



GCE A LEVEL MARKING SCHEME

SUMMER 2023

**A LEVEL
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1**

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2023 MARK SCHEME

Q	Solution	Mark	Notes
1	Let the first term = a Let the common difference = d		
	$a + (12 - 1)d = 41$	B1	oe
	$\frac{16}{2}(2a + (16 - 1)d) = 488$	B1	oe
	$2a + 15d = 61$		
	$2a + 22d = 82$		
	$7d = 21$	M1	a valid attempt to eliminate one variable, FT their linear equations
	$d = 3$	A1	cao
	$a (= 41 - 33) = 8$	A1	cao

Q	Solution	Mark	Notes
2(a)(i)	$5(\sin x + x^2)^4 f(x)$	M1	$f(x) \neq 0, 1$, condone lack of brackets round $f(x)$
	$= 5(\sin x + x^2)^4 (\cos x + 2x)$	A1	brackets required in $(\cos x + 2x)$, ISW

2(a)(ii)	$x^3 f(x) + g(x) \cos x$	M1	$f(x), g(x) \neq 0, 1$
	$= x^3(-\sin x) + (3x^2) \cos x$	A1	ISW

2(a)(iii)	$\frac{\sin 2x(f(x)) - e^{3x}(g(x))}{\sin^2 2x}$	M1	$f(x), g(x) \neq 0, 1$
	$= \frac{\sin 2x(3e^{3x}) - e^{3x}(2\cos 2x)}{\sin^2 2x}$	A1	$f(x) = 3e^{3x}$
		A1	$g(x) = 2\cos 2x$, ISW

OR

$$y = e^{3x}(\sin 2x)^{-1}$$

$$y = (f(x))(\sin 2x)^{-1} + e^{3x}(g(x)) \quad (\text{M1}) \quad f(x), g(x) \neq 0, 1$$

$$\frac{dy}{dx} = 3e^{3x}(\sin 2x)^{-1} + e^{3x}(-1)(\sin 2x)^{-2}(2\cos 2x) \quad (\text{A1}) \quad f(x) = 3e^{3x}$$

$$(\text{A1}) \quad g(x) = (-1)(\sin 2x)^{-2}(2\cos 2x),$$

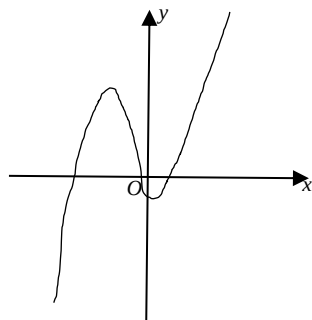
ISW

Q	Solution	Mark	Notes
2(b)	$8y \frac{dy}{dx} - (7x \frac{dy}{dx} + 7y) + 2x = 0$	B1	$8y \frac{dy}{dx}$
		B1	$(7x \frac{dy}{dx} + 7y)$ or $7x \frac{dy}{dx} \pm 7y$
		B1	$2x$ and 0
	$(8y - 7x) \frac{dy}{dx} = (7y - 2x)$		
	$\frac{dy}{dx} = \frac{7y - 2x}{8y - 7x}$		
	At $(2, 4)$, $\frac{dy}{dx} = \frac{28 - 4}{32 - 14} \left(= \frac{24}{18} = \frac{4}{3} \right)$	M1	oe, correctly substitute values for x and y , FT their dy/dx for equivalent expression
	Gradient of tangent at $(2, 4) = \frac{4}{3}$	m1	si
	Equation of tangent is		
	$y - 4 = \frac{4}{3}(x - 2)$	A1	oe, cao, ISW
	$3y = 4x + 4$		

