \mid car) = \frac{0.196169...}{0.2}$$

$= 0.98084... = 0.981$ (3 s.f.)

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(d) In the run-up to Christmas, Jake believes that the parcels he has to deliver are, on average, heavier. He assumes that the standard deviation is unchanged. He randomly selects 20 parcels and finds that their total mass is 46 kg. Test Jake's belief at the 5% level of significance. [7]

Let μ be the mean mass, in kg, of a parcel in the run-up to Christmas.

H_0: $\mu = 2.2$ H_1: $\mu > 2.2$

Under H_0: $\bar{X} \sim N(2.2, \frac{0.3^2}{20})$

$\bar{x} = \frac{46}{20} = 2.3$

p-value $= P(\bar{X} > 2.3 \mid H_0)$

$= P(Z > \frac{2.3 - 2.2}{0.3 \div \sqrt{20}}) = P(Z > 1.4907...)$

$= 0.068018...$

Since $0.0680 > 0.05$, there is insufficient evidence to reject H_0.

There is insufficient evidence at the 5% level to suggest that parcels are heavier, on average, in the run-up to Christmas.

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Jake delivers each parcel to one of three areas, *A*, *B* or *C*. The probabilities that a parcel has destination area *A*, *B* and *C* are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. All parcels are considered to be independent.

(e) On a particular day, Jake has three parcels to deliver. Find the probability that he will have to deliver to all three areas. [2]

$P(A, B, C \text{ in that order}) = \frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} = \frac{1}{36}$

In any order: $3! \times \frac{1}{36} = 6 \times \frac{1}{36} = \frac{1}{6}$

(f) On a different day, Jake has two parcels to deliver. Find the probability that he will have to deliver to more than one area. [4]

$P(AA) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(BB) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(CC) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$P(\text{only one area}) = \frac{1}{4} + \frac{1}{36} + \frac{1}{9} = \frac{14}{36} = \frac{7}{18}$

$P(\text{more than one area}) = 1 - \frac{7}{18} = \frac{11}{18} \quad (= 0.6111...)$

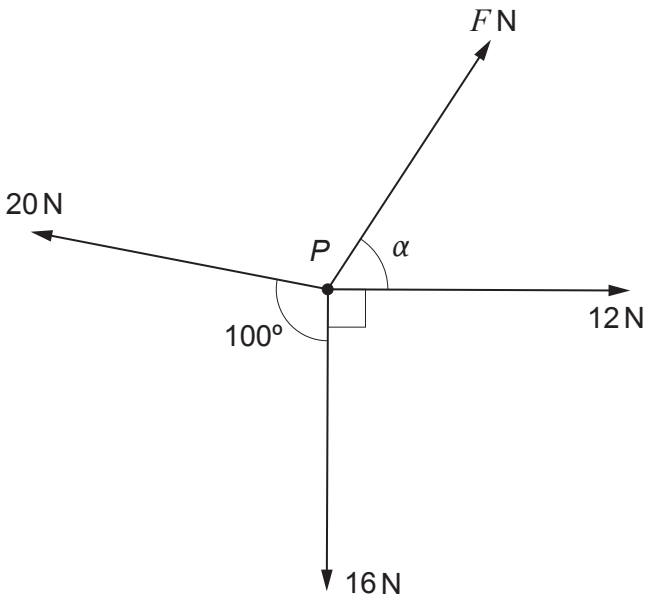
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Section B: Differential Equations and Mechanics

5. The diagram below shows four coplanar horizontal forces of magnitude FN, 12 N, 16 N and 20 N acting at a point P in the directions shown.



Given that the forces are in equilibrium, calculate the value of F and the size of the angle α. [7]

The 20 N force makes 100° − 90° = 10° with the horizontal.

Resolving parallel to the 12 N force (→):

$F \cos \alpha + 12 = 20 \cos 10°$

$F \cos \alpha = 20 \cos 10° - 12 = 7.6961...$

Resolving parallel to the 16 N force (↑):

$F \sin \alpha + 20 \sin 10° = 16$

$F \sin \alpha = 16 - 20 \sin 10° = 12.527...$

$$\tan \alpha = \frac{16 - 20 \sin 10°}{20 \cos 10° - 12} = \frac{12.527...}{7.6961...} = 1.6277...$$

$\alpha = 58.434...° = 58.4°$ (3 s.f.)

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$F^2 = 7.6961...^2 + 12.527...^2 = 216.16...$

$F = \sqrt{216.16...} = 14.702...$

$F = 14.7$ (3 s.f.)

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6. A ball is projected with velocity $(4w\mathbf{i} + 7w\mathbf{j})\text{ms}^{-1}$ from the top of a vertical tower. After 5 seconds, the ball hits the ground at a point that is 60 m horizontally from the foot of the tower.

The unit vectors **i** and **j** are horizontal and vertical respectively.

