Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

S25-1305U40-1

1305U40-1

FRIDAY, 6 JUNE 2025 – MORNING

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	5	
2	20	
3	7	
4	12	
5	6	
6	8	
7	12	
8	9	
9	11	
10	18	
11	12	
Total	120	

1305U401
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. (a) Write down the Maclaurin series expansion for e^{x^2} as far as the term in x^6. Simplify your answer. [2]

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

Replacing x by x^2:

$e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \ldots$

$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \ldots$

(b) Hence, find an approximate value for $\int_0^1 e^{x^2}\,dx$. Give your answer correct to three decimal places. [3]

$\int_0^1 e^{x^2}\,dx \approx \int_0^1 \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}\right) dx$

$= \left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42}\right]_0^1$

$= \left(1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}\right) - 0$

$= \frac{51}{35} = 1.457142\ldots$

≈ 1.457 (3 d.p.)

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2. (a) Using mathematical induction, prove that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

for positive integer values of n. [7]

When $n = 1$: LHS $= (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$

RHS $= \cos\theta + i\sin\theta =$ LHS, so the result is true for $n = 1$.

Assume the result is true for $n = k$, i.e.

$(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

Consider $n = k + 1$:

$(\cos\theta + i\sin\theta)^{k+1} = (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)$

$= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$

$= \cos k\theta\cos\theta + i\cos k\theta\sin\theta + i\sin k\theta\cos\theta + i^2\sin k\theta\sin\theta$

$= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$

$= \cos(k + 1)\theta + i\sin(k + 1)\theta$ (compound angle formulae)

which is the result for $n = k + 1$.

So if the result is true for $n = k$, then it is true for $n = k + 1$.

Since it is true for $n = 1$, by mathematical induction

it is true for all positive integers n.

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(b) Show that $\cos 4\theta$ can be written as $8\sin^4\theta - 8\sin^2\theta + 1$. [6]

By de Moivre: $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

Binomial expansion, with $c = \cos\theta$ and $s = \sin\theta$:

$(c + is)^4 = c^4 + 4c^3(is) + 6c^2(is)^2 + 4c(is)^3 + (is)^4$

$= c^4 + 4ic^3s - 6c^2s^2 - 4ics^3 + s^4$

Equating real parts:

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= (1 - \sin^2\theta)^2 - 6(1 - \sin^2\theta)\sin^2\theta + \sin^4\theta$

$= 1 - 2\sin^2\theta + \sin^4\theta - 6\sin^2\theta + 6\sin^4\theta + \sin^4\theta$

$\therefore \cos 4\theta = 8\sin^4\theta - 8\sin^2\theta + 1$ as required

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(c) (i) Hence, or otherwise, find the general solution of the equation $\cos 4\theta = 2\sin^2\theta - 1$. [6]

(ii) Write down the general solution as a single expression. [1]

$8\sin^4\theta - 8\sin^2\theta + 1 = 2\sin^2\theta - 1$

$8\sin^4\theta - 10\sin^2\theta + 2 = 0$

$4\sin^4\theta - 5\sin^2\theta + 1 = 0$

$(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$

$\sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta = \pm\frac{1}{2}$ or $\sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1$

$\sin\theta = \pm\frac{1}{2}$: $\theta = 30°, 150°, 210°, 330°, \ldots$

$\sin\theta = \pm 1$: $\theta = 90°, 270°, \ldots$

General solution (n an integer):

$\theta = 360n° + 30°$, $\theta = 360n° + 150°$,

$\theta = 360n° + 210°$, $\theta = 360n° + 330°$,

$\theta = 360n° + 90°$, $\theta = 360n° + 270°$

(i.e. $\theta = 180n° \pm 30°$ or $\theta = 180n° + 90°$)

(ii) $\theta = 60n° + 30°$, n an integer

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3. (a) By writing $\cosh x^2$ as $\frac{e^{x^2} + e^{-x^2}}{2}$, show that $\frac{d}{dx}\left(\cosh x^2\right) = 2x \sinh x^2$. [3]

$\cosh x^2 = \frac{e^{x^2} + e^{-x^2}}{2}$

$\frac{d}{dx}(\cosh x^2) = \frac{2x\, e^{x^2} - 2x\, e^{-x^2}}{2}$ (chain rule)

