Surname	StemUp Model Solution	Centre Number	Candidate Number
First name(s)			2

GCE A LEVEL

1305U40-1

S24-1305U40-1

MONDAY, 3 JUNE 2024 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	11	
2	13	
3	9	
4	21	
5	14	
6	8	
7	12	
8	11	
9	9	
10	12	
Total	120	

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01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. (a) Express the three cube roots of $5+10\mathrm{i}$ in the form $re^{i\theta}$, where $0 \leqslant \theta < 2\pi$. [6]

$r = \sqrt{(5^2 + 10^2)} = \sqrt{125} = 5\sqrt{5}$

$\arg(5 + 10i) = \tan^{-1}(\frac{10}{5}) = 1.107\ldots = 1.11$

$5 + 10i = \sqrt{125}\, e^{(1.107 + 2k\pi)i}$

$\sqrt[3]{(5 + 10i)} = (\sqrt{125})^{1/3}\, e^{(1.107 + 2k\pi)i/3}, \quad (\sqrt{125})^{1/3} = 125^{1/6} = \sqrt{5}$

$k = 0$: $z_1 = \sqrt{5}\, e^{0.369i} = \sqrt{5}\, e^{0.37i}$

$k = 1$: $z_2 = \sqrt{5}\, e^{(0.369 + 2\pi/3)i} = \sqrt{5}\, e^{2.46i}$

$k = 2$: $z_3 = \sqrt{5}\, e^{(0.369 + 4\pi/3)i} = \sqrt{5}\, e^{4.56i}$

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(b) The three cube roots of $5+10\mathrm{i}$ are plotted in an Argand diagram. The points are joined by straight lines to form a triangle. Find the area of this triangle, giving your answer correct to two significant figures. [5]

$z_1 = \sqrt{5}(\cos 0.369 + i \sin 0.369) = 2.086 + 0.807i$

$z_2 = \sqrt{5}(\cos 2.464 + i \sin 2.464) = -1.741 + 1.404i$

$z_3 = \sqrt{5}(\cos 4.558 + i \sin 4.558) = -0.344 - 2.209i$

The roots are equally spaced ($2\pi/3$ apart) on a circle radius $\sqrt{5}$,

so the triangle is equilateral.

$z_1z_2 = \sqrt{((-1.741 - 2.086)^2 + (1.404 - 0.807)^2)}$

$= \sqrt{(14.65... + 0.356...)} = \sqrt{15} = 3.8729...$

Area $= \frac{1}{2} \times 3.8729... \times 3.8729... \times \sin\frac{\pi}{3}$

$= 6.495...$

Area = 6.5 (2 s.f.)

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2. The function f is defined by $f(x) = \cosh\left(\frac{x}{2}\right)$.

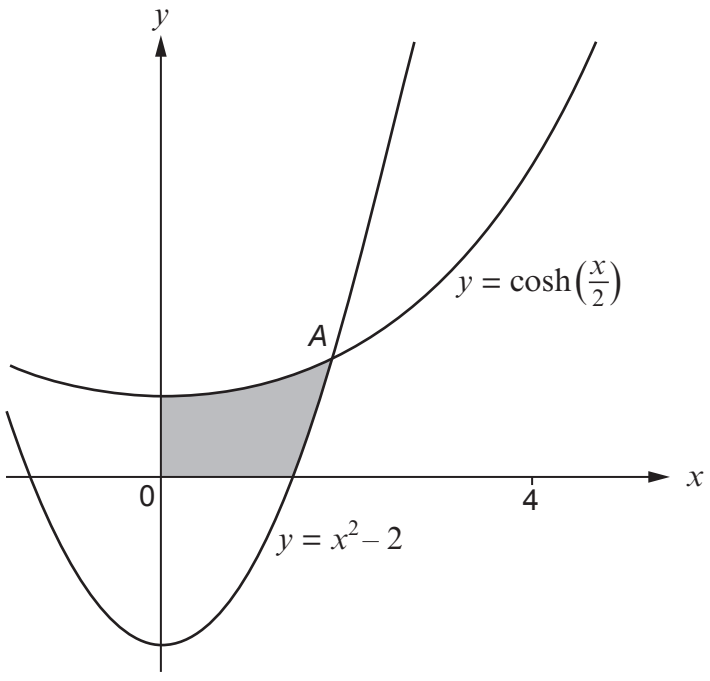
(a) State the Maclaurin series expansion for $\cosh\left(\frac{x}{2}\right)$ up to and including the term in x^4. [2]

$\cosh u = 1 + \frac{u^2}{2!} + \frac{u^4}{4!} + \ldots$ with $u = \frac{x}{2}$:

$\cosh\left(\frac{x}{2}\right) = 1 + \frac{(x/2)^2}{2!} + \frac{(x/2)^4}{4!} + \ldots$

$\cosh\left(\frac{x}{2}\right) \approx 1 + \frac{x^2}{8} + \frac{x^4}{384}$

Another function g is defined by $g(x) = x^2 - 2$. The diagram below shows parts of the graphs of $y = f(x)$ and $y = g(x)$.