(a) Find the value of w and hence determine the height of the tower. [5]

Horizontally (→), speed is constant:

$4w \times 5 = 60 \Rightarrow w = 3$

Vertically (↑ positive): $u = 7w = 21$, $a = -9.8$, $t = 5$

$s = ut + \frac{1}{2}at^2 = 21(5) + \frac{1}{2}(-9.8)(5)^2$

$s = 105 - 122.5 = -17.5$

The ball lands 17.5 m below its starting point,

so the height of the tower is 17.5 m.

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(b) Determine the proportion of the 5 seconds for which the ball is on its way down. [3]

Vertically (↑): at the highest point $v = 0$, $u = 21$, $a = -9.8$

$v = u + at \Rightarrow 0 = 21 - 9.8t$

$t = \frac{21}{9.8} = \frac{15}{7}$ s (time to reach maximum height)

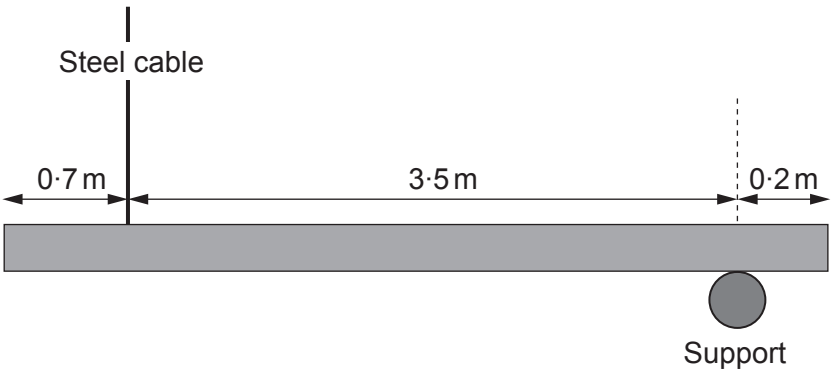
Proportion on the way down $= \frac{5 - 15/7}{5} = \frac{4}{7}$

$= 0.5714... = 0.571$ (3 s.f.)

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7. As part of a design for a new building, an architect wants to support a wooden beam in a horizontal position. The beam is suspended using a vertical steel cable and a smooth fixed support on its underside. The diagram below shows the architect's diagram and the adjacent table shows the categories of steel cable available.



Steel Cable	
Category	Supports forces up to (N)
A	3000
B	2500
C	2000

You may use the following modelling assumptions.

- The wooden beam is a rigid uniform rod of mass 100 kg.
- The force exerted on the beam by the support is vertical.
- The steel cable is inextensible.

SAFETY REQUIREMENT

Both the steel cable and the support must be capable of withstanding forces of **at least four times** those present in the architect's diagram above.

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The wooden beam is held in horizontal equilibrium.

(a) (i) Given that the support is capable of withstanding loads of up to 2000 N, show that the force exerted on the beam by the support satisfies the safety requirement. [3]

(ii) Determine which categories of steel cable in the table opposite could meet the safety requirements. [3]

(i) Beam length = 0.7 + 3.5 + 0.2 = 4.4 m, so the weight 100g acts at the midpoint, 2.2 m from the left end, i.e. 1.5 m from the cable.

Let R N be the support force and T N the tension in the cable.

Moments about the cable: $R \times 3.5 = 100g \times 1.5$

$3.5R = 1470$

$R = \frac{300}{7}g = 420$ N

Four times this force: $4 \times 420 = 1680$ N < 2000 N

Therefore the support satisfies the safety requirement.

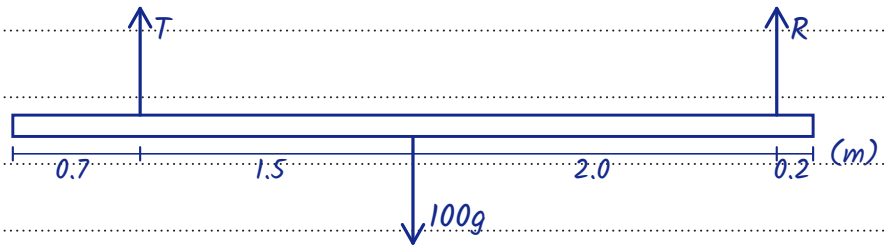
(ii) Resolving vertically: $T + R = 100g$

$T = 980 - 420 = 560$ N

Four times this force: $4 \times 560 = 2240$ N

Category A (3000 N) and category B (2500 N) can withstand 2240 N, category C (2000 N) cannot.

Therefore categories A and B could meet the safety requirements.



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(b) State how you have used the modelling assumption that the beam is a uniform rod. [1]

Uniform rod: the weight 100g acts at the centre of the rod, 2.2 m from each end, which gave the 1.5 m distance from the cable in the moments equation.

