Day 91 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_2 x - \log_2 y = 2$
$x + y = 40$

$\log_2(x/y) = 2$, so $x = 2^2 y = 4y$
$4y + y = 40$, so $5y = 40$
Answer: x = 32, y = 8

2 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = 3$ and $y = 2$, so $p = -3$ and $q = 2$.

$y = \frac{a}{x - 3} + 2$.

P(2, 5): $5 = \frac{a}{-1} + 2$, so $a = -3$.

Answer: a = −3, p = −3, q = 2

3 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-\mathbf{i})$ N, $(3\mathbf{i} + k\mathbf{j})$ N and $(3\mathbf{i} + 3\mathbf{j})$ N act on a particle of mass 3 kg. The resultant is parallel to $(\mathbf{i})$. Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(5)\mathbf{i} + (3 + k)\mathbf{j}$
Parallel to $(\mathbf{i})$: $\frac{3 + k}{5} = \frac{0}{1}$, so $k = -3$
$R = (5\mathbf{i})$ N, so $|R| = \sqrt{(5^2 + 0^2)} = 5.000$
$a = |R| \div 3$
Answer: (a) k = −3 (b) 1.67 m s^{-2}

4 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(51, \sigma^2)$ is shown; the shaded area is symmetrical. (a) Find σ. (b) Find P(X > 62).

The area 0.8 leaves 0.200 in total in the tails, 0.100 in each
$P(X < 43) = 0.100$ so $\frac{43 - 51}{\sigma} = -1.2816$
$\sigma = \frac{8}{1.2816} = 6.242$
$P(X > 62) = P(Z > 1.762) = 0.0390$
Answer: (a) 6.24 (b) 0.0390

5 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 10 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially θ = 95, and after 4 minutes θ = 61. Find A and k, then find when θ = 54 (3 s.f.).

t = 0: $10 + A = 95$ so $A = 85$
t = 4: $10 + 85e^{-4k} = 61$ so $e^{-4k} = \frac{3}{5}$
$k = \frac{1}{4} \ln \frac{5}{3} = 0.128$
$85\, e^{-kt} = 44$ so $t = \frac{1}{k} \ln \frac{85}{44}$
t = 5.16 minutes
Answer: A = 85, k = 0.128, t = 5.16 minutes

6 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x - y = -2$
$3x^2 - 2xy + 3y^2 = 12$

From the first equation $x = -2 + y$.
Substitute into the second equation and collect terms:
this gives $4y^2 - 8y = 0$.
$y = 0$ or $y = 2$, giving $x = -2$ or $x = 0$.
Answer: (x, y) = (−2, 0) and (0, 2)

7 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.
Find (a) the total number of runners, (b) an estimate of the number with $15 \leq t < 45$.

Frequency = frequency density × width. Bar 36 to 48: height 6 squares, width 12, frequency 72
So one square of frequency density = 1 runners per second
Class frequencies = 30, 12, 60, 42, 72
Total = 216
Estimate = 1 × [1(3) + 5(12) + 7(6) + 6(9)] = 159
Answer: (a) 216 (b) 159

8 Interception at constant speed (AQA, Edexcel, OCR, MEI, Eduqas)

At time t = 0 hours, ship A is at the point with position vector $(28\mathbf{i} + 28\mathbf{j})$ km relative to a port O, and moves with constant velocity $(-\mathbf{i} + \mathbf{j})$ km h^{-1}. Ship B leaves O at t = 0 with constant speed 10 km h^{-1} and intercepts A.
Find (a) the time at which B intercepts A, (b) the velocity of B as a vector.

Position of A at time t: $(28\mathbf{i} + 28\mathbf{j}) + t(-\mathbf{i} + \mathbf{j})$. B at time t: t v_B (unknown velocity)
At interception B has travelled 10 t, so $|(28\mathbf{i} + 28\mathbf{j}) + t(-\mathbf{i} + \mathbf{j})| = 10\,t$
Squaring and rearranging: $-98t^2 + 1568 = 0$, which gives t = 4 or a negative value
t = 4 (the other root is negative since B is faster than A)
A is then at $(28\mathbf{i} + 28\mathbf{j}) + 4(-\mathbf{i} + \mathbf{j}) = (24\mathbf{i} + 32\mathbf{j})$, so $v_B = \frac{1}{4}(24\mathbf{i} + 32\mathbf{j}) = (6\mathbf{i} + 8\mathbf{j})$
Answer: (a) 4 h (b) $(6\mathbf{i} + 8\mathbf{j})$ km h^{-1}

9 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^3 + xy^2 = 80$.
Find the equation of the tangent to C at the point (4, 2),
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x
At (4, 2): $dy/dx = -\frac{13}{4}$
$y - 2 = (-\frac{13}{4})(x - 4)$, then clear fractions
Answer: 13x + 4y − 60 = 0

10 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 336
inclusive that are divisible by 4 or by 6, but not by both.

Multiples of 4: 84 terms (4 to 336): sum = 14280.
Multiples of 6: 56 terms: sum = 9576.
Multiples of both, i.e. of 12: 28 terms: sum = 4872.
Required sum = 14280 + 9576 − 2 × 4872.
Answer: 14112