Day 98 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Segments: exact area and perimeter (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB of a circle, centre O and radius 2 cm, subtends an angle of $\frac{2\pi}{3}$ at O.

Find the exact area and the exact perimeter of the minor segment cut off by AB.

Sector OAB: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2^2 \times \frac{2\pi}{3}$; triangle OAB: $\frac{1}{2}r^2\sin\theta = \sqrt{3}$

Area = $\frac{4}{3}\pi - \sqrt{3}$

Chord AB = $2r\sin(\theta/2) = 2\sqrt{3}$

Arc AB = $r\theta = \frac{4}{3}\pi$

Perimeter = arc + chord

Answer: Area = $\frac{4}{3}\pi - \sqrt{3}$ cm^2, perimeter = $\frac{4}{3}\pi + 2\sqrt{3}$ cm

2 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = 1$ and $y = -1$, so $p = -1$ and $q = -1$.

$y = \frac{a}{x - 1} - 1$.

P(2, 5): $5 = \frac{a}{1} - 1$, so $a = 6$.

Answer: $a = 6$, $p = -1$, $q = -1$

3 Time above a given height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards from ground level with speed 39.2 m s^{-1}.

Find the length of time for which the ball is more than 58.8 m above the ground, and the greatest height it reaches. Take g = 9.8 m s^{-2}.

At height 58.8 m: $v^2 = 39.2^2 - 2 \times 9.8 \times 58.8 = 384.16$, so $v = 19.6$

Time from this height to the top: $t = \frac{19.6}{9.8} = 2$ s

By symmetry the time above this height is $2 \times 2 = 4$ s

Greatest height = $\frac{39.2^2}{(2 \times 9.8)} = 78.4$

Answer: 4 s, 78.4 m

4 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic $y = f(x)$, which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at $x = -2$ and touches at $x = -4$, so $f(x) = k(x + 2)(x + 4)^2$.

At (0, 32): $32 = k(2)(4)^2 = 32k$, so $k = 1$.

$f(x) = (x + 2)(x + 4)^2$, then expand.

Answer: $f(x) = x^3 + 10x^2 + 32x + 32$

5 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + 3y^2 - 2y$.

Find dy/dx and d^2y/dx^2 when $y = -1$.

$dx/dy = 3y^2 + 6y - 2 = -5$ when $y = -1$, so $dy/dx = \frac{1}{dx/dy} = -\frac{1}{5}$

$d^2x/dy^2 = 0$ when $y = -1$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{0}{(-5)^3}$

Answer: $dy/dx = -\frac{1}{5}$, $d^2y/dx^2 = 0$

6 Triangular pdf from a diagram (CIE)

The diagram shows the pdf of X.
Find (a) k, (b) E(X), (c) the median of X (3 s.f.).

(a) Area of triangle = ½ × 8 × k = 1, so k = $\frac{1}{4}$

(b) The mean of a triangular distribution is $\frac{0 + 1 + 8}{3} = 3$

F(1) = $\frac{1}{8} = \frac{1}{8} < \frac{1}{2}$, so the median is in the right part

Right-hand area $\frac{(8 - x)^2}{56} = \frac{1}{2}$, so $x = 8 - \sqrt{28} = 2.708$

Answer: (a) k = $\frac{1}{4}$ (b) E(X) = 3 (c) 2.71

7 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = t^3 + 9t$.
Find the coordinates of the points on the curve where the gradient is 3.

$dx/dt = 4t$, $dy/dt = 3t^2 + 9$, so $dy/dx = \frac{3t^2 + 9}{4t}$

Set equal to 3: $3t^2 - 12t + 9 = 0$
$t = 1$ or $t = 3$
Answer: (2, 10) and (18, 54)

8 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = 2f(x - 2) - 3$.

Inside f: replace x by $x - 2$, so x-coordinates become $(x + 2) \div 1$.
Outside: y-coordinates become $2 \times y - 3$.
A: $(-2, 5)$ B: $(0, -1)$
C: (4, 7) D: (6, 5)
Answer: A′(−2, 5), B′(0, −1), C′(4, 7), D′(6, 5)

9 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 17 and upper quartile 34. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 8 observations have mean 31. The largest is 67, an outlier, and it is removed. Find the new mean.

IQR = 34 − 17 = 17, so 1.5 × IQR = 25.5
Lower limit = 17 − 25.5 = −8.5, upper limit = 34 + 25.5 = 59.5
$\Sigma x = 8 \times 31 = 248$, so without 67: $\Sigma x = 181$

New mean = $\frac{181}{7}$

Answer: (a) −8.5 to 59.5 (b) 25.86

10 Closest approach of two ships (AQA, Edexcel, OCR, MEI, Eduqas)

Two ships A and B move with constant velocities. At time t = 0 hours, A is at $(-5\mathbf{i} - \mathbf{j})$ km relative to a fixed origin and B is at $(-12\mathbf{i} - 2\mathbf{j})$ km. A has velocity $(-2\mathbf{i} + 3\mathbf{j})$ km h^{-1} and B has velocity $(2\mathbf{i})$ km h^{-1}.
Find (a) the time at which the ships are closest together, (b) the least distance between them.

Position of B relative to A: $\mathbf{d} = (-7\mathbf{i} - \mathbf{j}) + t(4\mathbf{i} - 3\mathbf{j})$
$|\mathbf{d}|^2 = (4t - 7)^2 + (-3t - 1)^2 = 25t^2 - 50t + 50$
Least when $\frac{d}{dt}|\mathbf{d}|^2 = 0$: $t = -\frac{-25}{25} = 1$
$\mathbf{d} = (-7\mathbf{i} - \mathbf{j}) + 1(4\mathbf{i} - 3\mathbf{j}) = (-3\mathbf{i} - 4\mathbf{j})$, so least distance = 5
Answer: (a) t = 1 h (b) 5 km