GCE A LEVEL NEW

1300U30-1

S19-1300U30-1

WEDNESDAY, 5 JUNE 2019 – MORNING

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

1300U301
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0** **1** . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

0 1 **a)** Express $\dfrac{9}{(x-1)(x+2)^2}$ in terms of partial fractions. [4]

b) Find $\displaystyle\int \frac{9}{(x-1)(x+2)^2}\mathrm{d}x$. [3]

0 2 Expand $\dfrac{4-x}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^3. State the range of values of x for which the expansion is valid. [6]

0 3 The nth term of a number sequence is denoted by x_n. The $(n + 1)$th term is defined by $x_{n+1} = 4x_n - 3$ and $x_3 = 113$.

a) Find the values of x_2 and x_1. [2]

b) Determine whether the sequence is an arithmetic sequence, a geometric sequence or neither. Give reasons for your answer. [2]

0 4 **a)** Express $5\sin x - 12\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

b) Find the minimum value of $\dfrac{4}{5\sin x - 12\cos x + 15}$. [2]

c) Solve the equation

$$5\sin x - 12\cos x + 3 = 0$$

for values of x between 0° and 360°. [4]

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0 5 **a)** Find the range of values of x for which $|1 - 3x| > 7$. [3]

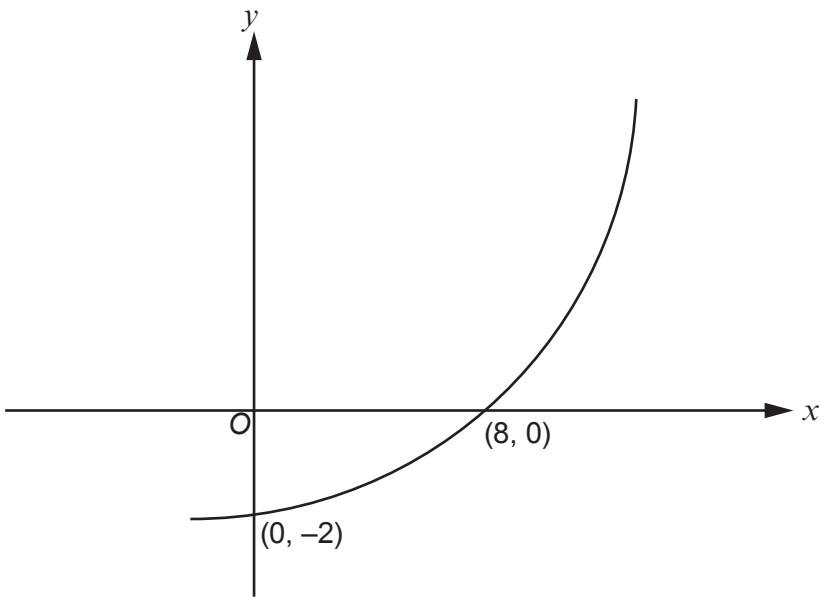
b) Sketch the graph of $y = |1 - 3x| - 7$. Clearly label the minimum point and the points where the graph crosses the x-axis. [4]

0 6 A curve C has parametric equations $x = \sin\theta$, $y = \cos 2\theta$.

a) The equation of the tangent to the curve C at the point P where $\theta = \frac{\pi}{4}$ is $y = mx + c$. Find the exact values of m and c. [6]

b) Find the coordinates of the points of intersection of the curve C and the straight line $x + y = 1$. [5]

0 7 The diagram below shows a sketch of the graph of $y = f(x)$. The graph crosses the y-axis at the point (0, –2), and the x-axis at the point (8, 0).



a) Sketch the graph of $y = -4f(x + 3)$. Indicate the coordinates of the point where the graph crosses the x-axis and the y-coordinate of the point where $x = -3$. [3]

b) Sketch the graph of $y = 3 + f(2x)$. Indicate the y-coordinate of the point where $x = 4$. [2]

TURN OVER

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0 8 **a)** The 3rd, 19th and 67th terms of an arithmetic sequence form a geometric sequence. Given that the arithmetic sequence is increasing and that the first term is 3, find the common difference of the arithmetic sequence. [5]

b) A firm has 100 employees on a particular Monday. The next day it adds 12 employees onto its staff and continues to do so on every successive working day, from Monday to Friday.

i) Find the number of employees at the end of the 8th week.

ii) Each employee is paid £55 per working day. Determine the total wage bill for the 8 week period. [6]