Q	Solution	Mark	Notes
3(a)	$\frac{9}{(1-x)(1+2x)^2} \equiv \frac{A}{(1-x)} + \frac{B}{(1+2x)} + \frac{C}{(1+2x)^2}$ $9 = A(1+2x)^2 + B(1-x)(1+2x) + C(1-x)$ Put $x = 1$ $9 = 9A, A = 1$ Put $x = -\frac{1}{2}$, $9 = \frac{3}{2}C, C = 6$ Coefficient $x^2, 0 = 4A - 2B, B = 2$	M1 m1 A1	 oe one correct coefficient
	$\frac{9}{(1-x)(1+2x)^2} \equiv \frac{1}{(1-x)} + \frac{2}{(1+2x)} + \frac{6}{(1+2x)^2}$		
3(b)	$(1-x)^{-1} = 1 + x + \frac{(-1)(-2)}{2}(-x)^2 + \dots$ $= 1 + x + x^2 + \dots$	B1	si
	$(1+2x)^{-1} = 1 - 2x + \frac{(-1)(-2)}{2}(2x)^2 + \dots$ $= 1 - 2x + 4x^2 + \dots$	B1	si
	$(1+2x)^{-2} = 1 - 4x + \frac{(-2)(-3)}{2}(2x)^2 + \dots$ $= 1 - 4x + 12x^2 + \dots$	B1	si
	$(1-x)^{-1} + 2(1+2x)^{-1} + 6(1+2x)^{-2}$ $= (1+x+x^2) + 2(1-2x+4x^2)$ $+ 6(1-4x+12x^2) + \dots$	M1	Adding candidate's 2 or 3 series, with their A, B, C
	$= 9 - 27x + 81x^2 + \dots$	A1	2 correct terms cao
		A1	all 3 correct cao, ISW
	Expansion is valid when $ x < \frac{1}{2}$	B1	$-\frac{1}{2} < x < \frac{1}{2}$ Mark final answer.

Q Solution**Mark Notes**

4(a)



$$f(-1) = 30, f(0) = -6, f(1) = 28$$

There are two roots in $[-1, 1]$.

M1 looking at signs of $f(x)$ in $[-1, 1]$ for at least 2 values, or attempt to find roots. (Correct roots are $-6, -\frac{1}{3}, \frac{1}{2}$).
Or sketch of graph between $x = -1$ and $x = 1$

A1 2 changes of signs or roots $-\frac{1}{3}, \frac{1}{2}$

Award M1A1 for stating '2 roots', provided not based on incorrect mathematics,
e.g. $30 - 28 = 2$ roots M1A0

Q	Solution	Mark	Notes
4(b)(i)	$f'(x) = 18x^2 + 70x - 7$	B1	seen anywhere
	$x_{n+1} = x_n - \frac{6x_n^3 + 35x_n^2 - 7x_n - 6}{18x_n^2 + 70x_n - 7}$	M1	si
	$x_0 = 1$		
	$x_1 = \frac{53}{81} = 0.6543209877$	A1	
4(b)(ii)	$(x_2 = 0.5234785163)$		
	$(x_3 = 0.5007059775)$		
	$(x_4 = 0.5000006736)$		
	root = 0.5	A1	
4(c)	e.g. $x_1 = 2.049390153 \left(= \frac{\sqrt{105}}{5} \right)$	B1	si
	$7x_1 + 6 - 6x_1^3 = -31.29890079 < 0,$	E1	$7x_1 + 6 - 6x_1^3 < 0$
	or $\frac{7x_1 + 6 - 6x_1^3}{35} = -0.89... < 0$		or $\frac{7x_1 + 6 - 6x_1^3}{35} < 0$
	so the square root will not be real. Hence the method cannot be used to find a root of $f(x) = 0$.		
			If no values seen, award B1E1 provided reference has been made to x_2 , explicitly or implicitly

