GCE A LEVEL – NEW

1300U40-1

S18-1300U40-1

MATHEMATICS – A2 unit 4
APPLIED MATHEMATICS B

WEDNESDAY, 13 JUNE 2018 – MORNING

1 hour 45 minutes

1300U401
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Write only within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. **0 1** .

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

Section A: Statistics

0 1 An architect bids for two construction projects. He estimates the probability of winning bid *A* is 0·6, the probability of winning bid *B* is 0·5 and the probability of winning both is 0·2.

a) Show that the probability that he does not win either bid is 0·1. [2]

b) Find the probability that he wins exactly one bid. [2]

c) Given that he does not win bid *A*, find the probability that he wins bid *B*. [3]

0 2 **a)** Marie is an athlete who competes in the high jump. In a certain competition she is allowed two attempts to clear each height, but if she is successful with the first attempt she does not jump again at this height. The probability that she is successful with her first jump at a height of 1·7 m is p. The probability that she is successful with her second jump is also p. The probability that she clears 1·7 m is 0·64. Find the value of p. [4]

b) The following table shows the numbers of male and female athletes competing for Wales in track and field events at a competition.

	Track	**Field**
Male	13	9
Female	7	4

Two athletes are chosen at random to participate in a drugs test. Given that the first athlete is male, find the probability that both are field athletes. [3]

0 3 Antonio arrives at a train station at a random point in time. The trains to his desired destination are scheduled to depart at 12-minute intervals.

a) Assume that Antonio gets on the next train.

i) Suggest an appropriate distribution to model his waiting time and give the parameters.

ii) State the mean and the variance of this distribution.

iii) State an assumption you have made in suggesting this distribution. [4]

b) Now assume that the probability that Antonio misses the next available train because he is distracted by his smartphone is 0·12. If he misses the next available train, he is sure to get on the one after that.

i) Find the probability that he waits between 9 and 19 minutes.

ii) Given that he waits between 9 and 19 minutes, find the probability that he gets on the first train. [6]

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0 4 Arwyn collects data about household expenditure on food. He records the weekly expenditure on food for 80 randomly selected households from across Wales.

Cost, x (£)	$x < 40$	$40 \leqslant x < 50$	$50 \leqslant x < 60$	$60 \leqslant x < 70$	$70 \leqslant x < 80$	$80 \leqslant x < 90$	$x \geqslant 90$
Number of households	5	11	16	18	15	9	6

a) Explain why a normal distribution may be an appropriate model for the weekly expenditure on food for this sample. [1]

Arwyn uses the distribution N(64, 15^2) to model expenditure on food.

b) Find the number of households in the sample that this model would predict to have weekly food expenditure in the range

i) $60 \leqslant x < 70$,

ii) $x \geqslant 90$. [4]

c) Use your answers to part *(b)*

i) to comment on the suitability of this model,

ii) to explain how Arwyn could improve the model by changing one of its parameters. [2]

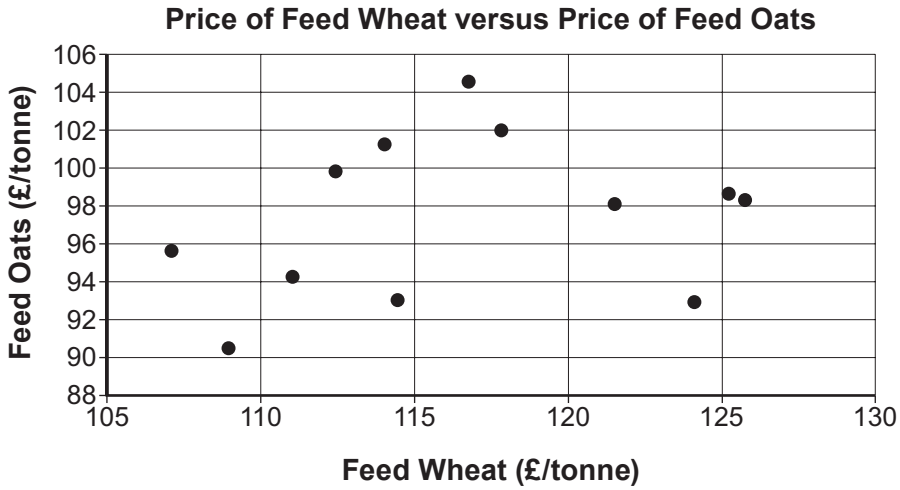
d) Arwyn's friend Colleen wishes to use the improved model to predict household expenditure on food in Northern Ireland. Comment on this plan. [1]