0 9 **a)** Given that α and β are two angles such that $\tan\alpha = 2\cot\beta$, show that

$$\tan(\alpha + \beta) = -(\tan\alpha + \tan\beta).$$ [2]

b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$4\tan\theta = 3\sec^2\theta - 7.$$ [6]

1 0 **a)** Differentiate each of the following functions with respect to x.

i) $x^5 \ln x$

ii) $\dfrac{e^{3x}}{x^3 - 1}$

iii) $(\tan x + 7x)^{\frac{1}{2}}$ [8]

b) A function is defined implicitly by

$$3y + 4xy^2 - 5x^3 = 8.$$

Find $\dfrac{dy}{dx}$ in terms of x and y. [3]

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1 1 The function $f(x)$ is defined by

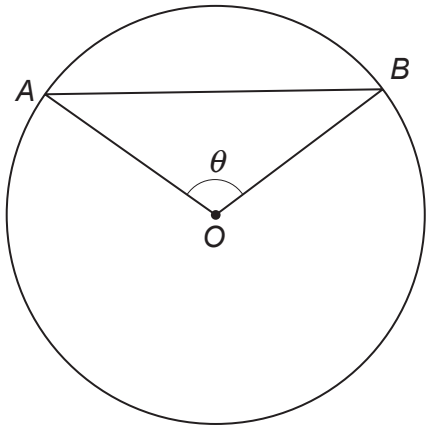
$$f(x) = \frac{\sqrt{x^2 - 1}}{x}$$

with domain $x \geqslant 1$.

a) Find an expression for $f^{-1}(x)$. State the domain for f^{-1} and sketch both $f(x)$ and $f^{-1}(x)$ on the same diagram. [6]

b) Explain why the function $ff(x)$ cannot be formed. [1]

1 2 A chord *AB* subtends an angle θ radians at the centre of a circle. The chord divides the circle into two segments whose areas are in the ratio 1 : 2.



a) Show that $\sin\theta = \theta - \frac{2\pi}{3}$. [4]

b) **i)** Show that θ lies between 2·6 and 2·7.

ii) Starting with θ_0 = 2·6, use the Newton-Raphson Method to find the value of θ correct to three decimal places. [6]

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13 Wildflowers grow on the grass verge by the side of a motorway. The area populated by wildflowers at time t years is $A\,\text{m}^2$. The rate of increase of A is directly proportional to A.

a) Write down a differential equation that is satisfied by A. [1]

b) At time $t = 0$, the area populated by wildflowers is $0{\cdot}2\,\text{m}^2$. One year later, the area has increased to $1{\cdot}48\,\text{m}^2$. Find an expression for A in terms of t in the form pq^t, where p and q are rational numbers to be determined. [7]

14 **a)** Find $\int (e^{2x} + 6\sin 3x)\,dx$. [2]

b) Find $\int 7(x^2 + \sin x)^6(2x + \cos x)\,dx$. [1]

c) Find $\int \frac{1}{x^2}\ln x\,dx$. [4]

d) Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{(2\cos x + 1)^2}\,dx \ .$$ [4]

15 Use proof by contradiction to show that $\sqrt{6}$ is irrational. [5]

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$$\frac{9}{(x-1)(x+2)^2} \equiv \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$9 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$x = 1$: $9 = 9A \Rightarrow A = 1$

$x = -2$: $9 = -3C \Rightarrow C = -3$

Coefficient of x^2: $0 = A + B \Rightarrow B = -1$

$$\frac{9}{(x-1)(x+2)^2} \equiv \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

1 (b)

$$\int \frac{9}{(x-1)(x+2)^2}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{1}{x+2}\,dx - \int \frac{3}{(x+2)^2}\,dx$$

$$= \ln|x-1| - \ln|x+2| + \frac{3}{x+2} + C$$

2

$$(1+2x)^{-\frac{1}{2}} = 1 + (-\tfrac{1}{2})(2x) + \frac{(-\frac{1}{2})(-3/2)}{2!}(2x)^2 + \frac{(-\frac{1}{2})(-3/2)(-5/2)}{3!}(2x)^3 + \ldots$$

$$= 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots$$

$$\frac{4-x}{\sqrt{(1+2x)}} = (4-x)\left(1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots\right)$$

$$= 4 - 4x + 6x^2 - 10x^3 - x + x^2 - \frac{3}{2}x^3 + \ldots$$

$$= 4 - 5x + 7x^2 - \frac{23}{2}x^3 + \ldots$$

Question number

2 cont.