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(b) The two graphs intersect at the point A, as shown in the diagram. Use your answer from part (a) to find an approximation for the x-coordinate of A, giving your answer correct to two decimal places. [5]

At A: $\cosh\left(\frac{x}{2}\right) = x^2 - 2$

$1 + \frac{x^2}{8} + \frac{x^4}{384} \approx x^2 - 2$

$\frac{x^4}{384} - \frac{7}{8}x^2 + 3 = 0$

$x^4 - 336x^2 + 1152 = 0$

Let $y = x^2$: $y^2 - 336y + 1152 = 0$

$y = \frac{336 \pm \sqrt{(336^2 - 4 \times 1152)}}{2} = \frac{336 \pm \sqrt{108288}}{2}$

$y = 332.535...$ or $y = 3.4643...$

$x = \pm 18.235...$ or $x = \pm 1.8613...$

Since $0 < x < 4$ at A, $x = 1.86$ (2 d.p.)

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(c) Using your answer to part (b), find an approximation for the area of the shaded region enclosed by the two graphs, the x-axis and the y-axis. [6]

$g(x) = 0 \Rightarrow x^2 = 2 \Rightarrow g$ crosses the x-axis at $x = \sqrt{2}$

$\text{Area} = \int_0^{1.86} \cosh\left(\frac{x}{2}\right) dx - \int_{\sqrt{2}}^{1.86} (x^2 - 2)\, dx$

$= \left[2 \sinh\left(\frac{x}{2}\right)\right]_0^{1.86} - \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$

$= (2 \sinh 0.93 - 0) - ((-1.5750\ldots) - (-1.8856\ldots))$

$= 2.1399\ldots - 0.3105\ldots$

$= 1.829$

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3. Given the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 4\cos^3 x \sin x + 5$$

and $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$, find an equation for y in terms of x. [9]

Divide by cos x:

$\frac{dy}{dx} + y\frac{\sin x}{\cos x} = 4\cos^2 x \sin x + \frac{5}{\cos x}$

Integrating factor $= e^{\int \tan x \, dx} = e^{\ln \sec x} = \sec x$

Multiply by sec x:

$\sec x \frac{dy}{dx} + y \tan x \sec x = 4 \sin x \cos x + 5\sec^2 x$

$\frac{d}{dx}(y \sec x) = 2 \sin 2x + 5\sec^2 x$

$y \sec x = \int (2 \sin 2x + 5\sec^2 x) \, dx$

$y \sec x = -\cos 2x + 5 \tan x + c$

$x = \frac{\pi}{4}$, $y = 3\sqrt{2}$: $3\sqrt{2} \times \sqrt{2} = -\cos\frac{\pi}{2} + 5 \tan\frac{\pi}{4} + c$

$6 = 0 + 5 + c \quad \Rightarrow \quad c = 1$

$y \sec x = -\cos 2x + 5 \tan x + 1$

$y = -\cos 2x \cos x + 5 \sin x + \cos x$

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4. (a) Given that $z^n + \frac{1}{z^n} = 2\cos n\theta$, where $z = \cos\theta + i\sin\theta$, express $16\cos^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$, where a, b, c are integers whose values are to be determined. [5]

$(z + \frac{1}{z})^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$

$= (z^4 + \frac{1}{z^4}) + 4(z^2 + \frac{1}{z^2}) + 6$

$(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$

$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$

$a = 2, \quad b = 8, \quad c = 6$

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The diagram below shows a sketch of the curve C with polar equation

$$r = 3 - 4\cos^2\theta, \qquad \text{where } \frac{\pi}{6} \leqslant \theta \leqslant \frac{5\pi}{6}.$$



(b) Calculate the area of the region enclosed by the curve *C*. [8]

Area $= \frac{1}{2}\int_{\pi/6}^{5\pi/6} r^2\, d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3 - 4\cos^2\theta)^2\, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (9 - 24\cos^2\theta + 16\cos^4\theta)\, d\theta$

