

Surname	Centre Number	Candidate Number
First name(s)		2



GCE A LEVEL

1300U30-1



S24-1300U30-1

TUESDAY, 4 JUNE 2024 – MORNING

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	11	
2	11	
3	7	
4	6	
5	4	
6	13	
7	7	
8	7	
9	9	
10	14	
11	10	
12	6	
13	3	
14	7	
15	5	
Total	120	

1300U301
01



JUN241300U30101

Examiner
only

- Examiner
only

Examiner
only

- [4]



1300U301
03



- [5]



- (b) By writing $24\cos x - 7\sin x$ in the form $R\cos(x + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$, solve the equation

$$24\cos x - 7\sin x = 16$$

for values of x between 0° and 360° .

[6]

1300U301
05



-
- The diagram shows a circular sector with center O . The full sector is OAC , with radius $OA = 5\text{ cm}$. A smaller sector OAB is removed from the top, where $OA = OB = r$. The remaining shaded region is a circular segment bounded by the radii OD and OC and the arc DC . The radius OD is labeled 5 cm .

[4]



1300U301
07

[3]



4. A function f is given by $f(x) = |3x + 4|$.

- (a) Sketch the graph of $y = f(x)$. Clearly label the coordinates of the point A , where the graph meets the x -axis, and the coordinates of the point B , where the graph cuts the y -axis. [3]

.....

.....

.....

.....



- (b) On a separate set of axes, sketch the graph of $y = \frac{1}{2}f(x) - 6$, where the points A and B are transformed to the points A' and B' .
Clearly label the coordinates of the points A' and B' . [3]

.....

.....

.....

.....



5. Prove by contradiction the following proposition:

When x is real and positive, $x + \frac{81}{x} \geq 18$.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**



6. (a) Differentiate $\cos x$ from first principles.

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Differentiate $e^{3x}\sin 4x$ with respect to x .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



1300U301
13

[5]



7. Showing all your working, evaluate

(a) $\sum_{r=3}^{50} (4r + 5),$

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



(b) $\sum_{r=2}^{\infty} \left(540 \times \left(\frac{1}{3} \right)^r \right).$

[3]



8. The function f is defined by

$$f(x) = x^3 + 4x^2 - 3x - 1.$$

- (a) Show that the equation $f(x) = 0$ has a root in the interval $[0, 1]$. [1]

- (b) Using the Newton-Raphson method with $x_0 = 0.8$,
- (i) write down in full the decimal value of x_1 as given in your calculator,
 - (ii) determine the value of this root correct to six decimal places.
- [4]



.....

.....

.....

.....

.....

.....

.....

.....

(c) Explain why the Newton-Raphson method does not work if $x_0 = \frac{1}{3}$. [2]

.....

.....

.....

.....

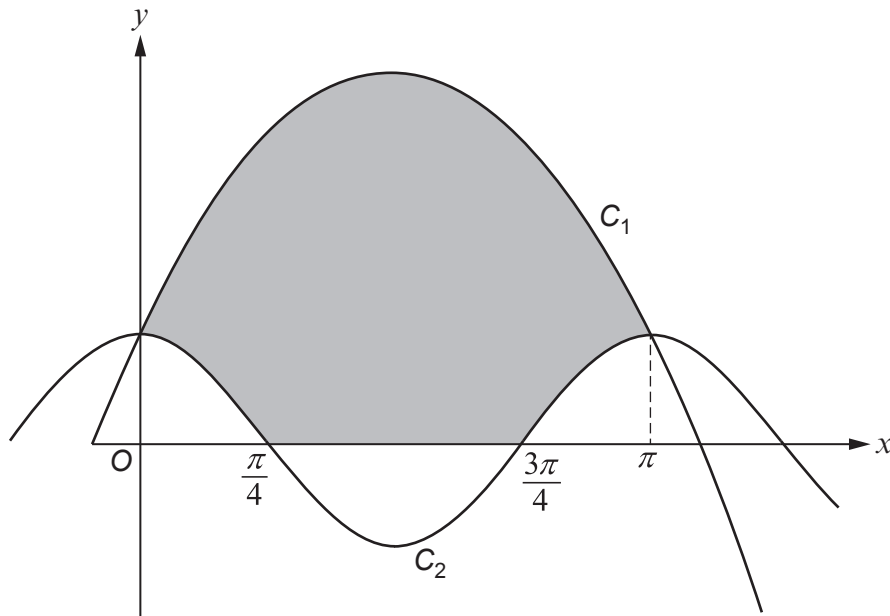
.....

