Day 95 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^{\frac{2}{3}} - 4x^{\frac{1}{3}} + 3 = 0$.

Let $u = x^{\frac{1}{3}}$: $u^2 - 4u + 3 = 0$.
$(u - 3)(u - 1) = 0$, so $u = 3$ or $u = 1$.
Cube: $x = 27$ or $x = 1$.
Answer: $x = 1$ or $x = 27$

2 Time above a given height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards from ground level. It is more than 39.2 m above the ground for 2 s.
Find the speed of projection and the greatest height reached. Take $g = 9.8$ m s^{-2}.

By symmetry, the ball takes 1 s to rise from height 39.2 m to its highest point
Speed at height 39.2 m: $v = 9.8 \times 1 = 9.8$ m s^{-1}
Speed of projection: $u^2 = v^2 + 2gh = 96.04 + 2 \times 9.8 \times 39.2 = 864.36$
$u = 29.4$. Greatest height $= \frac{u^2}{(2g)} = \frac{864.36}{19.6} = 44.1$
Answer: 29.4 m s^{-1}, 44.1 m

3 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (1, 3) and (3, 81).
Find the values of a and n.

$3 = a \times 1^n$ and $81 = a \times 3^n$
Divide: $3^n = 27$ so $n = 3$
$a = 3 \div 1^3 = 3$
Answer: $a = 3$, $n = 3$

4 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The sum of the first n terms of a sequence is
$S_n = 4n^2 + n$.
(a) Find an expression for u_n.
(b) Find the least n for which $u_n > 332$.

$u_n = S_n - S_{n-1} = 4n^2 + n - [4(n - 1)^2 + (n - 1)]$
$= 8n - 3$.
$u_1 = S_1 = 5$ also fits, so $u_n = 8n - 3$ for all n.
$8n - 3 > 332 \Rightarrow n > \frac{335}{8}$, so $n = 42$.
Answer: (a) $u_n = 8n - 3$ (b) $n = 42$

5 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 6 strips to estimate
$\int_1^3 \frac{1}{1 + x^2}\,dx$, to 4 s.f.
Is it an overestimate or an underestimate? Explain.

$h = \frac{1}{3}$; ordinates at $x = 1, \frac{4}{3}, \frac{5}{3}, 2, \ldots, 3$
Sum $= \frac{h}{2}[y_0 + y_6 + 2(y_1 + \ldots + y_5)] = 0.4677$
$f''(x) > 0$ on the interval, so the curve is convex and the trapezia lie above the curve
Answer: 0.4677; overestimate

6 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 11 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 60.8 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 11$, and $T_2 = \frac{4}{3} T_1$

$T_1(\frac{3}{5} + \frac{4}{3} \times \frac{4}{5}) = 11$, so $T_1 = 6.6$ and $T_2 = 8.8$

The tension in BP is the larger: it is 8.8 when W = 11

Tensions are proportional to W, so $W_{max} = 11 \times \frac{60.8}{8.8} = 76$

Answer: (a) AP: 6.6 N, BP: 8.8 N (b) 76 N

7 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ax^n$. The graph of lg y against lg x is the straight line shown.
Find a and n, giving a to 3 s.f.

lg y = lg a + n lg x. The intercept is lg a = 1, so $a = 10^1 = 10.00$

n = gradient $= \frac{7 - 1}{3} = 2$

Answer: a = 10.0, n = 2

8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{1}{35 + 20\sin\theta + 21\cos\theta}$

(a) Write $20\sin\theta + 21\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.
(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{20^2 + 21^2} = 29$, $\tan\alpha = \frac{21}{20}$, $\alpha = 46.4°$

Denominator $= 35 + 29\sin(\theta + 46.4°)$, which lies between 6 and 64

Greatest $f(\theta) = \frac{1}{6} = \frac{1}{6}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{64} = \frac{1}{64}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 46.4° = 270°$, so $\theta = 223.6°$

Answer: (a) $29\sin(\theta + 46.4°)$ (b) greatest $\frac{1}{6}$, least $\frac{1}{64}$, $\theta = 223.6°$

9 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When a polynomial f(x) is divided by $(x - 3)$ the remainder is -9; when divided by $(x + 2)$ the remainder is -4. Find the remainder when f(x) is divided by $(x - 3)(x + 2)$.

$f(x) = (x - 3)(x + 2)Q(x) + px + q$.
$f(3) = -9$: $3p + q = -9$.
$f(-2) = -4$: $-2p + q = -4$.
Subtracting: $5p = -5$, so $p = -1$, $q = -6$.
Answer: $-x - 6$

10 Normal approximation to a Poisson (CIE)

The number of customers arriving at a shop in one hour is Po(40).
Use a suitable approximation to find P(X ≥ 58), to 3 significant figures.

$\lambda = 40$ is large (greater than 15), so X is approximately N(40, 40)
Standard deviation = $\sqrt{40} = 6.325$
Continuity correction:
$P(X \geq 58) \approx P(Y > 57.5) = P(Z > 2.767) = 0.00283$
Answer: 0.00283