Surname	Centre Number	Candidate Number
First name(s)		2

GCE A LEVEL

S25-1305U40-1

1305U40-1

FRIDAY, 6 JUNE 2025 – MORNING

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**5**	
2	**20**	
3	**7**	
4	**12**	
5	**6**	
6	**8**	
7	**12**	
8	**9**	
9	**11**	
10	**18**	
11	**12**	
Total	**120**	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. (a) Write down the Maclaurin series expansion for e^{x^2} as far as the term in x^6. Simplify your answer. [2]

(b) Hence, find an approximate value for $\int_0^1 e^{x^2} dx$. Give your answer correct to three decimal places. [3]

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2. (a) Using mathematical induction, prove that

$$\left(\cos\theta + \mathrm{i}\sin\theta\right)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

for positive integer values of n. [7]

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(b) Show that $\cos 4\theta$ can be written as $8\sin^4\theta - 8\sin^2\theta + 1$. [6]

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(c) (i) Hence, or otherwise, find the general solution of the equation $\cos 4\theta = 2\sin^2\theta - 1$. [6]

(ii) Write down the general solution as a single expression. [1]

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3. (a) By writing $\cosh x^2$ as $\frac{e^{x^2} + e^{-x^2}}{2}$, show that $\frac{d}{dx}\left(\cosh x^2\right) = 2x \sinh x^2$. [3]

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(b) Hence, find $\int 2x^3 \sinh x^2 \mathrm{d}x$. [4]

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4. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 4 \\ 3 & 2 & -1 \\ 2 & 0 & 7 \end{pmatrix}.$$

(a) Find the inverse of $\mathbf{M}$. [5]

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(b) The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are such that

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & p \\ 3 & -1 & q \\ 2 & 5 & r \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & p \\ 6 & r \\ 0 & 2 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 25 & -6 \\ -3 & 11 \\ 32 & -17 \end{pmatrix}.$$

Given that $\mathbf{AB} = \mathbf{C}$, find the values of p, q, r. [7]

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5. The region R is bounded by the curve $x = \sinh 2y$, the y-axis and the line $y = 1$. Find the volume of the solid generated when R is rotated through four right angles about the y-axis. Give your answer correct to three decimal places. [6]

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6. Solve the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y = \sin x,$$

given that $y = 3$ when $x = 0$. [8]

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7. (a) Differentiate each of the following with respect to x, simplifying your answer where possible.

(i) $y = \cos^{-1}(2x-1)$ [3]

(ii) $y = 3x\sinh 4x$ [3]

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(b) Find the equation of the normal to the curve $y = \dfrac{\cos x}{\cosh x}$ at the point where $x = \dfrac{\pi}{2}$.

Give your answer in the form $y = mx + c$, with m, c correct to two decimal places. [6]

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8. (a) By putting $t = \tan\left(\frac{\theta}{2}\right)$, show that the equation $4\cos\theta + 7\sin\theta = 4 - \tan\left(\frac{\theta}{2}\right)$ can be written as

$$t^3 - 8t^2 + 15t = 0.$$ [3]

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(b) Hence, solve the equation $4\cos\theta + 7\sin\theta = 4 - \tan\left(\frac{\theta}{2}\right)$, for $0 \leqslant \theta < \pi$. [6]

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9. Find the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 16y = 3\sin x,$$

where (π, 3) is a stationary point. [11]

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10. (a) (i) Show that $\dfrac{2x^3+x^2+12}{x^3-2x^2-3x+6}$ can be written as $2+\dfrac{5x^2+6x}{x^3-2x^2-3x+6}$. [2]

(ii) Hence, find $\displaystyle\int \frac{2x^3+x^2+12}{x^3-2x^2-3x+6}\,\mathrm{d}x$. [9]

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(b) Evaluate $\int_{-2}^{0} \frac{1}{\sqrt{4-4x-x^2}}\,\mathrm{d}x$, giving your answer in the form $\frac{\pi}{k}$, where k is an integer. [4]

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(c) Calculate the mean value of $y = \tanh 2x$ between $x = 0$ and $x = 2$, giving your answer correct to two significant figures. [3]

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11. (a) Sketch the graph of the curve *C* with polar equation

$$r = \cos^2\theta, \quad \text{where} \quad 0 \leqslant \theta < 2\pi. \qquad [2]$$



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(b) The points on *C* where the tangents are parallel to the initial line are joined together to form a rectangle. Find the exact area of this rectangle. [10]

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