Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

S25-1305U60-1

1305U60-1

FRIDAY, 13 JUNE 2025 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 6
FURTHER MECHANICS B

1 hour 45 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	16	
2	9	
3	11	
4	17	
5	12	
6	15	
Total	80	

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01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A car, of mass 2250 kg, is moving along a straight horizontal road. The engine of the car is working at a constant rate of 96 kW. The car experiences a variable resistance to motion of kv^2 N, where k is a constant and v ms^{-1} is the speed of the car after it has travelled x metres. You may assume that the car has a maximum speed of 40 ms^{-1}.

(a) (i) Show that $k = 1{\cdot}5$. [3]

(ii) Show that v satisfies the differential equation

$$1500v^2 \frac{\mathrm{d}v}{\mathrm{d}x} = 64\,000 - v^3. \quad [3]$$

(i) Driving force $F = P/v$

At max speed, $a = 0$: $\frac{96000}{40} - k(40)^2 = 0$

$2400 = 1600k$

$k = \frac{2400}{1600} = 1.5$ as required

(ii) N2L along the road:

$\frac{96000}{v} - 1.5v^2 = 2250a$

Using $a = v\frac{dv}{dx}$:

$2250v\frac{dv}{dx} = \frac{96000}{v} - 1.5v^2$

Multiply by v: $2250v^2\frac{dv}{dx} = 96000 - 1.5v^3$

Divide by 1.5: $1500v^2\frac{dv}{dx} = 64000 - v^3$ as required

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(b) Find an expression for v^3 in terms of x, given that $v = 10$ when $x = 0$. [7]

Separating the variables:

$\int \frac{1500v^2}{64000 - v^3}\, dv = \int 1\, dx$

$-500 \ln|64000 - v^3| = x + C$

When $x = 0$, $v = 10$:

$-500 \ln|64000 - 1000| = 0 + C$

$C = -500 \ln 63000 \quad (= -5525.445...)$

$x = 500 \ln 63000 - 500 \ln|64000 - v^3|$

$\frac{x}{500} = \ln \frac{63000}{64000 - v^3}$

$\frac{63000}{64000 - v^3} = e^{x/500}$

$64000 - v^3 = 63000e^{-x/500}$

$v^3 = 64000 - 63000e^{-x/500}$

$= 1000(64 - 63e^{-x/500})$

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(c) Find the speed of the car when it has travelled a distance of 250 m. [2]

When $x = 250$:

$v^3 = 64000 - 63000e^{-250/500}$

$= 64000 - 63000e^{-0.5}$

$= 25788.568...$

$v = \sqrt[3]{25788.568...} = 29.5444...$

Speed $= 29.5\ ms^{-1}$ (3 s.f.)

(d) Explain why, according to this model, the car will not actually reach the maximum speed of 40 ms^{-1}. [1]

When $v = 40$, $v^3 = 64000$ so $64000 - v^3 = 0$, which would need $e^{-x/500} = 0$ (division by zero in the expression for x). This never happens for finite x, so 40 ms^{-1} is only a limiting value as $x \to \infty$.

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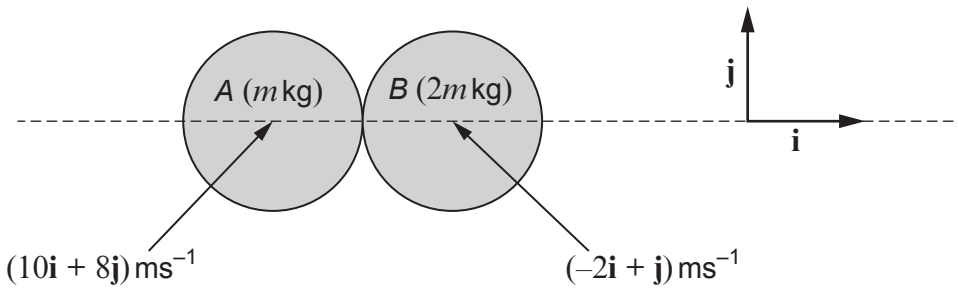
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2. Two smooth spheres *A* and *B*, of equal radii and of masses $m$kg and $2m$kg respectively, are moving on a smooth horizontal surface when they collide. Immediately before the collision, sphere *A* has velocity $(10\mathbf{i}+8\mathbf{j})\text{ms}^{-1}$ and sphere *B* has velocity $(-2\mathbf{i}+\mathbf{j})\text{ms}^{-1}$.

The coefficient of restitution between the spheres is $\frac{3}{4}$.

At the instant of collision, the line joining their centres is parallel to the vector $\mathbf{i}$, as shown in the diagram below.