$24\cos^2\theta = 12(1 + \cos 2\theta), \quad 16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$

Area $= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (9 - 12 - 12\cos 2\theta + 2\cos 4\theta + 8\cos 2\theta + 6)\, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3 - 4\cos 2\theta + 2\cos 4\theta)\, d\theta$

$= \frac{1}{2}\left[3\theta - 2\sin 2\theta + \frac{1}{2}\sin 4\theta\right]_{\pi/6}^{5\pi/6}$

$= \frac{1}{2}\left(\frac{5\pi}{2} + \sqrt{3} - \frac{\sqrt{3}}{4}\right) - \frac{1}{2}\left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{4}\right)$

$= \frac{1}{2}\left(2\pi + \frac{3\sqrt{3}}{2}\right)$

$= \pi + \frac{3\sqrt{3}}{4} \quad (= 4.44)$

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(c) Find the exact polar coordinates of the points on C at which the tangent is perpendicular to the initial line. [8]

$x = r\cos\theta = (3 - 4\cos^2\theta)\cos\theta = 3\cos\theta - 4\cos^3\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta$

Tangent perpendicular to the initial line $\Rightarrow \frac{dx}{d\theta} = 0$

$-3\sin\theta + 12\cos^2\theta\sin\theta = 0$

$-3\sin\theta(1 - 4\cos^2\theta) = 0$

$\sin\theta = 0$ or $\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$

$\sin\theta = 0$ gives $\theta = 0$ or π, outside $\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$, so the curve does not exist there.

$\cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$r = 3 - 4 \times \frac{1}{4} = 2$

Points: $(2, \frac{\pi}{3})$ and $(2, \frac{2\pi}{3})$

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5. Find each of the following integrals.

(a) $\int \frac{3-x}{x\left(x^2+1\right)}\mathrm{d}x$ [8]

$\frac{3-x}{x(x^2+1)} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$3 - x \equiv A(x^2+1) + x(Bx+C)$

$x = 0$: $3 = A$

$x = 1$: $2 = 2A + B + C \Rightarrow B + C = -4$

$x = -1$: $4 = 2A + B - C \Rightarrow B - C = -2$

Solving: $B = -3$, $C = -1$

$\int \frac{3-x}{x(x^2+1)}\,dx = \int \left(\frac{3}{x} - \frac{3x}{x^2+1} - \frac{1}{x^2+1}\right) dx$

$= 3\ln|x| - \frac{3}{2}\ln(x^2+1) - \tan^{-1}x + c$

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(b) $\displaystyle\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,dx$ [6]

$\frac{\sinh 2x}{\sqrt{(\cosh^4 x - 9\cosh^2 x)}} = \frac{2\sinh x \cosh x}{\cosh x \sqrt{(\cosh^2 x - 9)}} = \frac{2\sinh x}{\sqrt{(\cosh^2 x - 9)}}$

Let $u = \cosh x$, $\frac{du}{dx} = \sinh x$

$\int \frac{\sinh 2x}{\sqrt{(\cosh^4 x - 9\cosh^2 x)}}\,dx = \int \frac{2}{\sqrt{(u^2 - 9)}}\,du$

$= 2\cosh^{-1}\left(\frac{u}{3}\right) + c$

$= 2\cosh^{-1}\left(\frac{\cosh x}{3}\right) + c$

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6. The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{pmatrix} 12 & 30 & 8 \\ 18 & 25 & 20 \\ 19 & 50 & 16 \end{pmatrix}.$$

(a) Given that $\det \mathbf{M} = -1040$, give a geometrical interpretation of the solution to the following equation. [2]

$$\begin{pmatrix} 12 & 30 & 8 \\ 18 & 25 & 20 \\ 19 & 50 & 16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2668 \\ 3402 \\ 4581 \end{pmatrix}$$

det M = −1040 ≠ 0, so M^{-1} exists and there is a unique solution:

the three planes intersect at a single point.

(b) Three hotels A, B, C each have different types of room available to book: single, double and family rooms. For each type of room, the price per night is the same in each of the three hotels.

The table below gives, for each hotel, details of the number of each type of room and the total revenue per night when the hotel is full.

	Types of room			
Hotel	**Single**	**Double**	**Family**	**Total revenue**
A	12	30	8	£2,668
B	18	25	20	£3,402
C	19	50	16	£4,581

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Find the price per night of each type of room. [6]

Let £x, £y, £z be the prices of a single, double and family room.