$= 2x\left(\frac{e^{x^2} - e^{-x^2}}{2}\right)$

$= 2x \sinh x^2$ as required

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(b) Hence, find $\int 2x^3 \sinh x^2 \mathrm{d}x$. [4]

By parts: $u = x^2$, $\frac{dv}{dx} = 2x \sinh x^2$

$\frac{du}{dx} = 2x$, $v = \cosh x^2$ (from (a))

$\int 2x^3 \sinh x^2 \, dx = \int x^2(2x \sinh x^2) \, dx$

$= x^2 \cosh x^2 - \int 2x \cosh x^2 \, dx$

$= x^2 \cosh x^2 - \sinh x^2 + c$

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4. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 0 & 4 \\ 3 & 2 & -1 \\ 2 & 0 & 7 \end{pmatrix}.$$

(a) Find the inverse of $\mathbf{M}$. [5]

$\det M = 3(2 \times 7 - (-1) \times 0) - 0 + 4(3 \times 0 - 2 \times 2)$

$= 3 \times 14 + 4 \times (-4) = 26$

Matrix of cofactors $= \begin{pmatrix} 14 & -23 & -4 \\ 0 & 13 & 0 \\ -8 & 15 & 6 \end{pmatrix}$

Adjugate $= \begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}$

$M^{-1} = \frac{1}{26} \begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}$

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(b) The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are such that

$$\mathbf{A}=\begin{pmatrix}1 & 4 & p\\ 3 & -1 & q\\ 2 & 5 & r\end{pmatrix}, \qquad \mathbf{B}=\begin{pmatrix}1 & p\\ 6 & r\\ 0 & 2\end{pmatrix}, \qquad \mathbf{C}=\begin{pmatrix}25 & -6\\ -3 & 11\\ 32 & -17\end{pmatrix}.$$

Given that $\mathbf{AB} = \mathbf{C}$, find the values of p, q, r. [7]

$AB = \begin{pmatrix} 1+24+0 & p+4r+2p \\ 3-6+0 & 3p-r+2q \\ 2+30+0 & 2p+5r+2r \end{pmatrix} = \begin{pmatrix} 25 & -6 \\ -3 & 11 \\ 32 & -17 \end{pmatrix}$

Comparing entries: $3p + 4r = -6$, $3p + 2q - r = 11$, $2p + 7r = -17$

i.e. $\begin{pmatrix} 3 & 0 & 4 \\ 3 & 2 & -1 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -6 \\ 11 \\ -17 \end{pmatrix}$ (this is M)

$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = M^{-1}\begin{pmatrix} -6 \\ 11 \\ -17 \end{pmatrix} = \frac{1}{26}\begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}\begin{pmatrix} -6 \\ 11 \\ -17 \end{pmatrix}$

$= \frac{1}{26}\begin{pmatrix} -84+0+136 \\ 138+143-255 \\ 24+0-102 \end{pmatrix} = \frac{1}{26}\begin{pmatrix} 52 \\ 26 \\ -78 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

$p = 2$, $q = 1$, $r = -3$

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5. The region R is bounded by the curve $x = \sinh 2y$, the y-axis and the line $y = 1$. Find the volume of the solid generated when R is rotated through four right angles about the y-axis. Give your answer correct to three decimal places. [6]

$V = \pi\int_0^1 x^2\,dy = \pi\int_0^1 \sinh^2 2y\,dy$

$\cosh 4y = 1 + 2\sinh^2 2y \Rightarrow \sinh^2 2y = \frac{\cosh 4y - 1}{2}$

$V = \pi\int_0^1 \left(\frac{\cosh 4y - 1}{2}\right) dy$

$= \pi\left[\frac{\sinh 4y}{8} - \frac{y}{2}\right]_0^1$

$= \pi\left[\left(\frac{\sinh 4}{8} - \frac{1}{2}\right) - (0 - 0)\right]$

$= \pi(3.411239\ldots - 0.5)$

$= 9.145929\ldots$

$V = 9.146$ (3 d.p.)