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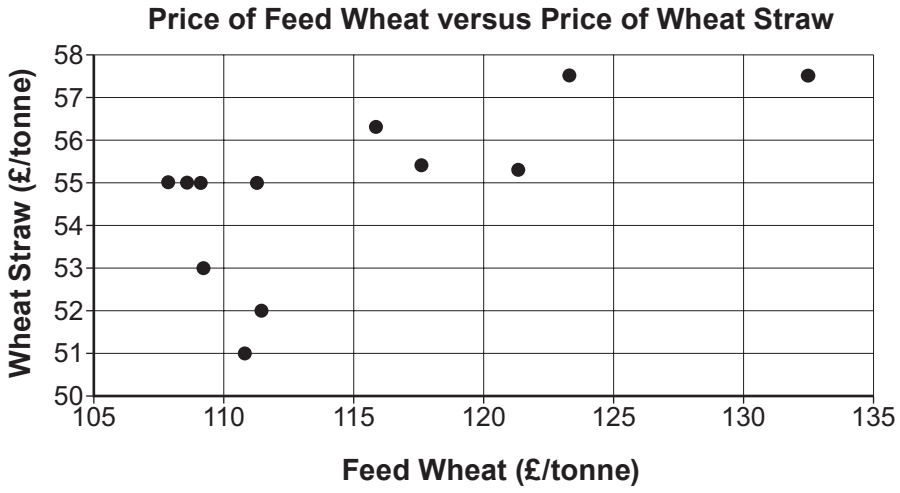
0 5 Rebecca is a farmer who is monitoring prices for products to use on her farm. She records the prices of two products made from **different** grains, wheat and oats, at random points in time, to investigate whether there is any correlation.



The product moment correlation coefficient for the data is 0·244. There are 12 data points, and the *p*-value is 0·4447.

a) Comment on the correlation between the prices of Feed Wheat and Feed Oats. [2]

Rebecca also records the prices of two wheat products at random points in time, to investigate whether there is any correlation.



The product moment correlation coefficient for the data is 0·653. There are 12 data points.

b) Stating your hypotheses clearly, test at the 5% level of significance whether there is any evidence of correlation between the prices of these two products. [5]

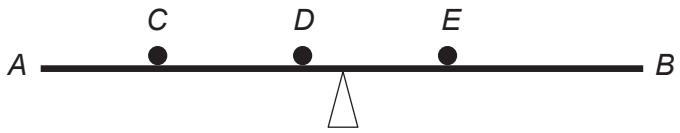
c) Without referring to the positioning of the points on the graphs, suggest why the product moment correlation coefficient is higher for the second set of data. [1]

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Section B: Differential Equations and Mechanics

0 6 The diagram shows a uniform plank *AB* of length 4 m supported in horizontal equilibrium by means of a central pivot. On the plank there are three objects of masses 8 kg, 2 kg and 15 kg placed in positions *C*, *D* and *E* respectively. The distance *AC* is 0·6 m and the distance *AE* is 2·8 m.



Find the distance *AD*. [4]

0 7 An object of mass 0·5 kg is thrown vertically upwards with initial speed 24 ms^{-1}. The velocity of the object at time t seconds is v ms^{-1}. During the upward motion, the object experiences a resistance to motion R N, where R is proportional to v. When the velocity of the object is 0·2 ms^{-1} the resistance to motion is 0·08 N.

a) Show that the upward motion of the object satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -9{\cdot}8 - 0{\cdot}8\,v.$$ [3]

b) Find an expression for v at time t. [6]

c) Determine the value of t when the object is at the highest point of the motion. [2]

0 8 An object of mass 60 kg is on a rough plane inclined at an angle of 20° to the horizontal. The coefficient of friction between the object and the plane is 0·3. Initially, the object is held at rest. A force which is parallel to the plane and of magnitude T N is applied to the object in an upward direction along the line of greatest slope. The object is then released.