Valid for $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

3 (a)

$x_3 = 4x_2 - 3$: $113 = 4x_2 - 3 \Rightarrow x_2 = 29$

$x_2 = 4x_1 - 3$: $29 = 4x_1 - 3 \Rightarrow x_1 = 8$

3 (b)

$x_2 - x_1 = 29 - 8 = 21$, $x_3 - x_2 = 113 - 29 = 84$

$21 \neq 84$, no common difference, so it is not an arithmetic sequence.

$\frac{x_2}{x_1} = \frac{29}{8} = 3.625$, $\frac{x_3}{x_2} = \frac{113}{29} = 3.896...$, $\frac{29}{8} \neq \frac{113}{29}$

No common ratio, so it is not a geometric sequence.

The sequence is neither arithmetic nor geometric.

4 (a)

$5\sin x - 12\cos x \equiv R \sin x \cos \alpha - R \cos x \sin \alpha$

$R \cos \alpha = 5$, $R \sin \alpha = 12$

$R = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$

$\tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.380...° = 67.38°$

$5\sin x - 12\cos x = 13 \sin(x - 67.38°)$

4 (b)

$\frac{4}{5\sin x - 12\cos x + 15} = \frac{4}{13\sin(x - 67.38°) + 15}$

Minimum when the denominator is a maximum, i.e. when $\sin(x - 67.38°) = 1$

Minimum value $= \frac{4}{13 + 15} = \frac{4}{28} = \frac{1}{7}$

Question number

4 (c) $13\sin(x - 67.38°) + 3 = 0 \Rightarrow \sin(x - 67.38°) = -\frac{3}{13}$

$x - 67.38° = -13.342°, \quad 180° + 13.342° = 193.342°$

(using $-67.38° < x - 67.38° < 292.62°$)

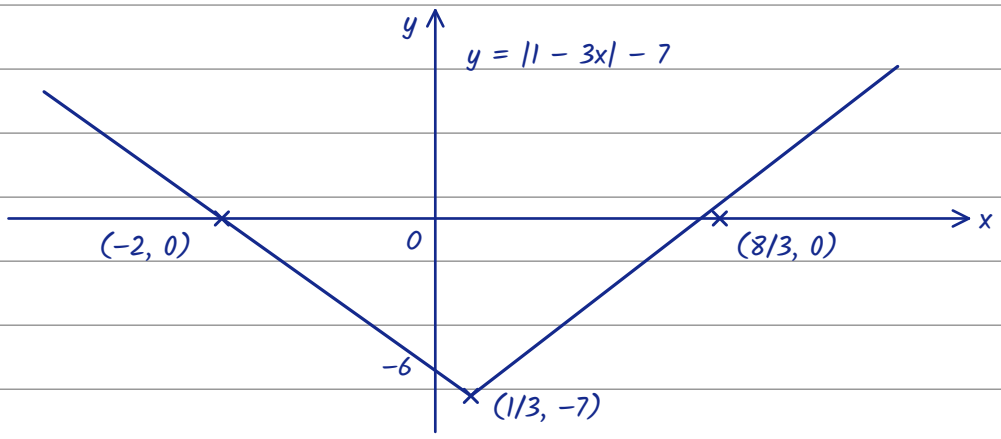
$x = 54.04°, \quad x = 260.72°$

5 (a) $1 - 3x > 7 \quad$ or $\quad 1 - 3x < -7$

$-3x > 6 \quad$ or $\quad -3x < -8$

$x < -2 \quad$ or $\quad x > \frac{8}{3}$

5 (b)



Minimum point $(1/3, -7)$; crosses the x-axis at $(-2, 0)$ and $(8/3, 0)$

6 (a) $\frac{dx}{d\theta} = \cos\theta, \quad \frac{dy}{d\theta} = -2\sin 2\theta, \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{2\sin 2\theta}{\cos\theta}$

When $\theta = \frac{\pi}{4}$: $\quad x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad y = \cos\frac{\pi}{2} = 0$

$m = -\frac{2\sin(\pi/2)}{\cos(\pi/4)} = -\frac{2}{1/\sqrt{2}} = -2\sqrt{2}$

Tangent: $\quad y - 0 = -2\sqrt{2}\left(x - \frac{1}{\sqrt{2}}\right)$

Question number

6 (a) cont.