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8. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are acting on an object of mass 3 kg such that

$$\mathbf{F}_1 = (\mathbf{i} + 8c\mathbf{j} + 11c\mathbf{k})\text{N},$$

$$\mathbf{F}_2 = (-14\mathbf{i} - c\mathbf{j} - 12\mathbf{k})\text{N},$$

$$\mathbf{F}_3 = ((15c + 1)\mathbf{i} + 2c\mathbf{j} - 5c\mathbf{k})\text{N},$$

where c is a constant. The acceleration of the object is parallel to the vector $(\mathbf{i} + \mathbf{j})$.

(a) Find the value of the constant c and hence show that the acceleration of the object is $(6\mathbf{i} + 6\mathbf{j})\text{ms}^{-2}$. [4]

$F_1 + F_2 + F_3 = (1 - 14 + 15c + 1)i + (8c - c + 2c)j + (11c - 12 - 5c)k$

$= (15c - 12)i + 9cj + (6c - 12)k$

$a = \frac{F_1 + F_2 + F_3}{3} = (5c - 4)i + 3cj + (2c - 4)k$

a is parallel to (i + j), so the k component is zero:

$2c - 4 = 0 \Rightarrow c = 2$

(check: i component $5(2) - 4 = 6$ and j component $3(2) = 6$, equal)

$a = (10 - 4)i + 3(2)j + (4 - 4)k$

$a = (6i + 6j)\ ms^{-2}$ as required

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(b) When $t = 0$ seconds, the object has position vector $\mathbf{r}_0$ m and is moving with velocity $(-17\mathbf{i} + 8\mathbf{j})\text{ms}^{-1}$. When $t = 4$ seconds, the object has position vector $(-13\mathbf{i} + 84\mathbf{j})$m. Find the vector $\mathbf{r}_0$. [3]

a is constant, so $r = r_0 + ut + \frac{1}{2}at^2$ with $u = -17i + 8j$, $t = 4$

$-13i + 84j = r_0 + (-17i + 8j)(4) + \frac{1}{2}(6i + 6j)(4)^2$

$-13i + 84j = r_0 + (-68i + 32j) + (48i + 48j)$

$-13i + 84j = r_0 - 20i + 80j$

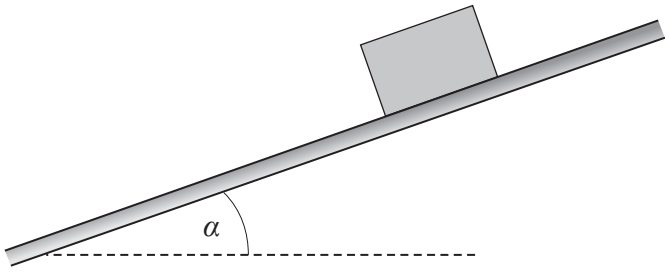
$r_0 = (7i + 4j)$ m

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9. The diagram below shows a parcel, of mass m kg, sliding down a rough slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{7}{25}$.



The coefficient of friction between the parcel and the slope is $\frac{1}{12}$. In addition to friction, the parcel experiences a variable resistive force of mv N, where v ms^{-1} is the velocity of the parcel at time t seconds.

(a) Show that the motion of the parcel satisfies the differential equation

$$5\frac{\mathrm{d}v}{\mathrm{d}t} = g - 5v. \qquad [5]$$

$\sin\alpha = \frac{7}{25} \Rightarrow \cos\alpha = \frac{24}{25}$

$R = mg\cos\alpha = \frac{24}{25}mg$

Friction $F = \mu R = \frac{1}{12} \times \frac{24}{25}mg = \frac{2}{25}mg$

N2L down the slope: $mg\sin\alpha - F - mv = ma$

$\frac{7}{25}mg - \frac{2}{25}mg - mv = m\frac{dv}{dt}$

$\div m$: $\frac{1}{5}g - v = \frac{dv}{dt}$

$\times 5$: $5\frac{dv}{dt} = g - 5v$ as required

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(b) Given that the parcel is initially at rest on the slope, find an expression for v in terms of t and g. [5]

$\int \frac{5}{g - 5v}\, dv = \int 1\, dt$

$-\ln|g - 5v| = t + C$

$t = 0,\ v = 0$: $-\ln g = C$

$t = \ln g - \ln|g - 5v| \Rightarrow -t = \ln \frac{g - 5v}{g}$

$e^{-t} = 1 - \frac{5v}{g}$

$v = \frac{g}{5}(1 - e^{-t})$

(c) To avoid damage, the speed of the parcel must be restricted to a maximum of $2\,ms^{-1}$ down the slope. Determine whether or not the speed of the parcel exceeds $2\,ms^{-1}$. [1]

As $t \to \infty$, $e^{-t} \to 0$ so $v \to g/5 = 1.96\ ms^{-1}$. Since $v < 1.96 < 2$ for all t, the speed of the parcel does not exceed $2\ ms^{-1}$.

END OF PAPER

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