a) Given that T = 15, calculate the acceleration of the object down the plane. [6]

b) Given that T = 350, determine whether or not the object moves up the plane. Give a reason for your answer. [3]

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0 9 Points *A* and *B* lie on horizontal ground. At time t = 0 seconds, an object *P* is projected from *A* towards *B* such that *AB* is the range of *P*. The speed of projection is 24·5 ms^{-1} in a direction which is 30° above the horizontal.

a) Calculate the range *AB* of the object *P*. [5]

At time t = 1 second, another object *Q* is projected from *B* towards *A* with the same speed of projection 24·5 ms^{-1} and in a direction which is also 30° above the horizontal.

b) Determine the height above the ground at which *P* and *Q* collide. [5]

1 0 A particle of mass 2 kg moves under the action of a constant force $\mathbf{F}$ N, where $\mathbf{F}$ is given by

$$\mathbf{F} = -3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

a) Find the magnitude of the acceleration of the particle. [3]

b) Given that at time t = 0 seconds, the position vector of the particle is $2\mathbf{i} - 7\mathbf{j} + 9\mathbf{k}$ and it is moving with velocity $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find the position vector of the particle when t = 2 seconds. [3]

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Surname	StemUp Model Solution
Answer booklet	Write the question number in the left-hand margin at the start of each answer.

Question number

1 (a)

$P(\text{neither}) = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.6 + 0.5 - 0.2)$

$= 1 - 0.9 = 0.1$ (as required)

1 (b)

$P(A \text{ only}) = 0.6 - 0.2 = 0.4$, $P(B \text{ only}) = 0.5 - 0.2 = 0.3$

$P(\text{exactly one}) = 0.4 + 0.3 = 0.7$

1 (c)

$P(\text{not } A) = 1 - 0.6 = 0.4$

$P(B \cap \text{not } A) = P(B \text{ only}) = 0.3$

$P(B \mid \text{not } A) = 0.3 \div 0.4 = 3/4$

2 (a)

$P(\text{clears 1.7 m}) = p + (1 - p)p = 0.64$

$p + p - p^2 = 0.64$

$p^2 - 2p + 0.64 = 0$

$(p - 0.4)(p - 1.6) = 0$

$p = 0.4$ or $p = 1.6$, but $p \leq 1$ so reject 1.6

$p = 0.4$

2 (b)

Total = 33 athletes, 22 male (9 field), 13 field altogether.

Given the 1st is male: P(1st is a field athlete) = 9/22

Then 32 athletes remain, of whom 12 are field: P(2nd field) = 12/32

P(both field | 1st male) = 9/22 × 12/32

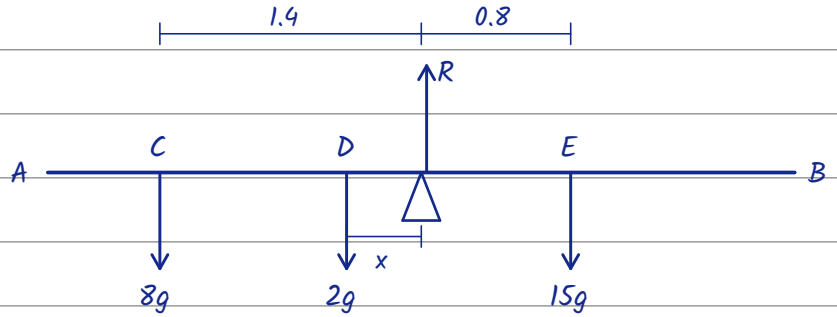
= 27/176 = 0.1534...