$y = -2\sqrt{2}x + 2$

$m = -2\sqrt{2}, \quad c = 2$

6 (b)

$x + y = 1 \quad \Rightarrow \quad \sin\theta + \cos 2\theta - 1 = 0$

$\sin\theta + 1 - 2\sin^2\theta - 1 = 0$

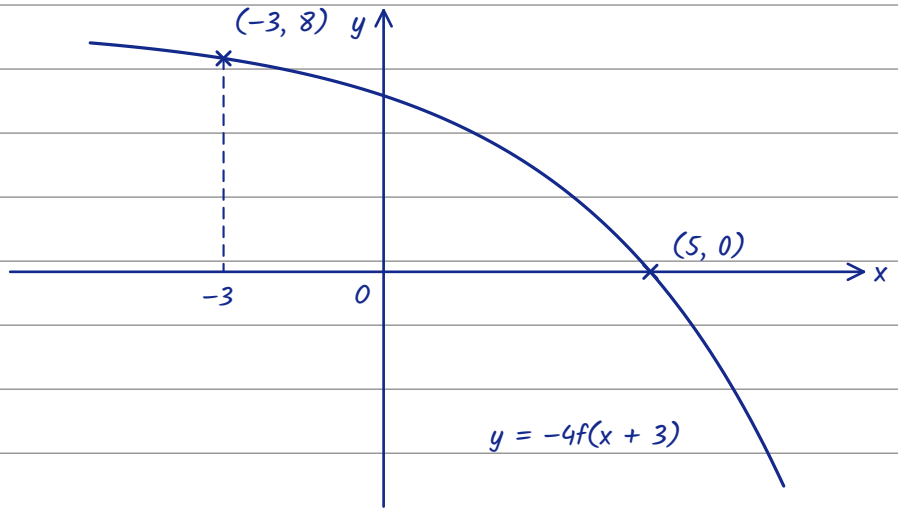
$2\sin^2\theta - \sin\theta = 0 \quad \Rightarrow \quad \sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$, so $x = 0$ or $x = \frac{1}{2}$

$y = 1 - x = 1$ or $\frac{1}{2}$

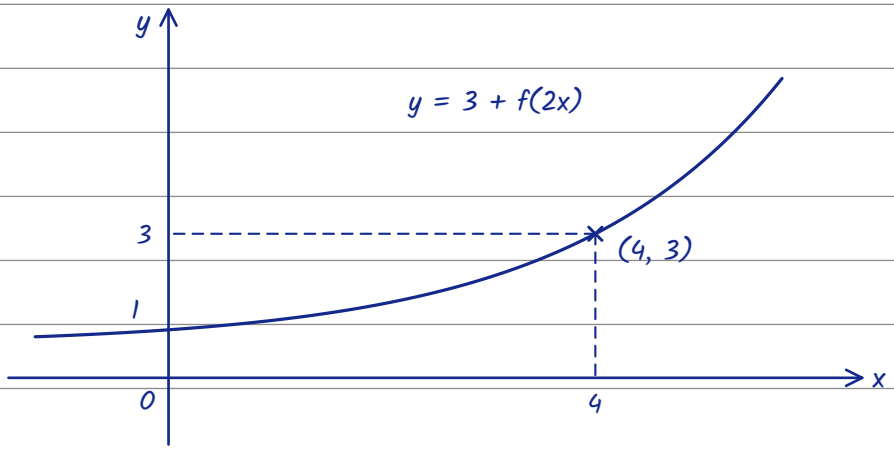
Points of intersection: $(0, 1)$ and $(\frac{1}{2}, \frac{1}{2})$

7 (a)



Crosses the x-axis at $(5, 0)$; when $x = -3$, $y = -4f(0) = -4(-2) = 8$

7 (b)