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6. Solve the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} + y = \sin x,$$

given that $y = 3$ when $x = 0$. [8]

Dividing by cos x: $\frac{dy}{dx} + \frac{y}{\cos x} = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} + y \sec x = \tan x$

Integrating factor $= e^{\int \sec x \, dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$

$(\sec x + \tan x)\frac{dy}{dx} + y \sec x(\sec x + \tan x) = \tan x(\sec x + \tan x)$

$\frac{d}{dx}[\, y(\sec x + \tan x)\,] = \sec x \tan x + \tan^2 x$

$= \sec x \tan x + \sec^2 x - 1$

$y(\sec x + \tan x) = \int (\sec x \tan x + \sec^2 x - 1)\, dx$

$y(\sec x + \tan x) = \sec x + \tan x - x + c$

$x = 0, y = 3$: $3(1 + 0) = 1 + 0 - 0 + c \Rightarrow c = 2$

$y(\sec x + \tan x) = \sec x + \tan x - x + 2$

$y = \frac{\sec x + \tan x - x + 2}{\sec x + \tan x}$

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7. (a) Differentiate each of the following with respect to x, simplifying your answer where possible.

(i) $y = \cos^{-1}(2x-1)$ [3]

Chain rule with $u = 2x - 1$, $\frac{du}{dx} = 2$:

$\frac{dy}{dx} = -\frac{1}{\sqrt{(1-(2x-1)^2)}} \times 2$

$= -\frac{2}{\sqrt{(1-4x^2+4x-1)}}$

$= -\frac{2}{\sqrt{(4x-4x^2)}} = -\frac{2}{2\sqrt{(x-x^2)}}$

$= -\frac{1}{\sqrt{(x-x^2)}}$

(ii) $y = 3x\sinh 4x$ [3]

Product rule:

$\frac{dy}{dx} = 3x \times 4\cosh 4x + 3\sinh 4x$

$= 12x\cosh 4x + 3\sinh 4x$

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(b) Find the equation of the normal to the curve $y = \dfrac{\cos x}{\cosh x}$ at the point where $x = \dfrac{\pi}{2}$.

Give your answer in the form $y = mx + c$, with m, c correct to two decimal places. [6]

Quotient rule: $\dfrac{dy}{dx} = \dfrac{\cosh x(-\sin x) - \cos x \sinh x}{\cosh^2 x}$

At $x = \dfrac{\pi}{2}$: $y = \dfrac{\cos(\pi/2)}{\cosh(\pi/2)} = 0$

$\dfrac{dy}{dx} = \dfrac{-\cosh(\pi/2) \times 1 - 0}{\cosh^2(\pi/2)} = -\dfrac{1}{\cosh(\pi/2)} = -0.398536...$

Gradient of normal $= \dfrac{-1}{-0.398536...} = 2.509178...$

$y - 0 = 2.509178...(x - \dfrac{\pi}{2})$

$y = 2.509178...x - 3.941408...$

$y = 2.51x - 3.94$

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8. (a) By putting $t = \tan\left(\frac{\theta}{2}\right)$, show that the equation $4\cos\theta + 7\sin\theta = 4 - \tan\left(\frac{\theta}{2}\right)$ can be written as

$$t^3 - 8t^2 + 15t = 0.$$ [3]

$t = \tan(\frac{\theta}{2})$: $\cos\theta = \frac{1-t^2}{1+t^2}$, $\sin\theta = \frac{2t}{1+t^2}$

$4(\frac{1-t^2}{1+t^2}) + 7(\frac{2t}{1+t^2}) = 4 - t$

Multiplying by $(1+t^2)$:

$4(1-t^2) + 14t = (4-t)(1+t^2)$

$4 - 4t^2 + 14t = 4 + 4t^2 - t - t^3$

$t^3 - 8t^2 + 15t = 0$ as required

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(b) Hence, solve the equation $4\cos\theta + 7\sin\theta = 4 - \tan\left(\frac{\theta}{2}\right)$, for $0 \leqslant \theta < \pi$. [6]

$t^3 - 8t^2 + 15t = 0 \Rightarrow t(t^2 - 8t + 15) = 0$

$t(t - 3)(t - 5) = 0$

$t = 0, \quad t = 3 \quad or \quad t = 5$

$0 \leq \theta < \pi \Rightarrow 0 \leq \frac{\theta}{2} < \frac{\pi}{2}$

$\tan(\frac{\theta}{2}) = 0 \Rightarrow \frac{\theta}{2} = 0 \Rightarrow \theta = 0$

$\tan(\frac{\theta}{2}) = 3 \Rightarrow \frac{\theta}{2} = 1.249045... \Rightarrow \theta = 2.498091...$

$\tan(\frac{\theta}{2}) = 5 \Rightarrow \frac{\theta}{2} = 1.373400... \Rightarrow \theta = 2.746801...$