Q	Solution	Mark	Notes
5(a)	Amount of growth after 10 years		
	$= 32 + 32(0.9) + 32(0.9)^2 + \dots + 32(0.9)^9$	B1	si, GP $a = 32$, $r = 0.9$, or at least 3 terms
	$= 32(1 + 0.9 + 0.9^2 + \dots + 0.9^9)$		
	$= 32\left(\frac{1(1 - 0.9^{10})}{1 - 0.9}\right)$	M1	correct use of correct formula, for their GP provided $r = 0.9$. Award M1 for $(A)\left(\frac{1(1 - 0.9^9)}{1 - 0.9}\right)$ or $(A)\left(\frac{1(1 - 0.9^{11})}{1 - 0.9}\right)$
	$= 32 \times 6.5132 = 208.42\dots$	A1	si, Award A1 for 521.06 ($A = 80$, $n = 10$) or 729.48 ($A = 112$, $n = 10$)
	Height after 10 years = $80 + 208.42$		
	Height after 10 years = 288(.42) (cm)	A1	oe, cao
5(b)	Maximum growth of tree = $32\left(\frac{1}{1 - 0.9}\right)$	M1	correct use of correct formula $(A)\left(\frac{1}{1 - 0.9}\right)$ gets M1
	(Maximum growth of tree = 320)		
	Maximum height of tree = 400 (cm)	A1	oe, cao

Q	Solution	Mark	Notes
6(a)	$\cos 75^\circ = \cos(30^\circ + 45^\circ)$ $= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{3} - 1}{2\sqrt{2}}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$	M1 A1 A1	oe convincing

Q	Solution	Mark	Notes
6(b)	$2\cot^2x + \operatorname{cosec}x = 4$		
	$2(\operatorname{cosec}^2x - 1) + \operatorname{cosec}x = 4$	M1	$\cot^2x + 1 = \operatorname{cosec}^2x$
	$2 \operatorname{cosec}^2x + \operatorname{cosec}x - 6 = 0$		
	$(2\operatorname{cosec}x - 3)(\operatorname{cosec}x + 2) = 0$		
	$\operatorname{cosec}x = \frac{3}{2}, -2$	A1	
	$\sin x = \frac{2}{3}$	B1	or $\sin x = -\frac{1}{2}$
	$x = 41.81^\circ, 138.19^\circ$	B1	$0.7297^c, 2.4119^c$
	$\sin x = -\frac{1}{2},$		
	$x = 210^\circ$	B1	$\frac{7\pi}{6}$
	$x = 330^\circ$	B1	$\frac{11\pi}{6}$
OR	$2\frac{\cos^2x}{\sin^2x} + \frac{1}{\sin x} = 4$	(B1)	
	$2(1 - \sin^2x) + \sin x = 4\sin^2x$	(M1)	use of $\sin^2x + \cos^2x = 1$
	$6\sin^2x - \sin x - 2 = 0$		
	$(3\sin x - 2)(2\sin x + 1) = 0$		
	$\sin x = \frac{2}{3}, -\frac{1}{2}$	(A1)	
	$x = 41.81^\circ, 138.19^\circ$	(B1)	$0.7297^c, 2.4119^c$
	$x = 210^\circ$	(B1)	$\frac{7\pi}{6}$
	$x = 330^\circ$	(B1)	$\frac{11\pi}{6}$

NOTES

Mark each branch separately.

FT 2 branches only if different signs.

Do not FT for other trig functions.

For each branch, -1 for a 3rd root in the range $0^\circ < \theta < 360^\circ$,
 -1 for a 4th root in the range $0^\circ < \theta < 360^\circ$.

Ignore roots outside the range $0^\circ < \theta < 360^\circ$.