Question number	
3 (a)(i)	Waiting time X ~ (continuous) uniform distribution, with parameters (0, 12), i.e. $X \sim U(0, 12)$
3 (a)(ii)	Mean = $(0 + 12) \div 2 = 6$ minutes Variance = $(12 - 0)^2 \div 12 = 12$ (minutes^2)
3 (a)(iii)	Assume the trains run exactly on time (they are never late or early).
3 (b)(i)	Gets the next train (prob 0.88): waits between 9 and 12 minutes $P(9 < X < 12) = 0.88 \times 3/12 = 0.88 \times 1/4 = 0.22$ Misses it (prob 0.12): waits between 12 and 24, needs $12 < X < 19$ $P(12 < X < 19) = 0.12 \times 7/12 = 0.07$ $P(9 < X < 19) = 0.22 + 0.07 = 0.29$
3 (b)(ii)	P(1st train | waits between 9 and 19 mins) $= 0.22 \div 0.29$ $= 22/29 = 0.7586...$
4 (a)	The data are approximately symmetrical: the frequencies are highest in the middle classes and tail off towards both ends.
4 (b)(i)	$X \sim N(64, 15^2)$ $P(60 \leq X < 70) = P(-0.2667 < Z < 0.4) = 0.26055...$ Predicted number = $80 \times 0.26055... = 20.84... \approx 21$ households
4 (b)(ii)	$P(X \geq 90) = P(Z \geq 1.7333) = 0.041518...$ Predicted number = $80 \times 0.041518... = 3.32... \approx 3$ households

Question number	
4 (c)(i)	The model predicts 21 but only 18 were observed in $60 \leq x < 70$, and it predicts 3 but 6 were observed for $x \geq 90$, so it may not be the best model: it could be improved.
4 (c)(ii)	Increase the standard deviation (variance). This flattens the curve, lowering the middle and lifting the tails.
4 (d)	It may not be suitable, since weekly household expenditure on food in Northern Ireland may have a different distribution from that in Wales (the model was based on a sample from Wales only).
5 (a)	The p-value 0.4447 > 0.05, so the correlation ($r = 0.244$) is not significant. The price of Feed Wheat does not seem to be correlated with the price of Feed Oats.
5 (b)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ (ρ = population pmcc) Test statistic $r = 0.653$, $n = 12$ Critical value (5%, two-tailed, $n = 12$) = 0.5760 Since $0.653 > 0.5760$, reject H_0. There is sufficient evidence to suggest there is a correlation between the prices of Feed Wheat and Wheat Straw.
5 (c)	The second graph compares two wheat products (from the same grain), whereas the first compares products made from different grains, so their prices are more likely to be related.

Question number

6



Let D be x m from the centre (pivot) of the plank, towards A.

Pivot at 2 m from A, so C is 2 − 0.6 = 1.4 m and E is 2.8 − 2 = 0.8 m from it.

Moments about the pivot (centre):

$8g \times 1.4 + 2g \times x = 15g \times 0.8$

$11.2 + 2x = 12$

$2x = 0.8 \Rightarrow x = 0.4$

$AD = 2 - 0.4 = 1.6$ m

7 (a)

$R = kv$: $0.08 = 0.2k \Rightarrow k = 0.4$, so $R = 0.4v$

N2L upwards (positive direction up), $a = dv/dt$:

$-0.5g - R = 0.5a$

$-0.5 \times 9.8 - 0.4v = 0.5\, dv/dt$

$dv/dt = -9.8 - 0.8v$ (as required)

7 (b)

$\int 1/(9.8 + 0.8v)\, dv = -\int 1\, dt$

$(1/0.8) \ln|9.8 + 0.8v| = -t + C$

When $t = 0$, $v = 24$: $C = (1/0.8) \ln|9.8 + 0.8 \times 24| = (1/0.8) \ln 29$

$(1/0.8) \ln|9.8 + 0.8v| = -t + (1/0.8) \ln 29$

$-0.8t = \ln((9.8 + 0.8v) \div 29)$

$29e^{-0.8t} = 9.8 + 0.8v$

$0.8v = 29e^{-0.8t} - 9.8$

$v = 36.25e^{-0.8t} - 12.25$

Question number

7 (c)

