Day 84 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population of bacteria is modelled by $P = P_0e^{kt}$, t in hours.
Initially P = 250 and after 6 hours P = 625.
Find k, and the time for the population to reach 1250, to 3 s.f.

$P_0 = 250$. $250\,e^{6k} = 625$ so $e^{6k} = \frac{5}{2}$

$k = \frac{1}{6}\ln\frac{5}{2} = 0.153$

$250\,e^{kt} = 1250$ so $e^{kt} = 5$, $t = \frac{1}{k}\ln 5$

t = 10.5 hours
Answer: k = 0.153, t = 10.5 hours

2 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation

$y = \frac{a}{x+p} + q$. Find a, p and q.

Asymptotes x = 3 and y = 3, so $p = -3$ and $q = 3$.

$y = \frac{a}{x-3} + 3$.

P(4, 7): $7 = \frac{a}{1} + 3$, so a = 4.

Answer: a = 4, p = −3, q = 3

3 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that

$\frac{\cos 2\theta - \cos 6\theta}{\theta^2}$

approaches as $\theta \to 0$.

$\cos 2\theta \approx 1 - 2\theta^2$ and $\cos 6\theta \approx 1 - 18\theta^2$
Difference $\approx 16\theta^2$, so the expression ≈ 16
Answer: 16

4 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x - y = -3$
$2x^2 - y^2 = -9$

From the first equation $x = -3 + y$.
Substitute into the second equation and collect terms:
this gives $y^2 - 12y + 27 = 0$.
y = 3 or y = 9, giving x = 0 or x = 6.
Answer: (x, y) = (0, 3) and (6, 9)

5 Two particles over a pulley: height reached (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 9 kg and 1 kg hang on a light string over a smooth fixed pulley. The 9 kg particle is 3.75 m above the ground; the 1 kg one is on the ground.
Released from rest, find (a) the tension, (b) the greatest height reached by the 1 kg particle. Take $g = 9.8\ \text{m s}^{-2}$.

$9g - T = 9a$ and $T - 1g = 1a$, so $a = \frac{9-1}{9+1} \times 9.8 = 7.84$

$T = 9(9.8 - 7.84) = 17.64$

The 1 kg particle rises 3.75 m before the 9 kg particle lands, reaching $v^2 = 2 \times 7.84 \times 3.75 = 58.8$

The string goes slack, so it then moves freely under gravity: extra height $= \frac{v^2}{2g} = \frac{58.8}{19.6} = 3$

Greatest height = 3.75 + 3
Answer: (a) 17.64 N (b) 6.75 m

6 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 60 students in Year 10, 180 in Year 11 and 120 in the sixth form.
A stratified sample of 60 students is taken.
(a) Find the number sampled from each group.
(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $60 \times \frac{60}{360}$, $60 \times \frac{180}{360}$, $60 \times \frac{120}{360}$ = 10, 30, 20

P(same) = 10 × 9 + 30 × 29 + 20 × 19/60 × 59

$= \frac{1340}{3540} = 0.379$

Answer: (a) 10, 30, 20 (b) 0.379

7 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the stationary points of
$y = \cos 2x + 2\cos x$ for $0 < x < 2\pi$,
and determine their nature.

$dy/dx = -2\sin 2x - 2\sin x = -2\sin x(2\cos x + 1)$

dy/dx = 0: sin x = 0 or $\cos x = -\frac{1}{2}$

$x = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ in the range

Test the sign of d^2y/dx^2 at each

Answer: $(\frac{2\pi}{3}, -\frac{3}{2})$ minimum; $(\pi, -1)$ maximum; $(\frac{4\pi}{3}, -\frac{3}{2})$ minimum

8 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 9\sqrt{x} + 20 = 0$.

Let $u = \sqrt{x}$: $u^2 - 9u + 20 = 0$.
$(u - 4)(u - 5) = 0$, so $\sqrt{x} = 4$ or $\sqrt{x} = 5$.
Square: x = 16 or x = 25.
Answer: x = 16 or x = 25

9 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table with $\mu = 0.2$, and is joined by a light string over a smooth pulley (see diagram) to particle B (7 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.

Released from rest, the string is cut when B has moved 0.5 m. Find the further distance A slides before stopping.

A: $T - 0.2 \times 3 \times 9.8 = 3a$. B: $7 \times 9.8 \times \frac{4}{5} - T = 7a$

Adding: $9.8(7 \times \frac{4}{5} - 0.2 \times 3) = 10a$, so a = 4.9

$T = 3(4.9 + 0.2 \times 9.8) = 20.58$

When cut: $v^2 = 2 \times 4.9 \times 0.5 = 4.9$. Then A decelerates at $\mu g = 1.96$

Further distance = $\frac{4.9}{2 \times 1.96} = 1.25$

Answer: 1.25 m

10 Conditional probability with a normal variable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A battery's life, in hours, is $N(78, 12^2)$.
Given that it exceeds 66, find the probability that it exceeds 75.

Required = $P(X > 75 \mid X > 66) = \frac{P(X > 75)}{P(X > 66)}$

P(X > 75) = P(Z > −0.250) = 0.5987
P(X > 66) = P(Z > −1.000) = 0.8413

$= \frac{0.5987}{0.8413} = 0.712$

Answer: 0.712