Day 94 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = 2t$, $y = 3t^2$. The line $y = 3x + 9$
meets C at A and B. Find the exact length AB.

Substitute: $3t^2 = 6t + 9$.
$3t^2 - 6t - 9 = 0 \Rightarrow t = 3$ or $t = -1$.
A(6, 27), B(−2, 3).
$AB^2 = 8^2 + 24^2 = 640$.
Answer: AB = $8\sqrt{10}$

2 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 4 strips to estimate
$\int_1^2 x^2 e^{-x}\,dx$, to 4 s.f.
Is it an overestimate or an underestimate? Explain.

$h = \frac{1}{4}$; ordinates at $x = 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \ldots, 2$

Sum = $\frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)] = 0.4841$

$f''(x) < 0$ on the interval, so the curve is concave and the trapezia lie below the curve
Answer: 0.4841; underestimate

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A javelin is projected from a point O on horizontal ground with speed 24.5 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 14.7, vertical 19.6 (as $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$)

Time of flight: $0 = 19.6\,t - 4.9t^2$, so $t = \frac{19.6}{4.9} = 4$

Range = $14.7 \times 4 = 58.8$

Greatest height: $0 = 19.6^2 - 2 \times 9.8 \times H$, so $H = \frac{384.16}{19.6} = 19.6$

Answer: (a) 4 s (b) 58.8 m (c) 19.6 m

4 Binomial: finding p from a ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(7, p). It is given that $P(X = 2) = \frac{9}{7} \times P(X = 1)$.

(a) Find the value of p.
(b) Find P(X = 0), correct to 3 significant figures.

$\frac{42}{2}\,p^2(1-p)^5 = \frac{9}{7} \times 7\,p(1-p)^6$

Divide by $p(1-p)^5$: $3\,p = \frac{9}{7}(1-p)$

$(3 + \frac{9}{7})p = \frac{9}{7}$

$p = \frac{3}{10}$

$P(X = 0) = (1-p)^7 = (\frac{7}{10})^7 = 0.0824$

Answer: (a) $p = \frac{3}{10}$ (b) 0.0824

5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-3x - 6| > |x + 6|$.

Square both sides: $(-3x - 6)^2 > (x + 6)^2$.
$(-4x - 12)(-2x) > 0$.
Critical values $x = -3$ and $x = 0$.
The quadratic has positive coefficient of x^2, so choose the outer region.
Answer: $x < -3$ or $x > 0$

6 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{3}{26 + 24 \sin\theta + 7 \cos\theta}$

(a) Write $24 \sin\theta + 7 \cos\theta$ as $R \sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.
(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{24^2 + 7^2} = 25$, $\tan\alpha = \frac{7}{24}$, $\alpha = 16.3°$

Denominator $= 26 + 25 \sin(\theta + 16.3°)$, which lies between 1 and 51

Greatest $f(\theta) = \frac{3}{1} = 3$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{3}{51} = \frac{1}{17}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 16.3° = 270°$, so $\theta = 253.7°$

Answer: (a) $25 \sin(\theta + 16.3°)$ (b) greatest 3, least $\frac{1}{17}$, $\theta = 253.7°$

7 Triangular pdf from a diagram (CIE)

The diagram shows the pdf of X.
Find (a) k, (b) E(X), (c) the median of X (3 s.f.).

(a) Area of triangle $= \frac{1}{2} \times 10 \times k = 1$, so $k = \frac{1}{5}$

(b) The mean of a triangular distribution is $\frac{0 + 2 + 10}{3} = 4$

$F(2) = \frac{2}{10} = \frac{1}{5} < \frac{1}{2}$, so the median is in the right part

Right-hand area $\frac{(10 - x)^2}{80} = \frac{1}{2}$, so $x = 10 - \sqrt{40} = 3.675$

Answer: (a) $k = \frac{1}{5}$ (b) E(X) = 4 (c) 3.68

8 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 155 N rests on a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$ and $\mu = 0.5$.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

$\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. Normal reaction: $R = 155 \cos\alpha + H \sin\alpha$

About to slide down: $H \cos\alpha + \mu R = 155 \sin\alpha$, so $H(\frac{3}{5} + 0.5 \times \frac{4}{5}) = 155(\frac{4}{5} - 0.5 \times \frac{3}{5})$

$H_{min} = 77.5$

About to slide up: $H \cos\alpha = 155 \sin\alpha + \mu R$, so $H(\frac{3}{5} - 0.5 \times \frac{4}{5}) = 155(\frac{4}{5} + 0.5 \times \frac{3}{5})$

$H_{max} = 852.5$
Answer: $77.5 \leq H \leq 852.5$

9 Partial fractions (improper) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{2x^3 - 6x^2 - 11x}{(x-4)(x+2)}$ in the form $ax + b + \frac{C}{(x-4)} + \frac{D}{(x+2)}$.

Divide: the numerator has degree 3, the denominator degree 2.

$(2x^3 - 6x^2 - 11x) \div (x^2 - 2x - 8)$ gives quotient $2x - 2$.

Remainder: $x - 16$.

Put $x = 4$: remainder $= C(6)$ so $C = -2$. Put $x = -2$: $D = 3$.

Answer: $2x - 2 - \frac{2}{(x-4)} + \frac{3}{(x+2)}$

10 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + y^2 + 4y$.

Find dy/dx and d^2y/dx^2 when $y = 1$.

$dx/dy = 3y^2 + 2y + 4 = 9$ when $y = 1$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{9}$

$d^2x/dy^2 = 8$ when $y = 1$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{8}{9^3}$

Answer: $dy/dx = \frac{1}{9}$, $d^2y/dx^2 = -\frac{8}{729}$