Q	Solution	Mark	Notes
6(c)(i)	$R\cos(\theta + \alpha) = 7\cos\theta - 24\sin\theta$ $R\cos\theta\cos\alpha - R\sin\theta\sin\alpha \equiv 7\cos\theta - 24\sin\theta$ $R\cos\alpha = 7, R\sin\alpha = 24$ $R = \sqrt{7^2 + 24^2} = 25$ $\alpha = \tan^{-1}\left(\frac{24}{7}\right) = 73.74^\circ$ $7\cos\theta - 24\sin\theta \equiv 25\cos(\theta + 73.74^\circ)$		
		M1	si, must be from correct identity M0 if $\cos\alpha = 7, \sin\alpha = 24$
		B1	
		A1	
6(c)(ii)	$25\cos(\theta + 73.74) = 5$ $\cos(\theta + 73.74) = \frac{5}{25}$ $\theta + 73.74 = \cos^{-1}(0.2)$ $\theta + 73.74 = 78.46, 281.54$ $\theta = 4.72^\circ,$ $\theta = 207.80^\circ$		
		M1	ft similar expressions if possible.
		A1	FT for one angle only
		A1	

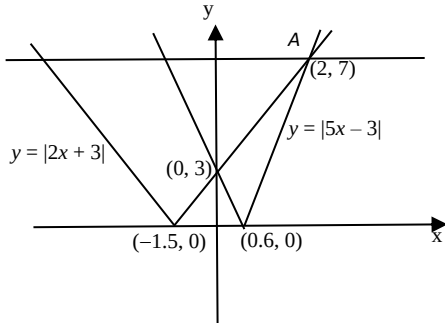
NOTES

Do not FT for other trig functions.

-1 for a 3rd root in the range $0^\circ < \theta < 360^\circ$,

-1 for a 4th root in the range $0^\circ < \theta < 360^\circ$.

Ignore roots outside the range $0^\circ < \theta < 360^\circ$.

Q	Solution	Mark	Notes
7(a)	$5x - 3 = 2x + 3$ $3x = 6$ $x = 2, y = 7$	M1 A1	AG allow verification (both lines for M1)
7(b)		B1 B1 B1 B1	correct sketch $y = 2x + 3 $ correct sketch $y = 5x - 3 $ $(-1.5, 0), (0.6, 0),$ $(0, 3), (2, 7)$
7(c)	Area = triangle in first quadrant = trapezium – 2 triangles $= \frac{1}{2}(3 + 7) \times 2 - \frac{1}{2} \times 0.6 \times 3 - \frac{1}{2} \times (2 - 0.6) \times 7$ $= 10 - 0.9 - 4.9$ $= 4.2$	M1 m1 A1 A1	identify correct area, allow shown on diagram, Only FT if graphs are in 1st and 2nd quadrants method for area, oe correct expression, si cao

Q	Solution	Mark	Notes
8(a)	$f(x) = \frac{(4x^2 + 12x + 9)}{(2x^2 + x - 3)}$ $= \frac{(2x + 3)(2x + 3)}{(2x + 3)(x - 1)}$ $= \frac{2x + 3}{x - 1} \left(= \frac{2(x - 1) + 2 + 3}{x - 1} \right)$ $= 2 + \frac{5}{x - 1}$	B1 B1 B1	correct factorisation of one quadratic common factor cancelled convincing
	OR		
	$f(x) = \frac{4x^2 + 12x + 9}{2x^2 + x - 3} = \frac{2(2x^2 + x - 3) + 10x + 15}{2x^2 + x - 3}$ $= 2 + \frac{10x + 15}{2x^2 + x - 3}$ $= 2 + \frac{5(2x + 3)}{(2x + 3)(x - 1)}$ $= 2 + \frac{5}{x - 1}$	(B1) (B1) (B1)	 correct factorisation of denominator common factor cancelled
8(b)	$\int_3^7 f(x) dx = \int_3^7 \left(2 + \frac{5}{x - 1} \right) dx$ $= [2x + 5\ln x - 1]_3^7$ $= (14 + 5\ln 6) - (6 + 5\ln 2)$ $= 8 + 5\ln 3$	M1 B1 m1 A1	 correct integration, condone omission of modulus signs correct use of limits cao. Note $5\ln 3 = \ln 243$ M0 for 13.493 not supported by workings.