$12x + 30y + 8z = 2668$ (R1)

$18x + 25y + 20z = 3402$ (R2)

$19x + 50y + 16z = 4581$ (R3)

$2R2 - 3R1$: $-40y + 16z = -1200$ (R4)

$12R3 - 19R1$: $30y + 40z = 4280$ (R5)

$4R5 + 3R4$: $208z = 13520$

Echelon form: $12x + 30y + 8z = 2668$

$-40y + 16z = -1200$

$208z = 13520$

$z = 65$

R4: $-40y + 16(65) = -1200 \Rightarrow -40y = -2240 \Rightarrow y = 56$

R1: $12x + 30(56) + 8(65) = 2668 \Rightarrow 12x = 468 \Rightarrow x = 39$

Check R3: $19(39) + 50(56) + 16(65) = 741 + 2800 + 1040 = 4581$ ✓

Single £39, Double £56, Family £65

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7. (a) A curve C is defined by the equation $y = \dfrac{1}{\sqrt{16 - 6x - x^2}}$ for $-3 \leqslant x \leqslant 1$.

(i) Find the mean value of $y = \dfrac{1}{\sqrt{16 - 6x - x^2}}$ between $x = -3$ and $x = 1$. [4]

$16 - 6x - x^2 = 25 - (x + 3)^2$

Mean value $= \frac{1}{1 - (-3)} \int_{-3}^{1} \frac{1}{\sqrt{(16 - 6x - x^2)}}\, dx$

$= \frac{1}{4} \int_{-3}^{1} \frac{1}{\sqrt{(25 - (x + 3)^2)}}\, dx$

$= \frac{1}{4} \left[\sin^{-1}\left(\frac{x + 3}{5}\right)\right]_{-3}^{1}$

$= \frac{1}{4} \left(\sin^{-1}\frac{4}{5} - \sin^{-1}0\right)$

$= \frac{1}{4} \times 0.92729\ldots$

$= 0.232$ (3 s.f.)

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(ii) The region R is bounded by the curve C, the x-axis and the lines $x = -3$ and $x = 1$. Find the volume of the solid generated when R is rotated through four right-angles about the x-axis. [5]

$V = \pi\int_{-3}^{1} y^2\,dx = \pi\int_{-3}^{1} \frac{1}{16 - 6x - x^2}\,dx$

$= \pi\int_{-3}^{1} \frac{1}{25 - (x+3)^2}\,dx$

$= \pi\left[\frac{1}{2\times 5}\ln\left|\frac{5+(x+3)}{5-(x+3)}\right|\right]_{-3}^{1}$

$= \pi\left(\frac{1}{10}\ln\frac{9}{1} - \frac{1}{10}\ln\frac{5}{5}\right)$

$= \frac{\pi}{10}\ln 9$

$= 0.690$ (3 s.f.)

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(b) Evaluate the improper integral

$$\int_1^{\infty} \frac{-8e^{-2x}}{4e^{-2x}-5}\,dx,$$

giving your answer correct to three decimal places. [3]

$\int_1^{\infty} \frac{-8e^{-2x}}{4e^{-2x}-5}\,dx = \lim_{a\to\infty} \int_1^{a} \frac{-8e^{-2x}}{4e^{-2x}-5}\,dx$

$= \lim_{a\to\infty} [\ln|4e^{-2x}-5|]_1^a$

$= \lim_{a\to\infty} (\ln|4e^{-2a}-5| - \ln|4e^{-2}-5|)$

As $a \to \infty$, $e^{-2a} \to 0$:

$= \ln 5 - \ln(5-4e^{-2})$

$= 1.6094\ldots - 1.4948\ldots = 0.11458\ldots$

$= 0.115$ (3 d.p.)

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8. (a) By writing $y = \sinh^{-1}(4x+3)$ as $\sinh y = 4x+3$, show that $\dfrac{dy}{dx} = \dfrac{4}{\sqrt{16x^2+24x+10}}$. [5]

$\sinh y = 4x + 3$

$\cosh y \dfrac{dy}{dx} = 4$

$\dfrac{dy}{dx} = \dfrac{4}{\cosh y} = \dfrac{4}{\pm\sqrt{(1+\sinh^2 y)}}$

$= \dfrac{4}{\pm\sqrt{(1+(4x+3)^2)}} = \dfrac{4}{\pm\sqrt{(16x^2+24x+10)}}$

$\cosh y > 0$ for all y (the graph of $\sinh^{-1}x$ is always increasing),

so the positive root is taken:

