Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

S25-1305U50-1

1305U50-1

FRIDAY, 13 JUNE 2025 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 5
FURTHER STATISTICS B

1 hour 45 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	9	
2	5	
3	18	
4	14	
5	6	
6	11	
7	17	
Total	80	

1305U501
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A manufacturer produces light bulbs and claims that the average lifespan of these bulbs is 800 hours with a standard deviation of 50 hours. A sample of 100 bulbs is randomly selected and tested. The mean lifespan is found to be 785 hours.

(a) Calculate a 95% confidence interval for the population mean lifespan of the bulbs based on the sample data and the standard deviation claimed by the manufacturer. [4]

Standard error $= \sigma/\sqrt{n} = 50/\sqrt{100} = 5$

95% CI $= \bar{x} \pm z \times SE$, $z = 1.96$

$= 785 \pm 1.96 \times 5$

$= 785 \pm 9.8$

95% confidence interval for μ: [775.2, 794.8] hours

(b) Interpret your confidence interval in context. [1]

The interval does not contain 800, so it does not support the manufacturer's claim: if $\sigma = 50$ is correct, the claimed mean lifespan of 800 hours appears to be too high (or the claimed σ is too low).

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(c) What is the smallest sample size that would be required to ensure that the 95% confidence interval for the mean lifespan is no wider than 10 hours? [3]

Width of CI $= 2 \times 1.96 \times 50/\sqrt{n} \leq 10$

$\Rightarrow \frac{1.96 \times 50}{\sqrt{n}} \leq 5$

$\Rightarrow \sqrt{n} \geq \frac{1.96 \times 50}{5} = 19.6$

$\Rightarrow n \geq 19.6^2 = 384.16$

Smallest sample size: $n = 385$

(d) What role has the Central Limit Theorem played in your calculations? [1]

The lifespans are not stated to be normally distributed, but because $n = 100$ is large, the CLT means the sample mean is approximately normally distributed, so we can use the normal distribution ($z = 1.96$) in the calculations.

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2. The continuous random variable X is uniformly distributed over the interval $(-\theta, \theta)$, where θ is an unknown constant.

(a) Find $\mathrm{E}\left(X^2\right)$. [4]

$X \sim U(-\theta, \theta)$

$E(X) = \frac{-\theta + \theta}{2} = 0$

$Var(X) = \frac{(\theta - (-\theta))^2}{12} = \frac{4\theta^2}{12} = \frac{\theta^2}{3}$

$E(X^2) = Var(X) + [E(X)]^2$

$E(X^2) = \frac{\theta^2}{3} + 0^2$

$E(X^2) = \frac{\theta^2}{3}$

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(b) Hence, write down an unbiased estimator for θ^2. [1]

$E(3X^2) = 3 \times \theta^2/3 = \theta^2$

Unbiased estimator for θ^2: $3X^2$

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3. Jeff, a psychology professor, is researching the effects of social media on teenagers. He sets up a trial, during which the participants are not allowed to use social media. Levels of wellbeing are self-reported at the beginning and end of the trial. Jeff wants to know whether or not participants' levels of wellbeing improve when they do not use social media.

The following results show the levels of wellbeing of the participants before and after the trial. A higher value corresponds to better wellbeing.

Participant	A	B	C	D	E	F	G	H	I
Wellbeing Before	67	72	56	49	50	90	10	75	55
Wellbeing After	75	81	57	46	60	88	30	80	59

(a) Conduct a Wilcoxon Signed-Rank test at a significance level as close to 5% as possible. [9]

H_0: median difference (after – before) in wellbeing = 0

H_1: median difference (after – before) in wellbeing > 0

Participant	A	B	C	D	E	F	G	H	I
d = after – before	8	9	1	−3	10	−2	20	5	4
Rank of |d|	6	7	1	3	8	2	9	5	4

W^+ = sum of ranks of positive differences

$= 6 + 7 + 1 + 8 + 9 + 5 + 4 = 40$ $\quad (W^- = 3 + 2 = 5)$

$n = 9$, one-tailed, significance level closest to 5%:

upper critical value = 37 (lower critical value = 8)

Since $40 > 37$ $(5 < 8)$, there is sufficient evidence to reject H_0.