$\theta = 0, \ 2.498, \ 2.747$

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9. Find the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 16y = 3\sin x,$$

where $(\pi, 3)$ is a stationary point. [11]

Auxiliary equation: $m^2 + 16 = 0 \Rightarrow m^2 = -16 \Rightarrow m = \pm 4i$

Complementary function: $y = A \sin 4x + B \cos 4x$

Particular integral: try $y = C \sin x + D \cos x$

$\frac{dy}{dx} = C \cos x - D \sin x, \quad \frac{d^2y}{dx^2} = -C \sin x - D \cos x$

$-C \sin x - D \cos x + 16(C \sin x + D \cos x) = 3 \sin x$

$15C \sin x + 15D \cos x = 3 \sin x$

$15C = 3 \Rightarrow C = \frac{1}{5}, \quad 15D = 0 \Rightarrow D = 0$

General solution: $y = A \sin 4x + B \cos 4x + \frac{1}{5}\sin x$

$\frac{dy}{dx} = 4A \cos 4x - 4B \sin 4x + \frac{1}{5}\cos x$

At $(\pi, 3)$: $3 = A \sin 4\pi + B \cos 4\pi + \frac{1}{5}\sin \pi$

$3 = 0 + B + 0 \Rightarrow B = 3$

Stationary point, so $\frac{dy}{dx} = 0$ when $x = \pi$:

$0 = 4A \cos 4\pi - 4B \sin 4\pi + \frac{1}{5}\cos \pi = 4A - \frac{1}{5}$

$\Rightarrow A = \frac{1}{20}$

$y = \frac{1}{20}\sin 4x + 3 \cos 4x + \frac{1}{5}\sin x$

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10. (a) (i) Show that $\dfrac{2x^3+x^2+12}{x^3-2x^2-3x+6}$ can be written as $2+\dfrac{5x^2+6x}{x^3-2x^2-3x+6}$. [2]

$2(x^3 - 2x^2 - 3x + 6) = 2x^3 - 4x^2 - 6x + 12$

so $2x^3 + x^2 + 12 = 2(x^3 - 2x^2 - 3x + 6) + 5x^2 + 6x$

$\therefore \dfrac{2x^3 + x^2 + 12}{x^3 - 2x^2 - 3x + 6} = 2 + \dfrac{5x^2 + 6x}{x^3 - 2x^2 - 3x + 6}$

(ii) Hence, find $\displaystyle\int \frac{2x^3+x^2+12}{x^3-2x^2-3x+6}\,dx$. [9]

$f(x) = x^3 - 2x^2 - 3x + 6$: $f(2) = 8 - 8 - 6 + 6 = 0$

so $(x - 2)$ is a factor: $x^3 - 2x^2 - 3x + 6 = (x - 2)(x^2 - 3)$

$\dfrac{5x^2 + 6x}{(x - 2)(x^2 - 3)} \equiv \dfrac{A}{x - 2} + \dfrac{Bx + C}{x^2 - 3}$

$5x^2 + 6x \equiv A(x^2 - 3) + (Bx + C)(x - 2)$

$x = 2$: $20 + 12 = A(1) \Rightarrow A = 32$

$x = 0$: $0 = -3A - 2C = -96 - 2C \Rightarrow C = -48$

x^2 coefficients: $5 = A + B \Rightarrow B = -27$

$\displaystyle\int \frac{2x^3 + x^2 + 12}{x^3 - 2x^2 - 3x + 6}\,dx = \int \left(2 + \frac{32}{x - 2} - \frac{27x}{x^2 - 3} - \frac{48}{x^2 - 3}\right) dx$

$\displaystyle\int \frac{27x}{x^2 - 3}\,dx = \frac{27}{2}\ln|x^2 - 3|$

$\displaystyle\int \frac{48}{x^2 - 3}\,dx = \frac{48}{2\sqrt{3}}\ln\left|\frac{x - \sqrt{3}}{x + \sqrt{3}}\right| = 8\sqrt{3}\ln\left|\frac{x - \sqrt{3}}{x + \sqrt{3}}\right|$

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$$\therefore \int \frac{2x^3 + x^2 + 12}{x^3 - 2x^2 - 3x + 6}\,dx = 2x + 32\ln|x - 2| - \frac{27}{2}\ln|x^2 - 3| - 8\sqrt{3}\ln\left|\frac{x - \sqrt{3}}{x + \sqrt{3}}\right| + c$$