Q	Solution	Mark	Notes
10(a)(i)	$fg(x) = f((x-2)^2)$ $= \frac{8}{(x-2)^2 - 4}$	M1 A1	si Mark final answer
10(a)(ii)	$fg(x)$ does not exist when $g(x) = 4$ $(x-2)^2 = 4$ $x-2 = (\pm) 2$ $x = 4$ $x = 0$	M1 A1 A1	si, FT their $fg(x)$ SC1 for unsupported answer of $x = 4$ only
10(b)	$y = \frac{8}{x-4}$ $xy - 4y = 8, \quad x = \frac{8+4y}{y}$ $f^{-1}(x) = \frac{8+4x}{x}$ or $f^{-1}(x) = \frac{8}{x} + 4$	M1 A1 A1	 put x as subject, or y as subject if x and y interchanged earlier. Allow one slip cao, ' $f^{-1}(x)$ ' required

Q	Solution	Mark	Notes
11(a)	$f'(x) = 15x^2 + 4x - 3 (= (5x + 3)(3x - 1))$ $f''(x) = 30x + 4$ When $f''(x) = 0$ $30x + 4 = 0$ $x = -\frac{2}{15}$ $[f''(x) < 0 \text{ if } x < -\frac{2}{15}, f''(x) > 0 \text{ if } x > -\frac{2}{15}, \text{ so } x = -\frac{2}{15} \text{ is the } x\text{-coordinate of a point of inflection.}]$	B1 B1 M1 A1	 FT their $f'(x)$ used cao, condone $-\frac{4}{30}$
	Valid reason		
	Eg. $f'(-\frac{2}{15}) (= -\frac{49}{15}) \neq 0$		or stationary points only at $x = -\frac{3}{5}, \frac{1}{3}$
	AND Non-stationary point of inflection.	B1	FT their x value
11(b)	If C is concave, $f''(x) < 0$.	M1	oe si allow $\leq$
	$30x + 4 < 0$		
	$x < -\frac{2}{15}$	A1	condone $x < -\frac{4}{30}$, allow $\leq$ FT linear $f''(x)$

Q	Solution	Mark	Notes
12(a)	$\frac{dy}{dx} = ky$	B1	allow $\frac{dy}{dx} = -ky$, allow $\frac{dy}{dx} = 4y$ Award if seen in (b).
12(b)	$x = 1, y = 0.5, \frac{dy}{dx} = 2$		
	$2 = 0.5k, k = 4$	B1	
	$\frac{dy}{dx} = 4y$		
	$\int \frac{dy}{y} = \int 4 \, dx$	M1	Separation of variables
	$\ln y = 4x (+ C)$	A1	
	$\ln 0.5 = 4 + C$	m1	Use of conditions
	$C = \ln 0.5 - 4 (= -4.693\dots)$		
	When $x = 3$,		
	$\ln y = 4 \times 3 + \ln(0.5) - 4 (= 7.3068\dots)$	m1	$\ln y - \ln(0.5) = 8, 2y = e^8$
	$y = 0.5e^8 = 1490(.478\dots)$	A1	Condone values of y that round to 1491
	OR		
	$\int \frac{dy}{y} = \int k \, dx$	(M1)	Separation of variables
	$\ln y = kx + C$	(A1)	
	$y = Ae^{kx}$		
	$x = 1, y = 0.5, \frac{dy}{dx} = 2; 2 = k \times 0.5; k = 4$	(B1)	
	$0.5 = Ae^4$	(m1)	Use of conditions
	$A = 0.5e^{-4}$		
	When $x = 3, y = 0.5e^{-4} \times e^{12}$	(m1)	
	$y = 0.5e^8 = 1490(.478\dots)$	(A1)	Condone values of y that round to 1491