(a) Find the velocity of *A* and the velocity of *B* immediately after the collision. [7]

Perpendicular to the line of centres (j) the components are unchanged:

A keeps 8j, B keeps 1j. Let the i-components after be v_A, v_B.

Conservation of momentum along the line of centres (i):

$m(10) + 2m(-2) = mv_A + 2mv_B$

$v_A + 2v_B = 6$ ...(1)

Restitution along the line of centres:

$v_B - v_A = \frac{3}{4}(10 - (-2))$

$v_B - v_A = 9$ ...(2)

(1) + (2): $3v_B = 15 \Rightarrow v_B = 5$

$v_A = 5 - 9 = -4$

Velocity of A after = $(-4i + 8j)$ ms^{-1}

Velocity of B after = $(5i + j)$ ms^{-1}

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(b) Find, in terms of m, the impulse exerted on sphere *B* by sphere *A*, stating your units clearly. [2]

Impulse on B = change in momentum of B

$I = 2m[(5i + j) - (-2i + j)]$

$= 2m(7i)$

$= 14m\ i\ \ Ns$

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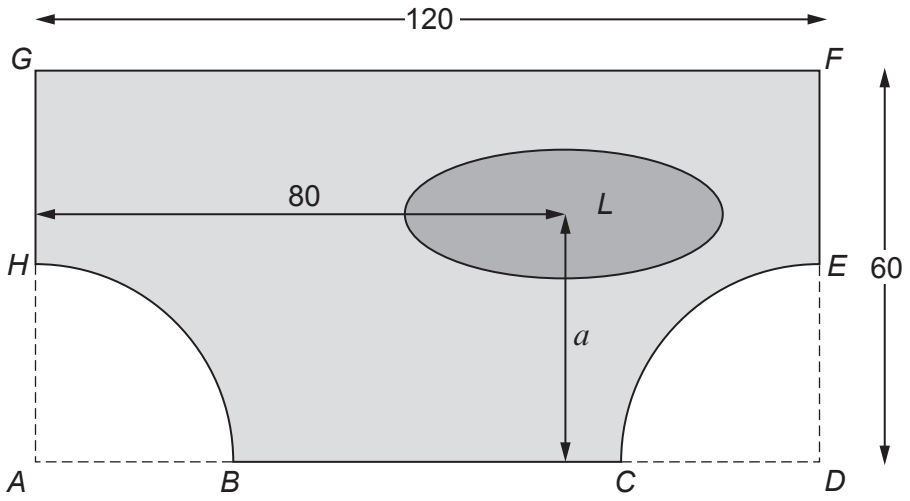
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3. The diagram below shows a sign which is made from uniform laminae. Quadrants *ABH* and *CDE*, centred at *A* and *D* respectively, are **removed** from a rectangular lamina *ADFG* which has density ρ kg/cm^2. Both quadrants have radius 30 cm.

Another uniform lamina, *L*, of surface area 225π cm^2 and density 2ρ kg/cm^2, is attached to *BCEFGH* so that its centre of mass is 80 cm from *AG* and a cm from *AD*.

The dimensions, in cm, are as shown in the diagram.



(a) (i) Calculate the distance of the centre of mass of the sign from *AG*.

(ii) Determine the value of a, given that the distance of the centre of mass of the sign from *AD* is 35 cm. [10]

Shape	Mass	Dist. from AG	Dist. from AD
ADFG	$60\times120 = 7200\rho$	60	30
Quad ABH	$-\frac{1}{4}\pi(30)^2\rho = -225\pi\rho$	$40/\pi$	$40/\pi$
Quad DCE	$-225\pi\rho$	$120 - 40/\pi$	$40/\pi$
L	$2\rho\times225\pi = 450\pi\rho$	80	a
Sign	7200ρ	$\bar{x}$	35

$(4r/3\pi = 4(30)/3\pi = 40/\pi$ for each quadrant)

Total mass $= 7200\rho - 450\pi\rho + 450\pi\rho = 7200\rho$

(i) Moments about AG:

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$7200\bar{x} = 7200(60) - 225\pi(40/\pi) - 225\pi(120 - 40/\pi) + 450\pi(80)$

$7200\bar{x} = 432000 - 9000 - 27000\pi + 9000 + 36000\pi$

$7200\bar{x} = 432000 + 9000\pi \quad (= 460274.33...)$

$\bar{x} = 60 + \frac{5\pi}{4} = 63.9269...$

Distance from AG = 63.9 cm (3 s.f.)