$\dfrac{dy}{dx} = \dfrac{4}{\sqrt{(16x^2+24x+10)}}$ as required

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(b) Show that the graph of $e^{-3x}y = \sinh 2x$ has only one stationary point. [6]

$y = \frac{\sinh 2x}{e^{-3x}} = e^{3x}\sinh 2x = e^{3x} \times \frac{e^{2x} - e^{-2x}}{2}$

$y = \frac{e^{5x}}{2} - \frac{e^{x}}{2}$

$\frac{dy}{dx} = \frac{5e^{5x}}{2} - \frac{e^{x}}{2}$

Stationary point: $\frac{dy}{dx} = 0$

$\frac{5e^{5x}}{2} - \frac{e^{x}}{2} = 0 \Rightarrow \frac{e^{x}}{2}(5e^{4x} - 1) = 0$

$e^{x} > 0$ for all x, so $\frac{e^{x}}{2} = 0$ has no solution.

$5e^{4x} - 1 = 0 \Rightarrow e^{4x} = \frac{1}{5} \Rightarrow x = \frac{1}{4}\ln\frac{1}{5}$ $(= -0.402...)$

This is the only solution, so there is only one stationary point.

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9. Find the general solution of the equation

$$\sin 6\theta + 2\cos^2\theta = 3\cos 2\theta - \sin 2\theta + 1.$$

[9]

$\sin 6\theta + 2\cos^2\theta = 3\cos 2\theta - \sin 2\theta + 1$

$\sin 6\theta + \sin 2\theta = 3\cos 2\theta - 2\cos^2\theta + 1$

LHS: $\sin 6\theta + \sin 2\theta = 2\sin\left(\frac{6\theta + 2\theta}{2}\right)\cos\left(\frac{6\theta - 2\theta}{2}\right) = 2\sin 4\theta \cos 2\theta$

RHS: $3\cos 2\theta - (2\cos^2\theta - 1) = 3\cos 2\theta - \cos 2\theta = 2\cos 2\theta$

$2\sin 4\theta \cos 2\theta = 2\cos 2\theta$

$2\sin 4\theta \cos 2\theta - 2\cos 2\theta = 0$

$2\cos 2\theta(\sin 4\theta - 1) = 0$

$\cos 2\theta = 0$: $2\theta = \frac{\pi}{2} + n\pi \Rightarrow \theta = \frac{\pi}{4} + \frac{n\pi}{2}$

$\sin 4\theta = 1$: $4\theta = \frac{\pi}{2} + 2n\pi \Rightarrow \theta = \frac{\pi}{8} + \frac{n\pi}{2}$

General solution: $\theta = \frac{\pi}{4} + \frac{n\pi}{2}$ or $\theta = \frac{\pi}{8} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

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10. The following simultaneous equations are to be solved.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4x + 2y + 6\mathrm{e}^{3t}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 6x + 8y + 15\mathrm{e}^{3t}$$

(a) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 20x = 0$. [5]

From the first equation: $2y = \frac{dx}{dt} - 4x - 6e^{3t}$

$y = \frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}$

$\frac{dy}{dt} = \frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t}$

Substitute into $\frac{dy}{dt} = 6x + 8y + 15e^{3t}$:

$\frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t} = 6x + 8(\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}) + 15e^{3t}$

$= 6x + 4\frac{dx}{dt} - 16x - 24e^{3t} + 15e^{3t}$

$\frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t} = 4\frac{dx}{dt} - 10x - 9e^{3t}$

$\frac{1}{2}\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$

$\times 2$: $\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$ as required

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(b) Given that $\frac{dx}{dt} = 9$ and $\frac{d^2x}{dt^2} = 10$ when $t = 0$, find the particular solution for x in terms of t. [7]

Auxiliary equation: $m^2 - 12m + 20 = 0$

$(m - 2)(m - 10) = 0 \Rightarrow m = 2$ or $m = 10$

$x = Ae^{2t} + Be^{10t}$

$\frac{dx}{dt} = 2Ae^{2t} + 10Be^{10t}$

$\frac{d^2x}{dt^2} = 4Ae^{2t} + 100Be^{10t}$

$t = 0$: $2A + 10B = 9$ (1)

$4A + 100B = 10$ (2)

(2) $- 2 \times$ (1): $80B = -8 \Rightarrow B = -\frac{1}{10}$

$2A - 1 = 9 \Rightarrow A = 5$

$x = 5e^{2t} - \frac{1}{10}e^{10t}$

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