Q	Solution	Mark	Notes
13	Attempt to eliminate p or q for C_1 or C_2	M1	si
	$C_1: y = (x - 1)^2$	A1	any correct form ISW
	$C_2: 2y = x$	A1	any correct form ISW
	Graphs meet when		
	$2(x - 1)^2 = x$ or $y = (2y - 1)^2$	m1	
	$2x^2 - 5x + 2 = 0$ or $4y^2 - 5y + 1 = 0$		
	$(2x - 1)(x - 2) = 0$ or $(4y - 1)(y - 1) = 0$	m1	si $ax^2 + bx + c = (dx + e)(fx + g)$ $df = a$ and $eg = c$
	$x = \frac{1}{2}, 2$ or $y = \frac{1}{4}, 1$	A1	cao, One correct pair
	$y = \frac{1}{4}, 1$ or $x = \frac{1}{2}, 2$	A1	cao, All correct

Points of intersection are $\left(\frac{1}{2}, \frac{1}{4}\right), (2, 1)$

Q	Solution	Mark	Notes
13.	OR		
	Graphs meet when x and y coordinates		
	for the two curves are equal	(M1)	si
	$3p + 1 = 4q$		
	$9p^2 = 2q$	(A1)	at least one correct equation
	Solving simultaneously	(m1)	one variable eliminated
	$3p + 1 = 2 \times 9p^2$ or $(4q - 1)^2 = 2q$		
	$18p^2 - 3p - 1 = 0$ or $16q^2 - 10q + 1 = 0$		
	$(6p + 1)(3p - 1) = 0$ or $(8q - 1)(2q - 1) = 0$	(m1)	attempt to solve quadratic equation
			$ax^2 + bx + c = (dx + e)(fx + g)$ $df = a$ and $eg = c$
	$p = -\frac{1}{6}, \frac{1}{3}$ or $q = \frac{1}{8}, \frac{1}{2}$	(A1)	cao
	When $p = -\frac{1}{6}$ or $q = \frac{1}{8}$, point is $(\frac{1}{2}, \frac{1}{4})$	(A1)	cao, $x = \frac{1}{2}$, $x = 2$ or $y = \frac{1}{4}$, $y = 1$
	When $p = \frac{1}{3}$ or $q = \frac{1}{2}$, point is $(2, 1)$	(A1)	cao, All correct

Q	Solution	Mark	Notes
14(a)	$\int_0^1 ((3x - 1)e^{2x}) dx$		
	$= [(3x - 1)Ae^{2x}]_0^1 - \int_0^1 Ae^{2x} \times 3 dx$	M1	attempt at integration by parts, at least one term correct. Limits not required. Only allow $A = 2$ or $\frac{1}{2}$
	$= \left[(3x - 1) \frac{e^{2x}}{2} \right]_0^1 - \int_0^1 \frac{e^{2x}}{2} \times 3 dx$		
	$= \left[(3x - 1) \frac{e^{2x}}{2} \right]_0^1 - \left[\frac{3}{4} e^{2x} \right]_0^1$	A1	all correct
	$= \left(e^2 + \frac{1}{2} \right) - \left(\frac{3e^2}{4} - \frac{3}{4} \right)$	m1	correct use of limits, evidence required
	$= \frac{1}{4}e^2 + \frac{5}{4} = 3.097(264025)$	A1	cao, at least 1dp

If M0, SC1 for

$$\left[\left(\frac{3x^2}{2} - x \right) e^{2x} \right]_0^1 - \int_0^1 \left(\frac{3x^2}{2} - x \right) 2e^{2x} dx$$

at least 1 correct term.
No more marks available.

Note

No marks for answer unsupported by workings.

14(b)	$u = 1 - 2\cos x$		
	$du = 2\sin x dx$	B1	$\frac{du}{dx} = 2\sin x$
	$\int \frac{\sin x}{1 - 2\cos x} dx = \frac{1}{2} \int \frac{1}{u} du$	M1	
	$= \frac{1}{2} \ln u (+ C)$	A1	oe, allow if modulus sign missing
	$= \frac{1}{2} \ln 1 - 2\cos x + C$	A1	oe, modulus sign and + C required.