There is sufficient evidence to suggest that not using social media improves (increases) teenagers' levels of wellbeing.

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In a further trial, Jeff wants to test whether there is a difference in levels of anxiety between teenagers who use social media as usual and those who are restricted in their use of social media. Participants are assigned to be in either a control group who can continue to use social media as usual, or an experimental group who are not allowed to use social media. There are 30 participants in the control group and 20 in the experimental group.

Participants are asked a series of questions and are assigned an anxiety score based on their responses.

Summary statistics for the anxiety scores, x for the control group and y for the experimental group, are shown below.

$$\sum x = 1560 \qquad \sum y = 940$$

You may assume that anxiety scores are normally distributed with a standard deviation of 5·5 for each group.

(b) State the hypotheses for Jeff's test. [1]

H_0: $\mu_X = \mu_Y$ H_1: $\mu_X \neq \mu_Y$

where μ_X, μ_Y are the population mean anxiety scores for the control group and the experimental group respectively.

(c) Calculate the *p*-value and state your conclusion in context. [8]

$\bar{x} = \frac{1560}{30} = 52$, $\bar{y} = \frac{940}{20} = 47$

$\bar{x} - \bar{y} = 5$

Under H_0, $\bar{X} - \bar{Y} \sim N(0, 5.5^2/30 + 5.5^2/20)$

$SE = \sqrt{(5.5^2/30 + 5.5^2/20)} = 11\sqrt{3}/12 = 1.5877...$

$z = 5 \div 1.5877... = 3.1491...$

$P(\bar{X} - \bar{Y} > 5) = P(Z > 3.1491...) = 0.00081863...$

p-value = $2 \times 0.00081863...$ (two-tailed)

p-value = 0.001637

Since $0.001637 < 0.01$ (and < 0.05), there is sufficient evidence to reject H_0.

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There is sufficient evidence to suggest that time off social media changes anxiety levels (it seems to suggest a decrease, since $\bar{y} < \bar{x}$).

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4. The time taken, X minutes, for people to complete puzzles of Type A can be modelled by the normal distribution $\mathrm{N}\left(10, 3\cdot 2^2\right)$. You may assume that the times taken to complete the puzzles are independent. Mali and Ela are both given several randomly selected puzzles of Type A to complete.

(a) Find the probability that Mali will be able to complete four puzzles in less time than it takes Ela to complete three puzzles. [4]

Let $X_1, \ldots, X_7 \sim N(10, 3.2^2)$ independently

(X_1 to X_4 Mali's times, X_5 to X_7 Ela's times)

$T = X_1 + X_2 + X_3 + X_4 - X_5 - X_6 - X_7$

$E(T) = 4 \times 10 - 3 \times 10 = 10$

$Var(T) = 7\ Var(X) = 7 \times 3.2^2 = 71.68$

$T \sim N(10, 71.68)$

$P(T < 0) = P(Z < (0 - 10)/\sqrt{71.68}) = P(Z < -1.1811\ldots)$

P(Mali's 4 quicker than Ela's 3) = 0.11877... = 0.1188 (4 s.f.)

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(b) Find the probability that Ela will complete a single puzzle at least twice as fast as Mali does. [6]

X_1 = time taken by Ela, X_2 = time taken by Mali, both $\sim N(10, 3.2^2)$

Ela at least twice as fast $\Leftrightarrow$ $X_1 \leq X_2/2$ $\Leftrightarrow$ $X_2 - 2X_1 \geq 0$

Let $U = X_2 - 2X_1$

$E(U) = 10 - 2 \times 10 = -10$

$Var(U) = Var(X_2) + 4Var(X_1) = 5 \times 3.2^2 = 51.2$

$P(U > 0) = P(Z > (0 - (-10))/\sqrt{51.2}) = P(Z > 1.3975...)$

$P(U > 0) = 0.08113$ (4 s.f.)

TURN OVER FOR QUESTION 4(c)

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Independently, the time taken, Y minutes, for people to complete a puzzle of Type B has the distribution $N\left(8, 2\cdot 8^2\right)$. Mali is given, at random, a puzzle of Type A and Ela is given, at random, a puzzle of Type B.

