Day 85 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where $\theta < 2\pi$.
Its perimeter is 26 cm and its area is 36 cm^2.
Find the two possible pairs of values of r and θ.

Arc = $r\theta$, so $2r + r\theta = 26$ and $\frac{1}{2}r^2\theta = 36$

$r\theta = 26 - 2r$, so $\frac{1}{2}r(26 - 2r) = 36$

$r^2 - 13r + 36 = 0$, so $(r - 4)(r - 9) = 0$

r = 4: $r\theta = 18$, $\theta = \frac{9}{2}$ r = 9: $r\theta = 8$, $\theta = \frac{8}{9}$

Answer: r = 4, $\theta = \frac{9}{2}$ or r = 9, $\theta = \frac{8}{9}$

2 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 12 kg is at rest on rough horizontal ground ($\mu = 0.25$). A horizontal force of 37.8 N acts on it for 7 s and is then removed.
Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take g = 9.8 $m\ s^{-2}$.

Friction = $\mu mg = 0.25 \times 12 \times 9.8 = 29.4$ N
While pushed: $37.8 - 29.4 = 12a$, so $a = 0.7$
$v = 0.7 \times 7 = 4.9$; $s_1 = \frac{1}{2} \times 0.7 \times 7^2 = 17.15$

After release: deceleration = $\mu g = 2.45$, so $s_2 = \frac{4.9^2}{2 \times 2.45} = 4.9$

Total distance = 17.15 + 4.9

Answer: (a) 4.9 $m\ s^{-1}$ (b) 22.05 m

3 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = -2$ and $y = 1$, so $p = 2$ and $q = 1$.

$y = \frac{a}{x + 2} + 1$.

P(−1, −3): $-3 = \frac{a}{1} + 1$, so $a = -4$.

Answer: a = −4, p = 2, q = 1

4 Speed decaying exponentially (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. The velocity of P is v $m\ s^{-1}$ and its acceleration is $-0.05v^2$ $m\ s^{-2}$. At t = 0, v = 8.
Find (a) the time taken for the speed of P to halve, (b) the distance P travels in this time, to 3 s.f.

$\frac{dv}{dt} = -0.05v^2$: $\int \frac{dv}{v^2} = -0.05\int dt$, so $\frac{1}{v} = 0.05t + c$, and $\frac{1}{v} = 0.05t + \frac{1}{8}$

v = 4: $\frac{1}{v} = 0.25$, so $0.05t = 0.25 - 0.125$, giving $t = 2.5$

$s = \int \frac{1}{(0.05t + 1/8)}\,dt = \frac{1}{0.05}\ln(1 + 0.05 \times 8t)$

At t = 2.5: $s = \frac{1}{0.05}\ln 2$

Answer: (a) 2.5 s (b) 13.9 m

5 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x\sqrt{75 - x}$, for $0 < x < 75$, has one stationary point. Find its exact coordinates and determine its nature.

$\mathrm{d}y/\mathrm{d}x = \dfrac{(150 - 3x)}{2\sqrt{75 - x}}$

$\mathrm{d}y/\mathrm{d}x = 0$: $150 - 3x = 0$, so $x = 50$

$y = 50 \times 5$

Gradient changes from positive to negative: a maximum

Answer: (50, 250), maximum

6 Coded regression line (AQA, Edexcel, OCR, MEI)

Data pairs (x, y) are coded by $u = x - 2$ and $v = \frac{y}{2}$. The regression line of v on u is $v = 19 + 0.8u$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Use it to estimate y when $x = 7$.

Substitute $u = x - 2$ and $v = y/2$: $\frac{y}{2} = 19 + 0.8(x - 2)$

$y = 2[19 + 0.8x - 1.6] = 2 \times 17.4 + 2 \times 0.8x$

$y = 34.8 + 1.6x$

At $x = 7$: $y = 34.8 + 1.6 \times 7 = 46$

Answer: (a) $y = 34.8 + 1.6x$ (b) 46

7 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = -2$ and

$u_{n+1} = \dfrac{3u_n + 1}{2u_n - 3}$.

Show that the sequence is periodic and find $\sum_{r=1}^{80} u_r$.

$u_1 = -2$, $u_2 = \frac{5}{7}$, $u_3 = -2$.

$u_3 = u_1$, so the sequence repeats with period 2.

One period sums to $-\frac{9}{7}$.

$80 = 2 \times 40 + 0$: 40 full periods plus the first 0 terms.

Answer: $-\frac{360}{7}$

8 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cos 2\theta - 14\cos\theta + 8 = 0$ for $0 \leq \theta < 360°$.

Use $\cos 2\theta \equiv 2\cos^2\theta - 1$: $4\cos^2\theta - 14\cos\theta + 6 = 0$

$2(2\cos\theta - 1)(\cos\theta - 3) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ (impossible, $|\cos\theta| \leq 1$)

Using the CAST diagram for $0 \leq \theta < 360°$

Answer: $\theta = 60°, 300°$

9 Poisson hypothesis test (CIE)

Faults in a cable occur at random at a mean rate of 4 per week. After a modification, 19 faults are found in 2 weeks. Test at the 5% level whether the rate of faults has increased.

H_0: $\lambda = 4$ per week, H_1: $\lambda > 4$ per week

Over 2 weeks: $X \sim Po(8)$

$P(X \geq 19) = 1 - P(X \leq 18) = 0.000650$

$0.000650 < 5\%$, so reject H_0

There is sufficient evidence that the rate has increased

Answer: Reject H_0 (p-value 0.000650)

10 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(9 + 3x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(9 + 3x)^{\frac{3}{2}} = 9^{\frac{3}{2}}(1 + \frac{1}{3}x)^{\frac{3}{2}} = 27(1 + u)^{\frac{3}{2}}$, $u = \frac{1}{3}x$.

$(1 + u)^{\frac{3}{2}} = 1 + \frac{3}{2}u + \frac{3}{8}u^2 + (-\frac{1}{16})u^3 + ...$

Substitute u and multiply by 27.

Valid when $|u| < 1$, i.e. $|x| < 3$.

Answer: $27 + \frac{27}{2}x + \frac{9}{8}x^2 - \frac{1}{16}x^3 + ...$; $|x| < 3$