(ii) Moments about AD:

$7200(35) = 7200(30) - 225\pi(40/\pi) - 225\pi(40/\pi) + 450\pi a$

$252000 = 216000 - 9000 - 9000 + 450\pi a$

$450\pi a = 54000$

$a = \frac{120}{\pi} = 38.197...$

a = 38.2 cm (3 s.f.)

(b) When the sign is freely suspended from a point *P* on *BC*, it hangs in equilibrium with *BC* horizontal. Write down the length *BP*. [1]

P is vertically below the centre of mass, so $BP = \bar{x} - 30$

$BP = 30 + \frac{5\pi}{4} = 33.9$ cm (3 s.f.)

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4. A particle *P* is moving along the x-axis with Simple Harmonic Motion with the origin *O* as its centre. Its displacement x metres from *O*, after t seconds, satisfies the following equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x,$$

where ω is a positive constant.

(a) The particle *P* is at *O* when $t = 0$ and it is instantaneously at rest when $x = a$.

(i) Use the above equation to show that v, the speed of *P* at time t, satisfies

$$v^2 = \omega^2\left(a^2 - x^2\right). \qquad [4]$$

(ii) Hence, or otherwise, prove that

$$x = a\sin\left(\omega t\right). \qquad [5]$$

(i) Using $\frac{d^2x}{dt^2} = v\frac{dv}{dx}$: $v\frac{dv}{dx} = -\omega^2 x$

$\int v\,dv = -\omega^2 \int x\,dx$

$\frac{1}{2}v^2 = -\frac{1}{2}\omega^2x^2 + C$

When $x = a$, $v = 0$: $0 = -\frac{1}{2}\omega^2a^2 + C \Rightarrow C = \frac{1}{2}\omega^2a^2$

$\frac{1}{2}v^2 = \frac{1}{2}\omega^2a^2 - \frac{1}{2}\omega^2x^2$

$v^2 = \omega^2(a^2 - x^2)$ as required

(ii) P starts at O moving in the positive direction, so

$v = \frac{dx}{dt} = \omega\sqrt{(a^2 - x^2)}$

$\int \frac{1}{\sqrt{(a^2 - x^2)}}\,dx = \int \omega\,dt$

$\sin^{-1}\frac{x}{a} = \omega t + C$

When $t = 0$, $x = 0$: $\sin^{-1}0 = C \Rightarrow C = 0$

$\frac{x}{a} = \sin(\omega t)$

$\therefore\ x = a\sin(\omega t)$ as required

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(b) When P is 3 m from O, its speed is 8 ms^{-1}, and when P is 4 m from O, its speed is 6 ms^{-1}.

(i) Show that the amplitude of the motion is 5 m and find the period of the motion. [6]

(ii) Find the distance of P from O when $t = 1{\cdot}8$. [2]

(i) Using $v^2 = \omega^2(a^2 - x^2)$:

$x = 3, v = 8$: $64 = \omega^2(a^2 - 9)$ ...(1)

$x = 4, v = 6$: $36 = \omega^2(a^2 - 16)$ ...(2)

(1) ÷ (2): $\frac{64}{36} = \frac{a^2 - 9}{a^2 - 16}$

$16(a^2 - 16) = 9(a^2 - 9)$

$16a^2 - 256 = 9a^2 - 81$

$7a^2 = 175 \Rightarrow a^2 = 25$

$a = 5$, so the amplitude is 5 m as required

From (1): $\omega^2 = \frac{64}{25 - 9} = 4 \Rightarrow \omega = 2$

Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s (= 3.14 s)

(ii) $x = 5\sin(2t)$

When $t = 1.8$: $x = 5\sin(3.6)$ (radians)

$= -2.2126...$

Distance of P from O = 2.21 m (3 s.f.)

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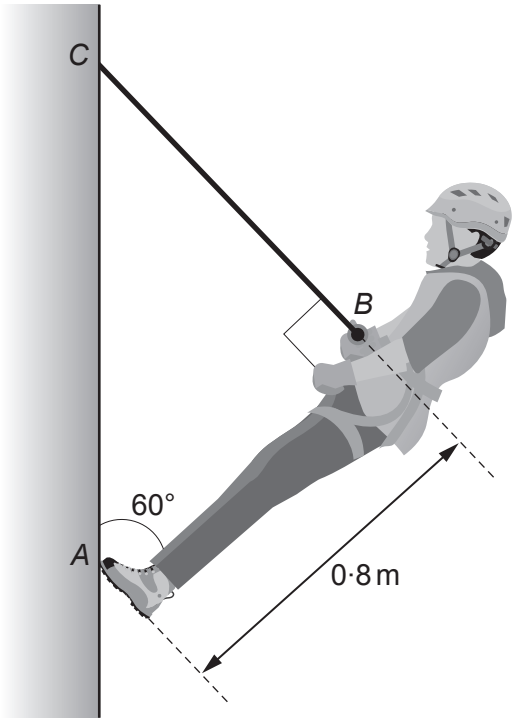
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5. A person of mass 72 kg is climbing down a cliff face using a rope that is fixed at a point C. As shown in the diagram below, the person's feet make contact with the cliff at A and the rope is attached to the person at B, where $AB = 0{\cdot}8$ m. The person is being held at rest by means of the rope BC, with AB making an angle of 60° with the vertical and $A\hat{B}C = 90°$.