.....

.....



9. The diagram below shows a sketch of the curve C_1 with equation $y = -x^2 + \pi x + 1$ and a sketch of the curve C_2 with equation $y = \cos 2x$. The curves intersect at the points where $x = 0$ and $x = \pi$.



Calculate the area of the shaded region enclosed by C_1 , C_2 and the x -axis. Give your answer in terms of π . [9]





$$f(x) = \frac{2(3x+1)}{x^2-2x-3} + \frac{x}{x+1}.$$


- (c) Find an expression for $f^{-1}(x)$ and write down the domain and range of f^{-1} . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) Find the value of x when $f(x) = f^{-1}(x)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



$$x = 2\theta + \sin 2\theta, \quad y = 1 + \cos 2\theta.$$

(a) Show that the gradient of the curve at the point with parameter θ is $-\tan \theta$. [6]



[4]



12. (a) Given that θ is small, show that $2\cos\theta + \sin\theta - 1 \approx 1 + \theta - \theta^2$.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence, when θ is small, show that

$$\frac{1}{2\cos\theta + \sin\theta - 1} \approx 1 + a\theta + b\theta^2,$$

where a, b are constants to be found.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

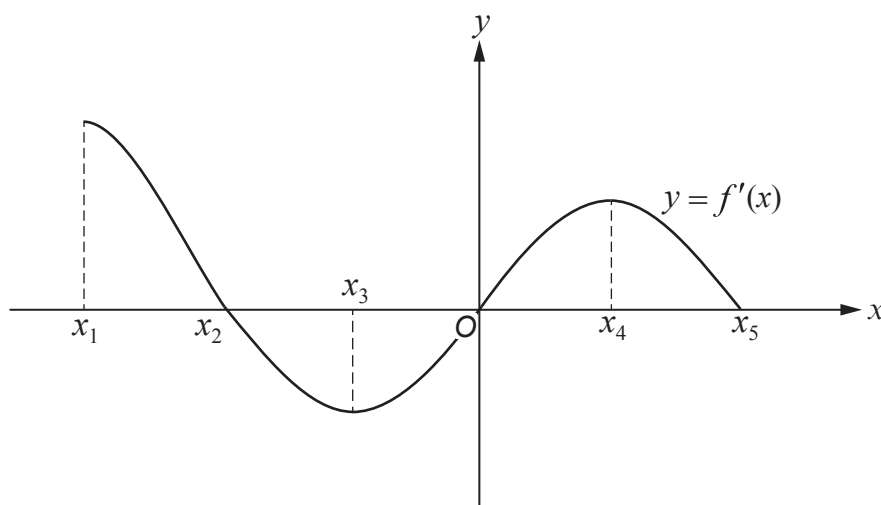
.....

.....

.....



13. The diagram below shows a sketch of the graph of $y = f'(x)$ for the interval $[x_1, x_5]$.



- (a) Find the interval on which $f(x)$ is both decreasing and convex. Give reasons for your answer. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Write down the x -coordinate of a point of inflection of the graph of $y = f(x)$. [1]

.....

.....

.....

.....



14. (a) Given that $y = \frac{1 + \ln x}{x}$, show that $\frac{dy}{dx} = \frac{-\ln x}{x^2}$. [2]

.....

.....

.....

.....

.....

.....

.....

- (b) Hence, solve the differential equation

$$\frac{dx}{dt} = \frac{x^2 t}{\ln x},$$

given that $t = 3$ when $x = 1$.

Give your answer in the form $t^2 = g(x)$, where g is a function of x . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



.....

.....

.....

.....

.....

.....

.....

.....



- Account A offers an annual compound interest rate of 1%.
- Account B offers an interest rate of 5% for the first year and an annual compound interest rate of 0.6% for each subsequent year.

After n years, account A is more profitable than account B. Find the smallest value of n . [5]

END OF PAPER



[illegible]

[illegible]

BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**



BLANK PAGE

**PLEASE DO NOT WRITE
ON THIS PAGE**