(c) Find the probability that Mali completes her puzzle in less time than Ela takes to complete her puzzle. [4]

Consider $V = X - Y$, $X \sim N(10, 3.2^2)$, $Y \sim N(8, 2.8^2)$

$E(V) = 10 - 8 = 2$

$Var(V) = 3.2^2 + 2.8^2 = 18.08$

$P(V < 0) = P(Z < (0 - 2)/\sqrt{18.08}) = P(Z < -0.4704...)$

P(Mali quicker than Ela) = 0.3190 (4 s.f.)

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5. A farmer believes that giving antibiotics regularly to his cows reduces the amount of milk they produce. The amounts of milk produced per day are not assumed to be normally distributed, so the farmer uses a Mann Whitney U test to test his belief.

The following results show the number of litres of milk produced in one day by randomly selected cows in two groups, one on antibiotics and the other not on antibiotics.

Cows on antibiotics:	12	18·5	24	17	13	22	19		
Cows not on antibiotics:	16	15	21	20	23	14	18	19·5	17·5

Conduct the farmer's test at a significance level as close as possible to 5% and comment on the results. [6]

H_0: the median amounts of milk produced by cows on antibiotics and not on antibiotics are the same.

H_1: the median amount of milk produced by cows on antibiotics is less than the median amount produced by cows not on antibiotics.

Ranking all 16 values (A = on antibiotics, B = not on antibiotics):

Value	12	13	14	15	16	17	17.5	18	18.5	19	19.5	20	21	22	23	24
Group	A	A	B	B	B	A	B	B	A	A	B	B	B	A	B	A
Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Sum of ranks for A: $1 + 2 + 6 + 9 + 10 + 14 + 16 = 58$

$U_A = 58 - (7 \times 8)/2 = 58 - 28 = 30$

Sum of ranks for B $= 136 - 58 = 78$, $U_B = 78 - (9 \times 10)/2 = 33$

(check: $30 + 33 = 63 = 7 \times 9$)

$n_A = 7$, $n_B = 9$, one-tailed, level closest to 5%:

lower critical value = 15 (upper critical value = 48)

Since $30 > 15$ ($33 < 48$), there is insufficient evidence to reject H_0.

There is insufficient evidence to suggest that cows produce less milk when they are on antibiotics, so the farmer's belief is not supported.

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6. An anthropologist selects a random sample of 12 women from The Netherlands and records their heights, in cm, as follows.

164 175 177 173 174 174 178 167 171 171 177 175

(a) Stating a necessary assumption, calculate a 90% confidence interval for the mean height, μ cm, of the population of women from The Netherlands. [9]

Assumption: the heights of women from The Netherlands are normally distributed.

$\Sigma x = 2076, \quad \Sigma x^2 = 359340, \quad n = 12$

$\bar{x} = \frac{2076}{12} = 173$

$s^2 = \frac{1}{n-1}(\Sigma x^2 - n\bar{x}^2) = \frac{1}{11}(359340 - 12 \times 173^2)$

$s^2 = \frac{192}{11} = 17.4545\ldots \qquad (s = 4.1778\ldots)$

Degrees of freedom = 11

90% CI, two-tailed: $t = 1.796$

$SE = \frac{s}{\sqrt{n}} = \sqrt{(17.4545\ldots/12)} = 1.20604\ldots$

$CI = \bar{x} \pm t \times SE = 173 \pm 1.796 \times 1.20604\ldots$

$= 173 \pm 2.1660\ldots$

90% confidence interval for μ: [170.8, 175.2] cm

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(b) A random sample of 15 women from Country A have a mean height of 165 cm. State, with a reason, whether you think that Country A is likely to be The Netherlands. [1]

Unlikely: 165 cm is outside (below) the confidence interval [170.8, 175.2],

so Country A is unlikely to be The Netherlands.

(c) Comment on the reasonableness of the assumption that you have made in part (a). [1]

The assumption is reasonable because heights (natural measurements)

are often normally distributed, and the data look roughly symmetric.

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7. The discrete random variable X denotes the score of a player in each round of a game.