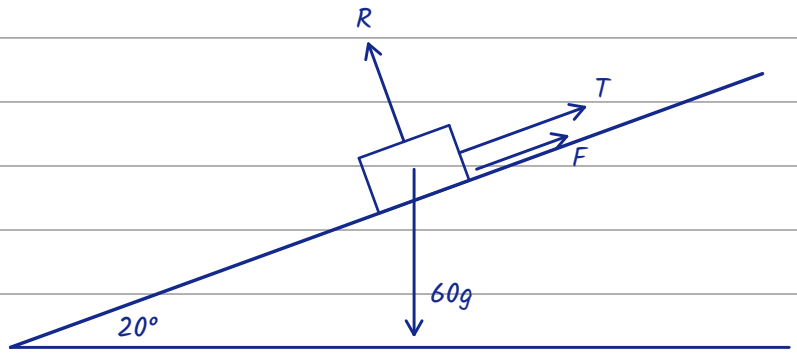
At the highest point, $v = 0$:

$36.25e^{-0.8t} = 12.25 \quad \Rightarrow \quad e^{-0.8t} = 9.8/29$

$t = 1.25(\ln 29 - \ln 9.8) = 1.3562...$

$t = 1.356$ s

8 (a)



Resolve perpendicular to the plane: $R = 60g \cos 20° = 552.54...$

Object slides down, so friction acts up the plane at its limiting value:

$F = \mu R = 0.3 \times 60g \cos 20° = 165.761...$

N2L down the plane (positive direction down the plane):

$60g \sin 20° - F - 15 = 60a$

$201.108... - 165.761... - 15 = 60a$

$60a = 20.347... \quad \Rightarrow \quad a = 0.33911...$

$a = 0.3391$ ms^{-2} (down the plane)

8 (b)

Resultant force up the plane (before friction)

$= 350 - 60g \sin 20° = 350 - 201.108... = 148.892...$ N

Limiting friction $= 0.3 \times 60g \cos 20° = 165.761...$ N

Since $148.892... < 165.761...$, friction can balance this force,

so the object does not move up the plane (it stays at rest).

Question number	
9 (a)	Initial horizontal velocity = $24.5 \cos 30° = 12.25\sqrt{3}$ ms^{-1}
	Initial vertical velocity = $24.5 \sin 30° = 12.25$ ms^{-1}
	Vertically (up +ve): $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 12.25$, $a = -9.8$
	$0 = 12.25t - 4.9t^2 \Rightarrow t(12.25 - 4.9t) = 0 \Rightarrow t = 2.5$ s
	Range AB = $12.25\sqrt{3} \times 2.5 = 30.625\sqrt{3}$
	AB = 53.04 m (53.044...)
9 (b)	Horizontal distance travelled by P: $D_P = 12.25\sqrt{3}\, t$
	Horizontal distance travelled by Q: $D_Q = 12.25\sqrt{3}\,(t - 1)$
	They collide when $D_P + D_Q = AB = 30.625\sqrt{3}$
	$12.25\sqrt{3}\, t + 12.25\sqrt{3}\,(t - 1) = 30.625\sqrt{3}$
	$12.25(2t - 1) = 30.625 \Rightarrow 2t - 1 = 2.5 \Rightarrow t = 1.75$ s
	Height of P at $t = 1.75$: $s = ut + \frac{1}{2}at^2$, $u = 12.25$, $a = -9.8$
	$s = 12.25 \times 1.75 + \frac{1}{2} \times (-9.8) \times 1.75^2$
	$s = 21.4375 - 15.00625 = 6.43125$
	Height = 6.43 m
10 (a)	$F = ma$: $-3i + 4j - 5k = 2a$
	$a = -(3/2)i + 2j - (5/2)k$
	$\lvert a \rvert = \sqrt{((-3/2)^2 + 2^2 + (-5/2)^2)} = \sqrt{(9/4 + 4 + 25/4)} = \sqrt{12.5}$
	$\lvert a \rvert = 5\sqrt{2} / 2 = 3.54$ ms^{-2} (3.5355...)
10 (b)	Use $r = ut + \frac{1}{2}at^2 + r_0$
	with $u = 3i - 2j + k$, $a = -(3/2)i + 2j - (5/2)k$, $t = 2$
	$r = (3i - 2j + k) \times 2 + \frac{1}{2}(-(3/2)i + 2j - (5/2)k) \times 2^2 + r_0$
	$r = (6i - 4j + 2k) + (-3i + 4j - 5k) + (2i - 7j + 9k)$
	Position vector = $5i - 7j + 6k$