You may use the following modelling assumptions.

- The person can be modelled as a rod with centre of mass at B.
- The rope is light and inextensible.
- The cliff face is vertical.
- The cliff face is rough and the coefficient of friction between the person's feet and the cliff is denoted by μ.

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(a) Find the tension in the rope *BC*. [3]

Moments about A (B is 0.8 sin 60° horizontally from A;

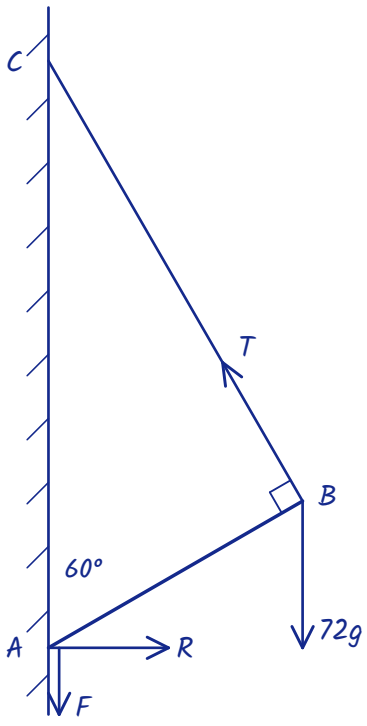
T is perpendicular to AB, so its moment arm is AB = 0.8):

$T \times 0.8 = 72g \times 0.8 \sin 60°$

$T = 72 \times 9.8 \times (\sqrt{3}/2) = 36\sqrt{3}\, g$

$T = 611.0675...$

Tension = 611 N (3 s.f.)



BC makes 30° with the vertical
(since AB makes 60° and ABC = 90°)

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(b) Determine the least value of μ such that the person can remain in the position shown in the diagram on page 14. [8]

From the diagram, T acts along BC at 30° to the vertical.

F acts vertically downwards at A, R horizontally away from the cliff.

Resolve horizontally: $R = T \sin 30°$

$R = 36\sqrt{3}\,g \times \frac{1}{2} = 18\sqrt{3}\,g = 305.5337...$ N

Resolve vertically: $F + T\cos 30° = 72g$

$F = 72g - 36\sqrt{3}\,g \times (\sqrt{3}/2)$

$F = 72g - 54g = 18g = 176.4$ N

Person remains at rest so $F \leq \mu R$. Least μ when friction is limiting:

$$\mu = \frac{F}{R} = \frac{18g}{18\sqrt{3}\,g} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\mu = 0.57735...$

Least value of $\mu = \sqrt{3}/3 = 0.577$ (3 s.f.)

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(c) State one reason as to why your answer to part (b) may be unreliable in reality. [1]

The person is not really a rigid rod with centre of mass at B (they can bend), so the modelling assumptions may not hold in reality.

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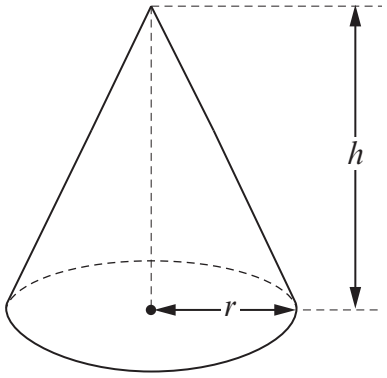
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6. (a) The diagram below shows a uniform solid right circular cone of height h and base radius r.

You are given that the volume of the cone is $\frac{1}{3}\pi r^2 h$.



Show, by integration, that the centre of mass of the cone is at a height of $\frac{1}{4}h$ above its base on its axis of symmetry. [4]

Place the vertex at the origin with the axis along the x-axis.

The slant edge is the line $y = \frac{r}{h}x$, $0 \leq x \leq h$ (rotated about the x-axis).