Question number	
7 (b) cont.	When $x = 4$: $y = 3 + f(8) = 3 + 0 = 3$
8 (a)	$T_3 = 3 + 2d$, $T_{19} = 3 + 18d$, $T_{67} = 3 + 66d$
	These form a GP: $\frac{3 + 66d}{3 + 18d} = \frac{3 + 18d}{3 + 2d}$ $(= r)$
	$(3 + 66d)(3 + 2d) = (3 + 18d)^2$
	$9 + 204d + 132d^2 = 9 + 108d + 324d^2$
	$192d^2 - 96d = 0 \Rightarrow 96d(2d - 1) = 0$
	$d = 0$ or $d = \frac{1}{2}$; the sequence is increasing so $d \neq 0$
	Common difference $d = \frac{1}{2}$
8 (b)(i)	Numbers of employees form an AP with $a = 100$, $d = 12$
	8 weeks $= 8 \times 5 = 40$ working days
	$T_{40} = 100 + 39 \times 12 = 568$ employees
8 (b)(ii)	Wage bill $= 55 \times (100 + 112 + 124 + \ldots$ to 40 terms$)$
	$= 55 \times \frac{40}{2}(2 \times 100 + 39 \times 12)$
	$= 55 \times 20 \times 668 = 55 \times 13360$
	$= £734\,800$
9 (a)	$\tan\alpha = 2\cot\beta = \frac{2}{\tan\beta} \Rightarrow \tan\alpha \tan\beta = 2$
	$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\tan\alpha + \tan\beta}{1 - 2}$
	$= -(\tan\alpha + \tan\beta)$ as required
9 (b)	$4\tan\theta = 3(1 + \tan^2\theta) - 7$
	$3\tan^2\theta - 4\tan\theta - 4 = 0$

Question number

9 (b) cont.

$(3\tan\theta + 2)(\tan\theta - 2) = 0$

$\tan\theta = 2 \quad \text{or} \quad \tan\theta = -\frac{2}{3}$

$\tan\theta = 2$: $\theta = 63.4°,\ 243.4°$

$\tan\theta = -2/3$: $\theta = 180° - 33.69° = 146.3°,\quad 360° - 33.69° = 326.3°$

$\theta = 63.4°,\ 146.3°,\ 243.4°,\ 326.3°$

10 (a)(i) Product rule: $\frac{d}{dx}(x^5 \ln x) = x^5 \times \frac{1}{x} + 5x^4 \ln x$

$= x^4 + 5x^4 \ln x$

10 (a)(ii) Quotient rule: $\frac{d}{dx}\left(\frac{e^{3x}}{x^3 - 1}\right) = \frac{(x^3 - 1)3e^{3x} - e^{3x}(3x^2)}{(x^3 - 1)^2}$

10 (a)(iii) Chain rule: $\frac{d}{dx}(\tan x + 7x)^{1/2} = \frac{1}{2}(\tan x + 7x)^{-1/2}(\sec^2 x + 7)$

10 (b) $3\frac{dy}{dx} + 4y^2 + 8xy\frac{dy}{dx} - 15x^2 = 0$

$\frac{dy}{dx}(3 + 8xy) = 15x^2 - 4y^2$

$\frac{dy}{dx} = \frac{15x^2 - 4y^2}{3 + 8xy}$

11 (a) $y = \frac{\sqrt{(x^2 - 1)}}{x} \quad \Rightarrow \quad x^2y^2 = x^2 - 1$

$x^2(1 - y^2) = 1 \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{(1 - y^2)}}$

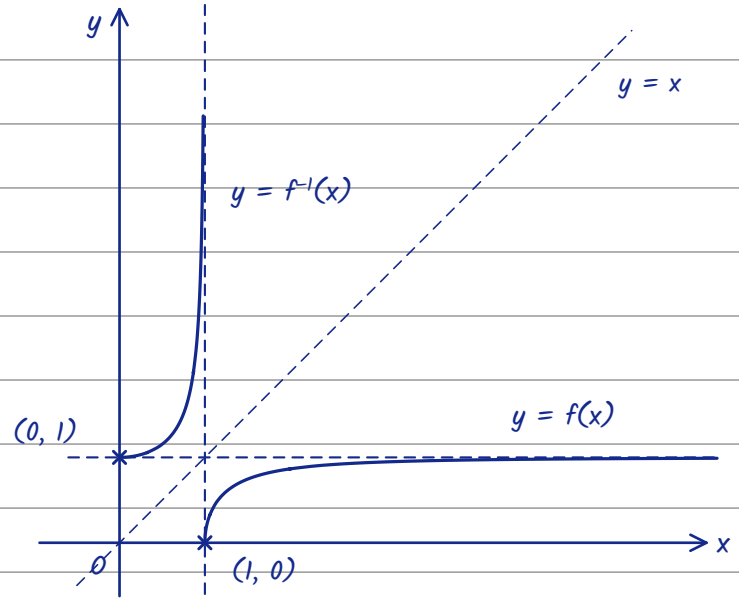
Question number

11 (a) cont.

