Day 88 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Compound change with logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A van costs £12000 and loses 10% of its value each year.
Find the least whole number of years after which its value is below £3000.

$12000 \times 0.90^n < 3000$, so $0.90^n < \frac{1}{4}$

$n > \frac{\ln \frac{1}{4}}{\ln 0.90} = 13.16$ (ln 0.90 is negative, so the inequality reverses)
$n = 14$
Answer: 14

2 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where
$v = t^2 - 8t + 12$ for $0 \le t \le 8$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 8 seconds.

$v = (t - 2)(t - 6)$, so $v = 0$ when $t = 2$ and $t = 6$
$v > 0$ for $0 < t < 2$, $v < 0$ for $2 < t < 6$, $v > 0$ for $t > 6$: direction changes at each
$s = \int v\,dt = \frac{1}{3}t^3 - 4t^2 + 12t$
Distances: 0 to 2: $\frac{32}{3}$; 2 to 6: $\frac{32}{3}$; 6 to 8: $\frac{32}{3}$
Total = $\frac{32}{3} + \frac{32}{3} + \frac{32}{3}$
Answer: (a) t = 2, t = 6 (b) 32 m

3 Logarithmic equation, reject a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\log_5(x + 3) + \log_5(x - 1) = 1$,
justifying clearly any root that you reject.

$(x + 3)(x - 1) = 5^1 = 5$
$x^2 + 2x - 8 = 0$
$(x - 2)(x + 4) = 0$, so $x = 2$ or $x = -4$
$x = -4$ makes $x + 3 = -1 < 0$ and log of a negative number is undefined
Answer: x = 2

4 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{1}{1} = 1$.
Gradient of $l_2 = -1$. Equation: $y - 1 = -1x$.
$y = 0$: $x = 1$, so C(1, 0).
AB = $\sqrt{2}$, BC = $\sqrt{2}$: area = ½ × AB × BC.
Answer: C(1, 0); area 1

5 Identity then equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Show that $\frac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$.

(b) Hence solve $\frac{\sin 2\theta}{1 + \cos 2\theta} = -3$ for $0 \le \theta < 360°$ (1 d.p.).

$\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2 \sin\theta \cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$
$\tan\theta = -3$, so $\theta = -71.6°$ (principal value) and add 180°
Answer: θ = 108.4°, 288.4°

6 Force up a rough slope: find weight and friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight W N is on a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{4}{5}$. A force P N acts up a line of greatest slope; the particle stays at rest for $31.8 \leq P \leq 64.2$.

Find (a) W, (b) the coefficient of friction, (c) the acceleration when P = 128.4. Take g = 9.8 m s^{-2}.

Limiting friction acts down the plane at P = 64.2 and up the plane at P = 31.8

$P_{max} = W\sin\alpha + F$ and $P_{min} = W\sin\alpha - F$, where $F = \mu W\cos\alpha$

Adding: $2W\sin\alpha = 96$, so $W = \frac{96}{2 \times \frac{4}{5}} = 60$

Subtracting: $2F = 32.4$, so $F = 16.2 = \mu \times 60 \times \frac{3}{5}$, giving $\mu = 0.45$

P = 128.4: net force up the plane = 128.4 − 64.2 = 64.2, and mass = $\frac{60}{9.8}$

Answer: (a) 60 N (b) 0.45 (c) 10.5 m s^{-2}

7 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.

Find (a) the total number of runners, (b) an estimate of the number with $9 \leq t < 35$.

Frequency = frequency density × width. Bar 32 to 36: height 1 squares, width 4, frequency 4

So one square of frequency density = 1 runners per second

Class frequencies = 4, 72, 8, 48, 4

Total = 136

Estimate = 1 × [6(7) + 2(4) + 4(12) + 1(3)] = 101

Answer: (a) 136 (b) 101

8 Transformations: new equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + 4x + 4$ is translated by $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$

and then a reflection in the y-axis.

Find the equation of the resulting curve in the form $y = ax^2 + bx + c$.

Translation by $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$: $y = f(x + 2) - 3 = x^2 + 8x + 13$.

Then a reflection in the y-axis: $x \to -x$.

Expand and simplify to give the quadratic.

Answer: $y = x^2 - 8x + 13$

9 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.

Given the second is a W, P(first was a W) = $\frac{1}{3}$. Find x.

P(W then W) = 0.6x, P(second W) = 0.6x + 0.4(1 − x)

P(first W | second W) = $\frac{0.6x}{0.6x + 0.4(1 - x)} = \frac{1}{3}$

$0.6x = \frac{1}{3}(0.6x + 0.4 - 0.4x)$

Solve the linear equation for x: $x = \frac{1}{4}$

Answer: $x = \frac{1}{4}$

10 Binomial expansion with partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-5x - 3}{(1 + 3x)(1 - x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) in ascending powers of x up to x^3, stating the range of validity.

(a) $-5x - 3 \equiv A(1 - x) + B(1 + 3x)$: $A = -1$, $B = -2$.

$f(x) = \frac{-1}{1 + 3x} - \frac{2}{1 - x}$.

(b) $\frac{1}{1 + cx} = 1 - cx + c^2x^2 - c^3x^3 + \ldots$ (use $c = 3$, then $c = -1$).

Coefficients of x^j: $-1(-3)^j - 2(1)^j$.

Valid for $|x| < \frac{1}{3}$ (both expansions).

Answer: (a) $A = -1$, $B = -2$ (b) $-3 + x - 11x^2 + 25x^3 + \ldots$; $|x| < \frac{1}{3}$