$V\bar{x} = \pi\int_0^h xy^2\,dx = \pi\int_0^h x\frac{r^2}{h^2}x^2\,dx = \frac{\pi r^2}{h^2}\int_0^h x^3\,dx$

$V\bar{x} = \frac{\pi r^2}{h^2}\left[\frac{x^4}{4}\right]_0^h = \frac{\pi r^2}{h^2} \times \frac{h^4}{4} = \frac{1}{4}\pi r^2 h^2$

Using $V = \frac{1}{3}\pi r^2 h$: $\bar{x} = \frac{\frac{1}{4}\pi r^2 h^2}{\frac{1}{3}\pi r^2 h} = \frac{3}{4}h$ from the vertex

Height above the base $= h - \frac{3}{4}h = \frac{1}{4}h$ as required

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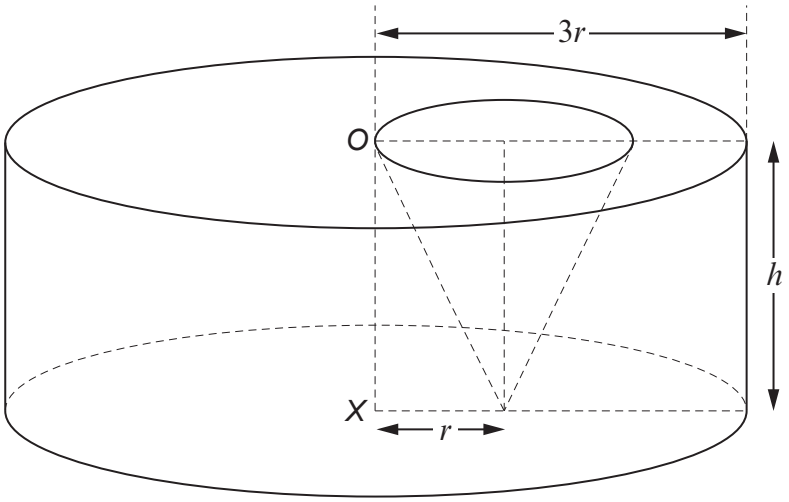
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(b) The diagram below shows a solid body which consists of a uniform right circular cylinder with an inverted right circular cone removed.

The cylinder has height h, radius $3r$ and the centre of its top plane face is O. The centre of its bottom plane face is X.

The cone has height h, radius r and the point O lies on the circumference of the base of the cone. The vertex of the cone lies on the base of the cylinder.



(i) Show that the vertical distance of the centre of mass of the solid from its top plane face is $\frac{53}{104}h$.

(ii) Find the horizontal distance of the centre of mass of the solid from OX.

(iii) The solid is suspended by a string which is attached to O. Given that OX is inclined at an angle of α to the vertical, find $\tan\alpha$ in terms of h and r. [11]

Shape	Volume (× ρ)	Dist. from OX	Dist. from top
Cylinder	$\pi(3r)^2h = 9\pi r^2h$	0	½h
Cone	$-\frac{1}{3}\pi r^2h$	r	¼h
Solid	$9\pi r^2h - \frac{1}{3}\pi r^2h = 26/3\ \pi r^2h$	$\bar{x}$	$\bar{y}$

(The cone's base is in the top face, so its c.o.m. is ¼h below the top.)

(i) Moments about the top plane face:

$\frac{26}{3}\pi r^2h \times \bar{y} = 9\pi r^2h \times \frac{1}{2}h - \frac{1}{3}\pi r^2h \times \frac{1}{4}h$

$\frac{26}{3}\bar{y} = \frac{9}{2}h - \frac{1}{12}h = \frac{53}{12}h$

$\bar{y} = \frac{53}{12}h \times \frac{3}{26} = \frac{53}{104}h$ as required

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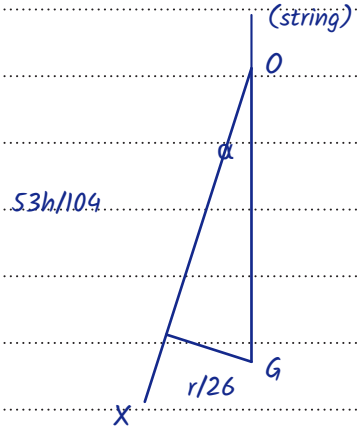
(ii) Moments about OX (taking the cone side as positive):

$\frac{26}{3}\pi r^2 h \times \bar{x} = 9\pi r^2 h \times 0 - \frac{1}{3}\pi r^2 h \times r$

$\bar{x} = -\frac{1}{3} r \times \frac{3}{26} = -\frac{r}{26}$

Horizontal distance from OX = $\frac{r}{26}$

(iii) Hanging in equilibrium from O, the centre of mass G lies vertically below O.



$\tan\alpha = \frac{r/26}{53h/104} = \frac{104r}{1378h}$

$\tan\alpha = \frac{4r}{53h}$

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