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(b) Evaluate $\int_{-2}^{0} \frac{1}{\sqrt{4-4x-x^2}}\,dx$, giving your answer in the form $\frac{\pi}{k}$, where k is an integer. [4]

$4 - 4x - x^2 = 8 - (x^2 + 4x + 4) = 8 - (x + 2)^2$

$\int_{-2}^{0} \frac{1}{\sqrt{(4 - 4x - x^2)}}\,dx = \int_{-2}^{0} \frac{1}{\sqrt{(8 - (x + 2)^2)}}\,dx$

$= \left[\sin^{-1}\left(\frac{x + 2}{\sqrt{8}}\right) \right]_{-2}^{0}$

$= \sin^{-1}\left(\frac{2}{\sqrt{8}}\right) - \sin^{-1}(0)$

$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 0 = \frac{\pi}{4}$

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(c) Calculate the mean value of $y = \tanh 2x$ between $x = 0$ and $x = 2$, giving your answer correct to two significant figures. [3]

Mean value $= \frac{1}{2-0}\int_0^2 \tanh 2x \, dx$

$= \frac{1}{2}\left[\frac{1}{2}\ln(\cosh 2x)\right]_0^2$

$= \frac{1}{4}(\ln \cosh 4 - \ln \cosh 0)$

$= \frac{1}{4}(3.307188\ldots - 0) = 0.826797\ldots$

$= 0.83$ (2 s.f.)

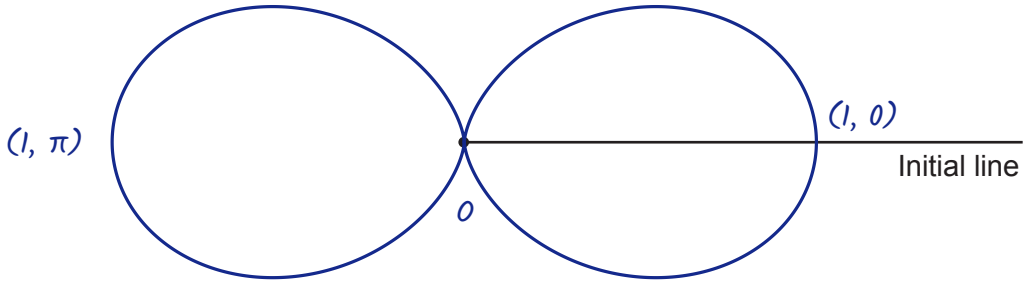
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11. (a) Sketch the graph of the curve C with polar equation

$$r = \cos^2\theta, \quad \text{where} \quad 0 \leqslant \theta < 2\pi. \qquad [2]$$



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(b) The points on C where the tangents are parallel to the initial line are joined together to form a rectangle. Find the exact area of this rectangle. [10]

$y = r\sin\theta = \cos^2\theta \sin\theta$

$\frac{dy}{d\theta} = \cos^2\theta\cos\theta + \sin\theta(-2\cos\theta\sin\theta) = \cos^3\theta - 2\cos\theta\sin^2\theta$

Tangent parallel to initial line: $\frac{dy}{d\theta} = 0$

$\cos\theta(\cos^2\theta - 2\sin^2\theta) = 0$

$\cos\theta = 0$ gives $r = 0$ (the pole): no tangent parallel to the initial line

so $\cos^2\theta - 2(1 - \cos^2\theta) = 0 \Rightarrow 3\cos^2\theta = 2$

$\cos\theta = \pm\frac{\sqrt{2}}{\sqrt{3}}$, $\sin\theta = \pm\frac{1}{\sqrt{3}}$, $r = \cos^2\theta = \frac{2}{3}$

$x = r\cos\theta = \frac{2}{3} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{6}}{9}$ (in size)

$y = r\sin\theta = \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$ (in size)

By symmetry the four points are $(\pm\frac{2\sqrt{6}}{9}, \pm\frac{2\sqrt{3}}{9})$ in Cartesian form

Rectangle has width $2x$ and height $2y$:

Area $= 2 \times \frac{2\sqrt{6}}{9} \times 2 \times \frac{2\sqrt{3}}{9} = \frac{16\sqrt{18}}{81}$

$= \frac{48\sqrt{2}}{81} = \frac{16\sqrt{2}}{27}$

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