The score X has the following probability distribution, where θ is an unknown parameter such that $0 < \theta < 0{\cdot}4$.

x	–3	–1	2	4
$P(X = x)$	θ	0·2	θ	$0{\cdot}8 - 2\theta$

(a) (i) Obtain an expression for $E(X)$. [2]

(ii) Show that $Var(X) = 4 + 35\theta - 81\theta^2$. [2]

(i)

$E(X) = -3 \times \theta + (-1) \times 0.2 + 2 \times \theta + 4 \times (0.8 - 2\theta)$

$= -3\theta - 0.2 + 2\theta + 3.2 - 8\theta$

$E(X) = 3 - 9\theta$

(ii)

$E(X^2) = (-3)^2 \times \theta + (-1)^2 \times 0.2 + 2^2 \times \theta + 4^2 \times (0.8 - 2\theta)$

$= 9\theta + 0.2 + 4\theta + 12.8 - 32\theta = 13 - 19\theta$

$Var(X) = E(X^2) - [E(X)]^2 = 13 - 19\theta - (3 - 9\theta)^2$

$= 13 - 19\theta - (9 - 54\theta + 81\theta^2)$

$= 13 - 19\theta - 9 + 54\theta - 81\theta^2$

$Var(X) = 4 + 35\theta - 81\theta^2$ as required

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To estimate θ, a random sample of n observations of X is taken and $\bar{X}$ denotes the sample mean.

(b) (i) Show that $T_1 = \dfrac{3-\bar{X}}{9}$ is an unbiased estimator for θ. [2]

(ii) Find an expression for Var(T_1) in terms of n and θ. [2]

(i)

$E(T_1) = E\left(\frac{3-\bar{X}}{9}\right) = \frac{3-E(\bar{X})}{9}$

$E(\bar{X}) = E(X) = 3 - 9\theta$

$E(T_1) = \frac{3-(3-9\theta)}{9} = \frac{9\theta}{9} = \theta$

$\therefore T_1$ is an unbiased estimator for θ.

(ii)

$Var(T_1) = \frac{1}{81} Var(\bar{X}) = \frac{1}{81} \times \frac{Var(X)}{n}$

$Var(T_1) = \frac{4 + 35\theta - 81\theta^2}{81n}$

TURN OVER FOR QUESTIONS 7(c) & 7(d)

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A positive value of X wins the game. Let Y denote the number of times, in a random sample of n values of X, that the player wins the game.

(c) Show that $T_2 = \dfrac{4n-5Y}{5n}$ is an unbiased estimator for θ. [4]

$P(X > 0) = P(X = 2) + P(X = 4) = \theta + 0.8 - 2\theta = 0.8 - \theta$

$Y \sim B(n, 0.8 - \theta)$

$E(Y) = n(0.8 - \theta)$

$E(T_2) = \dfrac{4n - 5E(Y)}{5n}$

$E(T_2) = \dfrac{4n - 5n(0.8 - \theta)}{5n}$

$E(T_2) = \dfrac{4n - 4n + 5n\theta}{5n} = \theta$

$\therefore T_2$ is an unbiased estimator for θ.

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(d) By considering Var(T_1) – Var(T_2), determine which of T_1 and T_2 is the better estimator for θ. [5]

$\text{Var}(Y) = n(0.8 - \theta)(0.2 + \theta)$

$\text{Var}(T_2) = \frac{(-5)^2\,\text{Var}(Y)}{(5n)^2} = \frac{25n(0.8 - \theta)(0.2 + \theta)}{25n^2}$

$\text{Var}(T_2) = \frac{0.16 + 0.6\theta - \theta^2}{n}$

$\text{Var}(T_1) - \text{Var}(T_2) = \frac{4 + 35\theta - 81\theta^2}{81n} - \frac{0.16 + 0.6\theta - \theta^2}{n}$

$= \frac{4 + 35\theta - 81\theta^2 - 12.96 - 48.6\theta + 81\theta^2}{81n}$

$= \frac{-8.96 - 13.6\theta}{81n}$

Since $\theta > 0$ and $n > 0$, $-8.96 - 13.6\theta < 0$, so $\text{Var}(T_1) - \text{Var}(T_2) < 0$

i.e. $\text{Var}(T_1) < \text{Var}(T_2)$ and both estimators are unbiased.

$\therefore T_1$ is the better estimator for θ.

END OF PAPER

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