Take the positive root since $x \geqslant 1$: $f^{-1}(x) = \dfrac{1}{\sqrt{(1 - x^2)}}$

$f(1) = 0$ and $f(x) \to 1$ as $x \to \infty$, so the range of f is $0 \leqslant f(x) < 1$

Domain of f^{-1}: $0 \leqslant x < 1$



11 (b) The range of f, [0, 1), is not contained in the domain of f ($x \geqslant 1$),

e.g. $f(2) = \sqrt{3}/2 < 1$, so $f(f(2))$ is not defined. Hence $ff(x)$ cannot be formed.

12 (a) Area of sector $OAB = \frac{1}{2}r^2\theta$, area of triangle $OAB = \frac{1}{2}r^2\sin\theta$

Area of minor segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

Ratio 1 : 2, so the minor segment is $\frac{1}{3}$ of the circle:

$3(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta) = \pi r^2$

$\theta - \sin\theta = \dfrac{2\pi}{3} \quad \Rightarrow \quad \sin\theta = \theta - \dfrac{2\pi}{3}$ as required

12 (b)(i) Let $f(\theta) = \theta - \sin\theta - \dfrac{2\pi}{3}$

$f(2.6) = -0.009896\ldots < 0$, $f(2.7) = 0.178225\ldots > 0$

Change of sign (f is continuous), so $2.6 < \theta < 2.7$

Question number

12 (b)(ii) $f(\theta) = 1 - \cos\theta$

$\theta_{n+1} = \theta_n - \dfrac{\theta_n - \sin\theta_n - 2\pi/3}{1 - \cos\theta_n}$

$\theta_0 = 2.6$

$\theta_1 = 2.6053296...$

$\theta_2 = 2.6053256...$

$\theta = 2.605$ (correct to 3 d.p.)

13 (a) $\dfrac{dA}{dt} = kA$ (k a positive constant)

13 (b) $\int \dfrac{dA}{A} = \int k\,dt \quad \Rightarrow \quad \ln A = kt + C$

$t = 0, A = 0.2$: $C = \ln 0.2$, so $\ln\dfrac{A}{0.2} = kt$

$t = 1, A = 1.48$: $k = \ln\dfrac{1.48}{0.2} = \ln 7.4 = 2.00148...$

$A = 0.2e^{kt} = 0.2(e^k)^t$ with $e^k = 7.4$

$A = 0.2(7.4)^t$ ($p = 0.2$, $q = 7.4$)

14 (a) $\int (e^{2x} + 6\sin 3x)\,dx = \dfrac{1}{2}e^{2x} - 2\cos 3x + C$

14 (b) $\int 7(x^2 + \sin x)^6(2x + \cos x)\,dx = (x^2 + \sin x)^7 + C$

14 (c) By parts: $u = \ln x$, $\dfrac{dv}{dx} = x^{-2} \quad \Rightarrow \quad \dfrac{du}{dx} = \dfrac{1}{x}$, $v = -x^{-1}$

$\int \dfrac{1}{x^2}\ln x\,dx = -\dfrac{1}{x}\ln x - \int\left(-\dfrac{1}{x}\right) \times \dfrac{1}{x}\,dx$

Question number

14 (c) cont.

$= -\frac{1}{x} \ln x + \int x^{-2}\, dx$

$= -\frac{1}{x} \ln x - \frac{1}{x} + C$

14 (d)

$u = 2\cos x + 1 \quad \Rightarrow \quad du = -2\sin x\, dx$

$x = 0: \ u = 3; \qquad x = \frac{\pi}{3} : \ u = 2\cos\frac{\pi}{3} + 1 = 2$

$I = \int_3^2 -\frac{1}{2u^2}\, du = \frac{1}{2}\int_2^3 u^{-2}\, du$

$= \frac{1}{2}\left[-\frac{1}{u}\right]_2^3$

$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3}\right)$

$= \frac{1}{12}$

15

Assume that $\sqrt{6}$ is rational. Then $\sqrt{6} = a/b$ where a and b are integers with no common factor (other than 1).

Squaring: $6 = a^2/b^2 \quad \Rightarrow \quad a^2 = 6b^2$

So a^2 is even, and therefore a is even. Let $a = 2k$:

$6b^2 = (2k)^2 = 4k^2 \quad \Rightarrow \quad 3b^2 = 2k^2$

So $3b^2$ is even, so b^2 is even, and therefore b is even.

So a and b have a common factor 2. This is a contradiction.

Hence $\sqrt{6}$ is irrational.